

# Functional Range of Movement of the Hand: Declination Angles to Reachable Space

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**Abstract**—The measurement of the range of hand joint movement is an essential part of clinical practice and rehabilitation. Current methods use three finger joint declination angles of the metacarpophalangeal, proximal interphalangeal and distal interphalangeal joints. In this paper we propose an alternate form of measurement for the finger movement. Using the notion of reachable space instead of declination angles has significant advantages. Firstly, it provides a visual and quantifiable method that therapists, insurance companies and patients can easily use to understand the functional capabilities of the hand. Secondly, it eliminates the redundant declination angle constraints. Finally, reachable space, defined by a set of reachable fingertip positions, can be measured and constructed by using a modern camera such as Creative Sens3D or built-in hand gesture sensors such as the Leap Motion Controller. Use of cameras or optical-type sensors for this purpose have considerable benefits such as eliminating and minimal involvement of therapist errors, non-contact measurement in addition to valuable time saving for the clinician. A comparison between using declination angles and reachable space were made based on Hume’s experiment on functional range of movement to prove the efficiency of this new approach.

## I. INTRODUCTION

The measurement of the range of hand joint movement is an essential part of clinical practice and rehabilitation. Doctors and therapist often use goniometers, inclinometers, or electro-goniometers to measure the declination of hand joint angles which encode joint movement range[1]. One challenge with current methods is that active hand movement measurement can be affected by the fact that most sensors touch the hand and therefore may affect the joint measurements [2]. Moving to optical solutions provides a non-touch measurement eliminating such issues. However, one limitation of such optical devices is that it is impossible to measure all joint angles due to occlusions. Indeed multiple sensors can help to overcome this problem although it creates addition complexities in the system design.

Measuring declination joint angles is not the only way to evaluate the range of hand movement. Historically, subjective visual examination has been used to examine the range

of movement before the use of goniometers [3]. Despite the advantage of goniometers declination angles fall short of a complete dextral profile. According to Kendall [4], “For muscles that pass over two or more joints, the normal range of muscle length will be less than the total range of movement of the joints over which the muscle passes”. So, in measuring joint movement in which a two-joint muscle is involved, the second joint should be placed in a shortened position - demonstrating that hand movement is not just simply summing measured declination angles or separately considering each angle.

Many dexterous daily activities are determined by fingertip trajectories rather than finger joint angles and therefore it makes sense to measure dexterity using these parameters. In recent years this has become quite feasible via the development of a number of pervasive devices such as Microsoft Kinect Sensor, Leap Motion Controller, Creative Sens3D Camera offering a better solution for measuring joint movements. These devices can measure the position of fingertips accurately. The set of all reachable fingertip positions is defined as the “*reachable space*”. Compared with subjective visual assessment of reachable space, employing visual recording devices, provides a descriptive visualization while it delivers highly accurate non-contact objective measurements.

Accordingly, the reachable space of fingertips deserves further attention in areas such as rehabilitation. In the study of the characteristics of planar fingertip movements, Cruz [5] used the Optotrak system to measure fingertip location. Using 10 subjects (six males, four females), he arrived at an approximation to the reachable space. In an earlier study, Venema [6] tried to find the workspace by recursively sweeping the range of hand movement. The limitation of such work is that they lack a mathematical model for the reachable space - the purpose of this study.

To this end, a precise definition of “*reachable space*” is required. First of all, the “*reachable space*” is a set of points that a fingertip of one finger can reach. This has already been used in robotics [7], [8] but, to our knowledge, it has not been used in clinical practice. In current clinical terminology, the normal range of movement which is the standard or average range over a given population. The functional range of movement is the range of movement measured while people do daily activities - the common position of fingertips when people do daily activities. Task-specific declination angles are the common declination angles for finger joints while performing specific tasks such as power grip, precision grip, key pinch, tip pinch. In reachable space, subspaces

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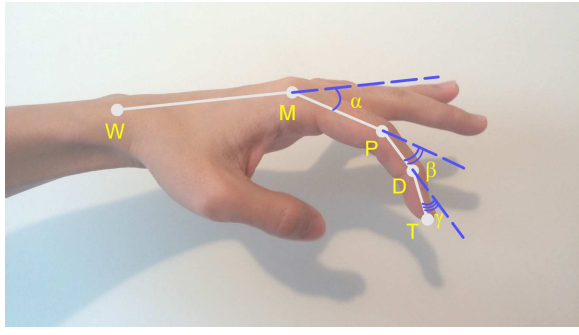


Fig. 1. Simple finger model.

associated with range of movement for specific tasks are subspaces called task-specific subspace.

## II. REACHABLE SPACE

The joints of the human hand are classified into three kinds: flexion, directive or spherical joints, which consist of one DOF (extension/flexion), two DOFs (one for extension/flexion and one for adduction/abduction) and three DOFs (rotation), respectively. For each finger and thumb, there is 4 DOFs. Considering the three DOFs for the rotation of the wrist, the model has 23 DOFs [9], [10]. We use a mathematical model to calculate the fingertip position. Assume that we know the length of the phalangeal bone and the angles of phalangeal joints. The origin of the axis is at the wrist point. We also assume that the metacarpus bone lies on the X axis.  $p_M$  denotes the coordinate of M.  $l_1, l_2, l_3, l_4$  are the length of metacarpus, proximal, middle, and distal bones respectively.

Metacarpophalangeal joint (MCP) position is computed from wrist position. As the axis is attached to the palm plane, the coordinate of MCP is depended only on the length of metacarpus and unchanged during the movement.

$$p_M = p_W + p_{M/W} \quad (1)$$

$$\begin{bmatrix} x_M \\ y_M \\ z_M \end{bmatrix} = 0 + \begin{bmatrix} l_1^1 \\ 0 \\ l_1^2 \end{bmatrix} \quad (2)$$

Proximal interphalangeal joint (PIP) position is computed from MCP position. The MCP has 2 degree of freedom: extension/flexion and adduction/abduction.  $R(\alpha_1, \alpha_2)$  is the rotation operator with respects to extension/flexion and adduction/abduction.

$$p_P = R(\alpha_1, \alpha_2) p_{P/M} + p_M \quad (3)$$

$$\begin{bmatrix} x_P \\ y_P \\ z_P \end{bmatrix} = \begin{bmatrix} \cos \alpha_1 \cos \alpha_2 & -\cos \alpha_2 \sin \alpha_1 & \sin \alpha_2 \\ \sin \alpha_1 & \cos \alpha_1 & 0 \\ -\cos \alpha_1 \sin \alpha_2 & \sin \alpha_1 \sin \alpha_2 & \cos \alpha_2 \end{bmatrix} \times \begin{bmatrix} l_2 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} x_M \\ y_M \\ z_M \end{bmatrix} \quad (4)$$

Distal interphalangeal joint (DIP) position is computed from proximal interphalangeal joint coordinate. PIP has only

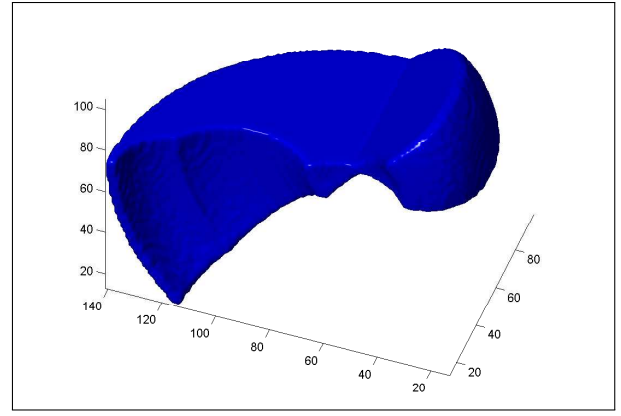


Fig. 2. Reachable space of fingertips associated with normal range of movement (Table I) in 3 dimensions.

extension/flexion movement.  $R(\beta)$  is the rotation operator which respects to extension/flexion of PIP.

$$p_D = R(\alpha_1, \alpha_2) R(\beta) p_{D/P} + p_P \quad (5)$$

$$\begin{bmatrix} x_D \\ y_D \\ z_D \end{bmatrix} = \begin{bmatrix} \cos \alpha_1 \cos \alpha_2 & -\cos \alpha_2 \sin \alpha_1 & \sin \alpha_2 \\ \sin \alpha_1 & \cos \alpha_1 & 0 \\ -\cos \alpha_1 \sin \alpha_2 & \sin \alpha_1 \sin \alpha_2 & \cos \alpha_2 \end{bmatrix} \times \begin{bmatrix} \cos \beta & -\sin \beta & 0 \\ \sin \beta & \cos \beta & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} l_3 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} x_P \\ y_P \\ z_P \end{bmatrix} \quad (6)$$

Fingertip is computed from DIP coordinate. DIP has only extension/flexion movement.  $R(\gamma)$  is the rotation operator with respect to extension/flexion of DIP.

$$p_T = R(\alpha_1, \alpha_2) R(\beta) R(\gamma) p_{T/D} + p_D \quad (7)$$

$$\begin{bmatrix} x_T \\ y_T \\ z_T \end{bmatrix} = \begin{bmatrix} \cos \alpha_1 \cos \alpha_2 & -\cos \alpha_2 \sin \alpha_1 & \sin \alpha_2 \\ \sin \alpha_1 & \cos \alpha_1 & 0 \\ -\cos \alpha_1 \sin \alpha_2 & \sin \alpha_1 \sin \alpha_2 & \cos \alpha_2 \end{bmatrix} \times \begin{bmatrix} \cos \beta & -\sin \beta & 0 \\ \sin \beta & \cos \beta & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} \cos \gamma & -\sin \gamma & 0 \\ \sin \gamma & \cos \gamma & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} l_4 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} x_D \\ y_D \\ z_D \end{bmatrix} \quad (8)$$

By running through all declination angles in the range of movement of hand using the proposed mathematical model, one can achieve the 3 dimension reachable space depicted in Figure 2.

TABLE I  
NORMAL ACTIVE RANGE OF MOVEMENT OF FINGER

MCP	0-100
PIP	0-105
DIP	0-85
Total arc:	0-290

TABLE II  
FUNCTIONAL ACTIVE RANGE OF MOVEMENT OF FINGER

	Range	Average	SD	Median
MCP	33-73	61	$\pm 12$	62
PIP	36-86	60	$\pm 12$	63
DIP	20-61	39	$\pm 14$	39
Total functional arc	96-208	164	$\pm 27$	165

TABLE III  
TASK-SPECIFIC POSITIONS OF THE JOINTS OF THE HAND (FINGER)

	Task-specific positions (fingers)			
	Key pinch	Tip pinch	Grasp	Grip
MCP	62 ( $\pm 8$ )	58 ( $\pm 7$ )	33 ( $\pm 6$ )	72 ( $\pm 12$ )
PIP	76 ( $\pm 8$ )	76 ( $\pm 13$ )	39 ( $\pm 7$ )	28 ( $\pm 5$ )
DIP	46 ( $\pm 8$ )	33 ( $\pm 12$ )	26 ( $\pm 5$ )	50 ( $\pm 5$ )
Total arc	185 ( $\pm 20$ )	167 ( $\pm 19$ )	96 ( $\pm 14$ )	208 ( $\pm 23$ )

### III. FROM RANGE OF MOTION TO REACHABLE SPACE

Data from Hume [11] has been used to compare declination angles and reachable space. In this clinical trial, 35 right handed men, aged 26 to 28 years, none with a history of antecedent hand injury were studied. The trial recorded the maximum active range of movement (Table I), daily activities (functional) range of movement (Table II) and the position of finger joints when subjects did some specific tasks (Table III). In daily activities test, there were 11 activities recorded: holding a telephone, holding a can, using a zipper, holding a toothbrush, turning a key, using a comb, writing with a pen, holding a fork, holding a scissor, unscrewing a jar and holding a harmer. The task-specific test included key pinch, tip pinch, precision grip (grasp) and power grip. To further illustrate reachable space, we calculate the reachable space for the general case. We use the bone length values as follows: Metacarpal bone length is 68 mm, Proximal phalanx bone length is 40 mm, Medial phalanx bone length is 22 mm, and Distal phalanx bone is 15 mm. After computation which takes no longer than 1 second, we obtain the results given in (Table IV and Figure 3, 4, 5, 6, 7). Figure 8 is the combination of figures 3, 4, 5, 6, 7 to help readers understand the relative positions of subspaces. From the computational outcomes, we can see the position of specific task and it gives us an idea of the patients's hand status. For example, if one reachable space of a finger covers the functional subspace, then we can expect that the patient is able to do daily activities after passing the strength tests. Otherwise, daily activities cannot be preformed and the strength tests are redundant. In another example, if one patent has the reachable space that cover the key pinch task subspace, we can expect that the patient can perform the key pinch task. Table IV shows the area of the 2D reachable space that we computed from the subspaces using Scanline Fill algorithm in Euclidean metric.

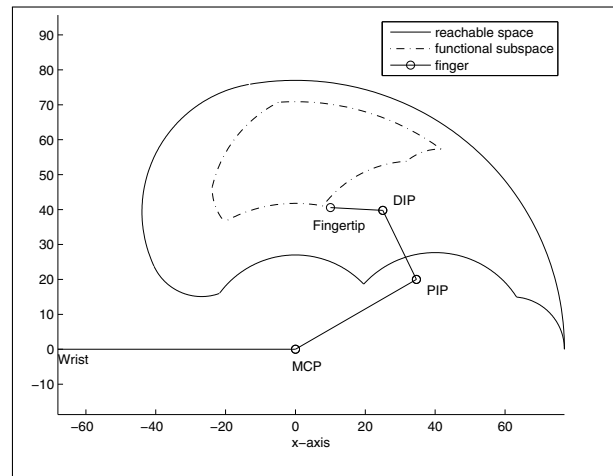


Fig. 3. The functional subspace (daily activities) in reachable space

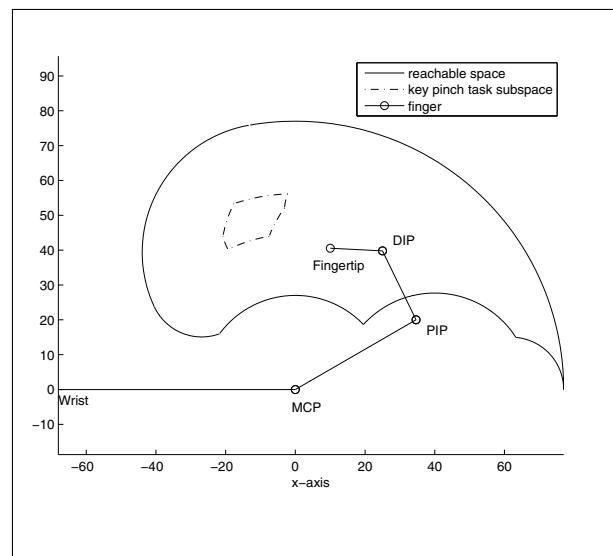


Fig. 4. The key pinch task subspace in reachable space

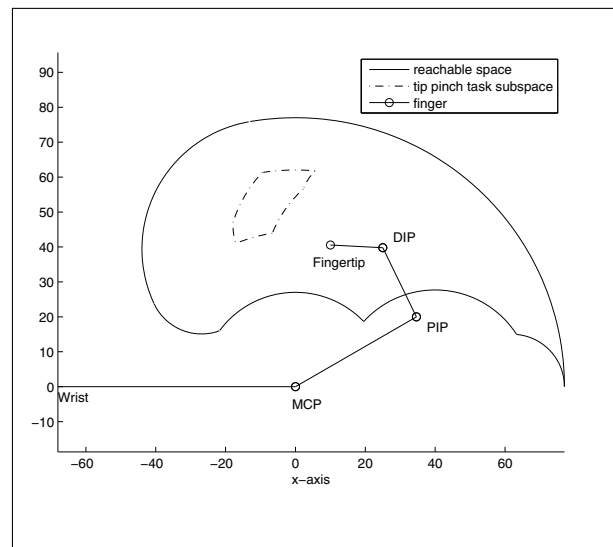


Fig. 5. The tip pinch task subspace in reachable space

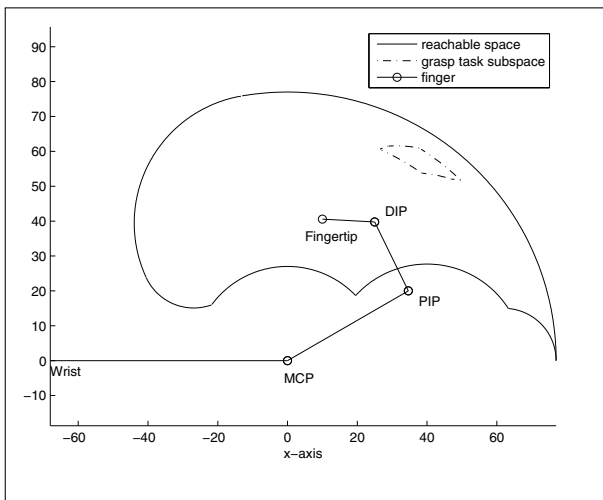


Fig. 6. The grasp (power grip) task subspace in reachable space

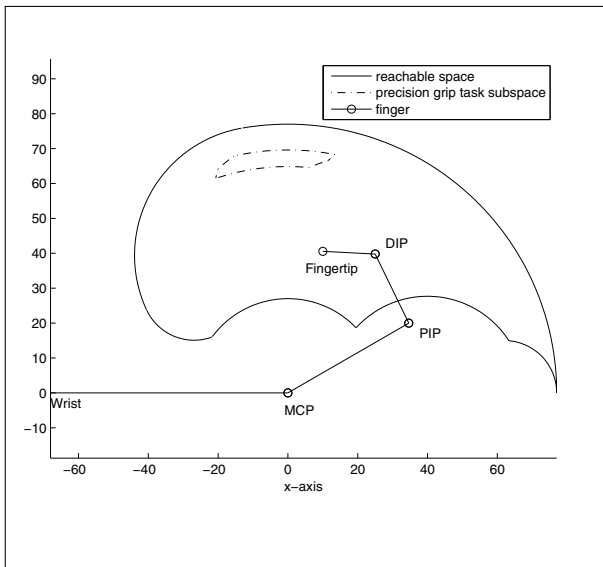


Fig. 7. The grip (precision grip) task subspace in reachable space

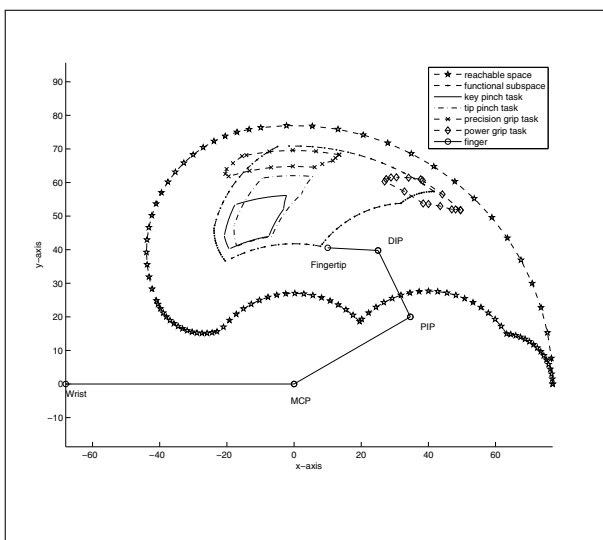


Fig. 8. Various subspace in reachable space.

TABLE IV  
REACHABLE SPACE AND TASK-SPECIFIC SUBSPACE

	Area ( $mm^2$ )	Percentage (%)
Full space	5089.6	100
Functional space	1275.5	25.1
Key pinch space	181.6	3.6
Tip pinch space	263.2	5.2
Grasp space	87.1	1.7
Grip space	141.7	2.8

#### IV. CONCLUSION

Building reachable space models requires bone lengths and range of movement that make the reachable space more descriptive and specific for individuals. By focusing only on reachable ability of fingertip regardless of declination angles, the reachable space can avoid the redundancy in angle configurations because for a reachable point, there are many combinations of declination angles for finger to reach that point. Moreover, reachable space is easy to visualize and forms a more refined and accurate description of the flexibility of the hand. In addition, modern camera such as Leap Motion Controller or Creative Senz3D Camera can measure the reachable space quickly, accurately and the conversion of current declination angle profiles to reachable space can be easily achieved. The inverse process of obtaining the declination angles from the reachable space require additional joint angle constrains.

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