Distributions in the Error Space: Goal-Directed Movements Described in Time and State-Space Representations

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Abstract— **Manipulation of error feedback has been of great interest to recent studies in motor control and rehabilitation. Typically, motor adaptation is shown as a change in performance with a single scalar metric for each trial, yet such an approach might overlook details about how error evolves through the movement. We believe that statistical distributions of movement error through the extent of the trajectory can reveal unique patterns of adaption and possibly reveal clues to how the motor system processes information about error. This paper describes different possible ordinate domains, focusing on representations in time and state-space, used to quantify reaching errors. We hypothesized that the domain with the lowest amount of variability would lead to a predictive model of reaching error with the highest accuracy. Here we showed that errors represented in a time domain demonstrate the least variance and allow for the highest predictive model of reaching errors. These predictive models will give rise to more specialized methods of robotic feedback and improve previous techniques of error augmentation.**

I. Introduction

Error feedback is critical for supporting motor adaptation in rehabilitation, sports, piloting, and skilled manual tasks [1, 2]. During goal-directed movements, predictions of sensory outcomes are compared with feedback errors in order to updated subsequent motor plans. Researchers have proposed that in order to compensate for a visual-motor rotation, an internal model is used to compare the desired goal with a motor plan. Error is calculated by comparing the goal trajectory and motor output, allowing for either online or trial-to-trial corrections [3-5]. This process involves many areas of the brain including the cerebellum, anterior cingulate cortex, and basal ganglia [6]. While error information in the human nervous system is clearly essential, little is known about their statistical properties or how they might be related to the learning process.

During motor adaptation, feedback errors are commonly classified into three categories: absolute error (the absolute deviation from a target), constant error (movement bias of the subject), and variable error (movement variability) [7]. To measure performance change of a subject across trials, many studies use a scalar metric to represent the absolute error from each trajectory (e.g. maximum perpendicular error or initial offset angle). While such metrics effectively indicate changes in absolute error or any subject-specific constant error, they do not fully characterize variable errors. We believe that the variability in error, i.e. those occurring throughout the entire trajectory can reveal additional insight as to how the executed motor plan varies from trial to trial. Recent work by Wu et al. also suggests that movement variability is a key marker in the ability of the subject to learn, where there is greater learning as a result of a larger extent of exploration in the error space [8].

Many studies have demonstrated the occurrence of error-based learning, yet it is unclear what representation of error is most relevant to the motor system. Conditt et al. suggests that adaptation to a dynamic force environment involves an internal representation of the structure of the field [9]. Hudson and Landy suggest that movement representations consist of both position and vector coding, where each one uses different aspects of sensory feedback [10]. Generalization studies can also provide some insight as to how these errors our processed, where subjects experience a variation of a learned skill at new target locations, different speeds, or hand configurations. Goodbody and Wolpert found that generalization of learning a novel dynamically environment (such as a force field) was best when the force field was represented in state-space [11]. What is not yet known is how to best describe error, particularly which domain, or metric representation, should be used to most reliably predict error tendencies.

Recent work by Huang and Patton showed how distribution analysis during free exploration can be used to

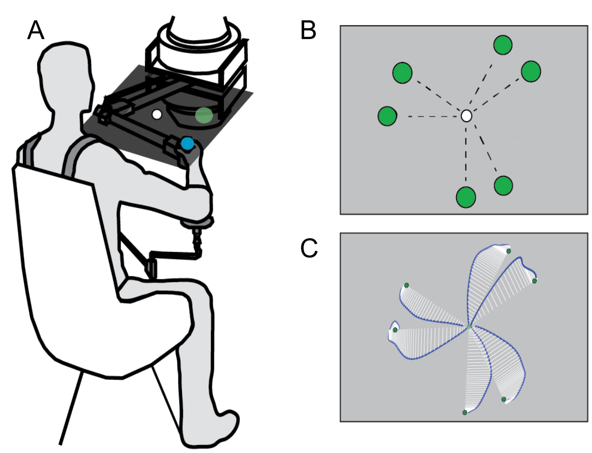

Figure 1: A) Planer manipulandum with horizontal display, B) Representative trajectories for a typical subject reaching to six target directions (green circles), C) Maximum perpendicular error (shown with white lines) is calculated as the distance between the cursor position (blue line) and an ideal straight-line path.

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identify patterns of deficit for stroke survivor subjects that

might not be explained with analyzing velocity or hand-position alone [12]. This approach provides a subject-specific picture of individual movement tendencies. One obvious speculation is whether individuals also produce unique profiles in the space of errors. Here, we performed a similar distribution analysis on *error* in targeted reaching movements. We focused on whether the error distributions were best characterized in either position or time domains. While there might be infinite possible coding mechanisms of error, we entertained versions of those most commonly used such as time or state-space (path length, distance along x-axis, distance to origin, and distance to target). As a first attempt at characterizing how the variability in error is represented across an entire trajectory, we compared the mean and standard deviations of perpendicular error across each ordinate domain. We hypothesized that errors represented in time would be the least variable and allow for the best predictive models of error distributions.

II. METHODS

A. Human Subjects

Subject data was analyzed from a previous experiment in which 9 healthy subjects were intermittently exposed to a 30° visual feedback rotation while holding a planar manipulandum with a horizontal display (Fig. 1A). Subjects were instructed to reach from the center position to 6 target directions (Fig. 1B) and received feedback (colored targets) pertaining to their movement speed once they hit the target where yellow was too slow, red was too fast, and green was within the ideal velocity range. During the *intermittent exposure phase,* subjects experienced rotated visual feedback 1 in 7 trials, during the *learning phase* subjects experienced rotated visual feedback continuously with catch trials (brief removal of the distortion) every 1 in 7 trials. All participants provided informed consent in accordance with Northwestern University Institutional Review Board. Participants were aged 21-40 (Mean age of 25 ± 3). Position of the robot cursor and handle were recorded at 500 Hz.

B. Ordinate Domains

Ordinate domains were defined to be the dependent measure for the sequence of error calculations. Since we were unsure what type of space ordinate to use, we further divided space into two candidate representations: *Path length* was determined as the Euclidian distance between the current and previous sample (representative of the distance along the actual trajectory). To calculate *distance along x-axis*, trajectories were rotated to align with the x-axis and the distance was determined to be the respective change along the horizontal axis (representative of the distance along the intended trajectory).

C. Mean and Variability of Movement Error

For each trajectory, perpendicular error was measured as the distance from cursor position to the corresponding point along the ideal straight-line path (Fig. 1C). Perpendicular errors were then graphed with respect to time, path length, and distance along x-axis. The mean and standard deviation of perpendicular error were determined at each sample and fit with a 5^{th} order polynomial to produce a smooth, continuous representation of error across the trajectory for each ordinate (Fig. 2). Confidence intervals were calculated across subjects for all proposed ordinate domains using a significance level of 0.05 (Fig. 3).

D. Predictive Model of Error

A Gaussian distribution was used to represent the magnitude and range of perpendicular error *(e)* at each sample, calculated with time (t) or path length (d), using Eq. 1 and Eq. 2 respectively, using continuous functions of mean (μ) and standard deviation of error (σ). The Gaussian function was scaled by a value, a, such that the integral of each Gaussian in time and space was equal to 1; likewise, the sum of the frequency of error occurring at each point in time and space was equal to 1.

$$P_{error}(t) = a(t) * \exp\left(-\frac{(e-\mu(t))^2}{2\sigma(t)^2}\right) \quad \text{(Eq-1)}$$

$$P_{error}(d) = a(d) * \exp\left(-\frac{(e-\mu(d))^2}{2\sigma(d)^2}\right) \quad \text{(Eq-2)}$$

Histograms showing the density of perpendicular error vs. time and perpendicular error vs. path length were calculated using a 50 x 50 grid. A model predicting the probability of error was constructed for each subject as a function of time and distance from origin spanning the same range as the raw data (Fig. 4).

E. Model Goodness of Fit

The coefficient of determination (R^2) was calculated to test how well the prediction of reaching errors (modeled using a Gaussian distribution for at each ordinate sample) explained the experimental data for a given reach. We performed two-way within-subject repeated measures ANOVA with factors being the ordinate domain and phase.

III. RESULTS

Variability in Error

Changes in mean perpendicular error and standard deviation of perpendicular error were detected between intermittent exposure, early learning and late learning (Fig.

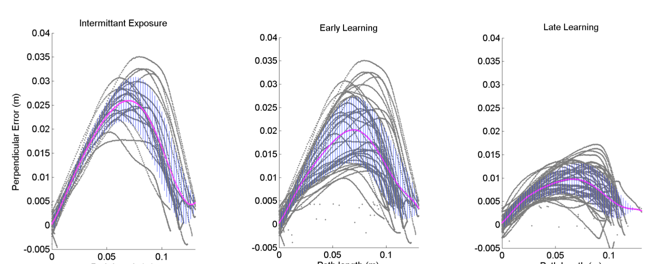

Figure 2: Examples of trajectories of perpendicular error from a typical subject's center-out reaching movements during intermittent exposure (left), beginning of learning (center) and late in learning (right) were computed in terms of several ordinates, shown here with respect to path length. While the mean errors (magenta) exhibit evidence of learning across trials, the standard deviation (blue lines) successfully captures the trial-to-trial variation of error.

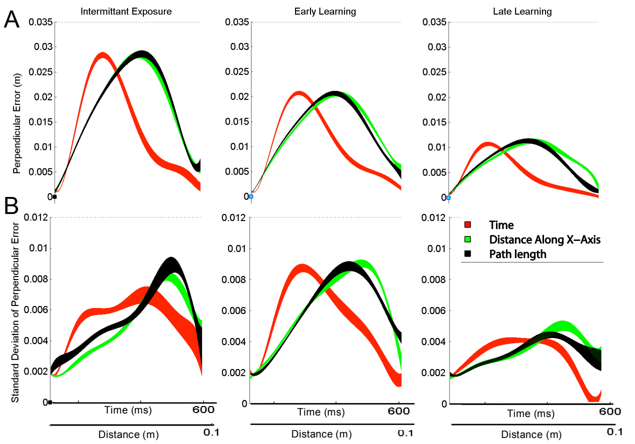

Figure 3: A) Mean and B) standard deviation of perpendicular error for each subject during intermittent exposure (left), early (center) and late learning (right) during center-out reaching movements. Mean error trajectories exhibit similar magnitudes in terms of time (plotted in milliseconds), path length, and distance (plotted as distance), but with differences peak location. In contrast, the variability of error in terms of time is markedly lower than the other ordinate descriptions during intermittent exposure and late learning. In addition, the peak of error variability in time occurs within a lower percentage of task completion compared to the ordinates associated with state space. Shaded region represents the 95% confidence interval for 9 subjects.

2) for the three proposed ordinates—time, path length, and distance along x-axis.

The distribution of mean perpendicular error across subjects (Fig. 3A) showed significantly different locations of peak error, where peak occurred at 33±3% of task completion for time, and 59±3% and 57±4% for path length and distance along x-axis respectively. These trends were consistent across all experimental phases tested.

The peak location of standard deviation of perpendicular error across subjects (Fig. 3B) was significantly different between time, path length, and distance along x-axis during intermittent exposure, early learning, and late learning. The overall variability was less for time as compared to path length (p=0.0105) and distance along x-axis (p=0.0018) during intermittent exposure. We found no significant difference in overall variability between ordinates during early learning and late learning.

Predictability of error

We calculated the coefficient of determination (R^2) between the probabilistic error model and the experimental data (Fig. 4, Fig. 5) for three proposed ordinate domains. Using a 2-factor within subject repeated measures ANOVA, we found significant differences between the three proposed ordain across phases (p=0.0458) and between ordinates (p=0.0042). Further post-hoc analysis using paired t-tests (with Bonferroni corrections for 9 possible comparisons) showed significantly different R^2 values between ordinates of time and distance along x-axis during intermittent exposure and early learning (p=0.0057, p=0.0468) and significantly different R^2 values between ordinates of path length and distance-along x-axis during intermittent exposure and early learning (p= 2.28 e-04, p= 0.0291).

IV. Discussion

The purpose of this study was to test how variability in movement errors was reflected in different ordinate domains. We analyzed data from a previous experiment in which healthy subjects performed center-out reaching while experiencing a 30° rotated feedback condition. We examined how the distribution of perpendicular error varies based on the proposed ordinate domain, primarily focusing on time and state-space based ordinates.

We found that the average perpendicular errors across subjects exhibit the same magnitude regardless of ordinate domain, though there were differences in peak location. The similarity of the maximum values of perpendicular error across domains demonstrates how the use of trajectory norms is a justified tool for tracking learning in motor control studies. However, the difference in peak location further motivates the need to investigate in what other ways the ordinate domains may differ. It is possible that the differences in peak location might be attributed to direct relationships between variables, for example between position, time, and velocity. However, the trial-to-trial variation is not be constrained by such dynamic relationships, allowing for peak velocity to occur at any point in time.

During intermittent exposure, we found that the overall variability of error across subjects was lowest for the time domain than the two candidate ordinates based in state-space. This trend was also true, though not significant, during late

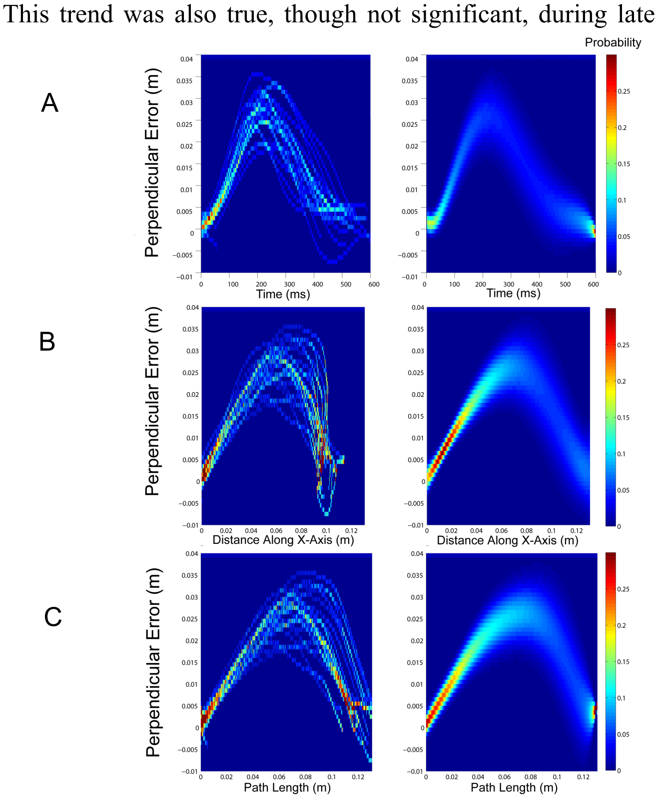

Figure 4: Histogram shows distribution of perpendicular errors during a reaching movement (left) and model predictions of error (right) during intermittent exposure to a visual rotation based on ordinates of A) time, B) distance along x-axis, and C) path length for a typical subject.

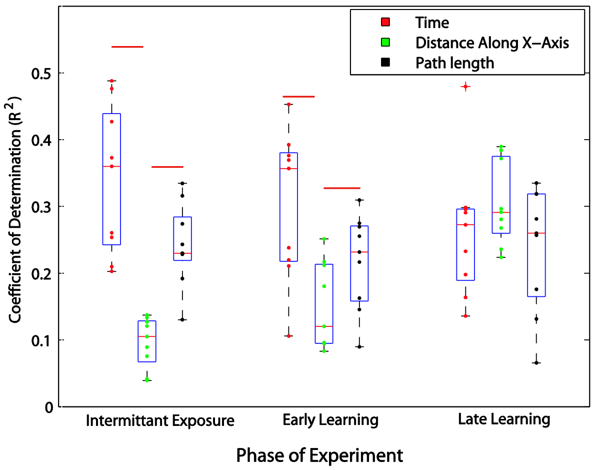

Figure 5: Coefficient of determination (R^2) values indicate the goodness of fit between the proposed error distribution model and experimental data for three possible ordinate domains during intermittent exposure (left), early learning (center) and late learning (right). Colored points denote individual subjects for each domain type, boxes denote subject means. Red bars indicate significant difference between pairs after post-hoc corrections.

learning. During intermittent exposure, where there is high error and low variability, time is the least noisy of the proposed ordinates. In phases of high variability, such as in early learning, it is difficult to discern differences in ordinates domain. Further, the variability of error during intermittent exposure and late learning appear to be scaled versions of each other, possibly suggesting that patterns of variability are unique to each phase of learning.

To demonstrate the effect of how variability in each ordinate domain contributes to the predictability of motor error distributions, we constructed a simple predictive model (Fig. 4) using Gaussian distributions across each ordinate domain with the mean and standard deviation of perpendicular error (with continuous polynomials for each ordinate domain). This modelling approach was able to explain approximately 40% of experimental data for each phase. The time ordinate domain showed the highest result during intermittent exposure. Possible iterations of this predictive model would allow for multiple Gaussian distributions at each sampling point or use a different distribution that does not enforce normality. Since the first half of movement has the highest probability, there is also possibility that such a model would better describe the experimental data if we broke up the trajectory into the ballistic phase and subsequent corrective phases.

It remains uncertain whether multiple domains are used to update feedback in order to adapt to a visual-motor discrepancy. It is possible that the central nervous system could use multiple ordinate frames (including the ones we proposed) to update the motor plan or none of them. When observing the goodness of fit across phases, the ordinate of distance along x-axis had an upward improvement from intermittent exposure to late learning (Fig. 5). We believe that this ordinate is the most aligned to a state-representation and this improvement corresponds to previous evidence that a state space representation takes over once learning has plateaued [9, 10].

Our findings that time offers the best ordinate domain for representing errors is in agreement with known constraints in sensory-motor control. Because sensory feedback is time-delayed, the motor system cannot react instantaneously to the state of the limb. Accordingly, our suggest results (Fig. 3) that during the ballistic phase of goal-directed reaching movements errors align in time. If we only consider the feed-forward motor plan it is also possible that a spatial path is generated but is variable in time, Nashner and Berthoz showed that movement latencies associated with visual feedback is approximately 100 milliseconds, where peak error is shown to be with respect to the time domain [13].

We conclude time to be the most substantive basis for a predictive model of performance error, since time-based errors demonstrated the least variance (Fig. 3) and led to the best fit using our simple modeling approach (Fig. 5). The ability to predict movement errors using the variability of subjects gives rise to better training techniques that are motivated by subjects' reaching errors, such as error augmentation [4].

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