Adaptive Coherent Averaging for Real-time Electrocardiogram Enhancement

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***Abstract*— This paper presents an adaptive coherent averaging structure capable of removing broad-band interference from the electrocardiogram (ECG) while preserving the morphological features of the signal. The proposed structure improves the signal-to-noise ratio (SNR) of the adaptive line enhancer (ALE) while maintaining robustness to quasi-periodic signals. The least mean-square (LMS) and recursive least-square (RLS) adaptive algorithms are implemented. Analysis and comparison of the results is provided, leading to an optimized hybrid implementation. The cascade nature of the proposed structure is both scalable and suitable for implementation in real-time hardware.**

I. Introduction

The electrocardiogram (ECG) is a recording of the heart's electrical potentials over time. It provides physicians with a graphical representation of the heart's functionality and is used to expose physiological and pathological irregularities of the cardiac rhythm and circulatory system. Unfortunately, multiple sources of interference contaminate the ECG signal during acquisition, corrupting the morphology and making it difficult for physicians to provide an accurate diagnosis. These sources of interference can be grouped into two categories: narrow-band and broad-band.

The major contributing narrow-band interference are as follows: power line interference (PLI), baseline wander (BW), motion artifacts (MA) and electrode motion artifacts (EM). The remaining broad-band interference are as follows: electromyographic (EMG), and electrode contact noise. [1]

Typically, the frequency of the narrow-band interference is either known in advance or can be determined through analysis. This allows for the interference to be attenuated by either a simple FIR notch filter or a slightly more complex adaptive noise canceller (ANC) as shown in [2]. The advantage of using an ANC over the more simplistic solution is that the filter can track the frequency of the interference if it drifts over time. It is discussed in [3] that even when the narrow-band interference exhibits spectral overlap with the ECG, a notch filter can be used with minimal effect to the ECG morphology.

It is more difficult, however, to attenuate the broad-band interference when it shares the frequency spectra of the ECG. The reason for this is because the ECG is composed of multiple harmonic components, as shown in the lower plot of Fig. 1. Many linear adaptive approaches have been applied to remove the broad-band interference from the ECG including the use of an adaptive noise canceller by Yelderman *et al.* [4], the time-sequenced adaptive filter by Ferrara [5], the adaptive recurrent filter by Thakor and Zhu [3], the Fourier linear combiner (FLC) by Vaz *et al.* [6], the wavelet transform by Li *et al.* [7], and the adaptive comb filter (ACF) by Cyrill *et al.* [8]. There are advantages and disadvantages to each of these techniques, but all are sensitive to the quasi-periodic nature of the ECG. This makes their performance highly dependent on accurately determining the reoccurrence interval of the ECG complex, shown in the upper plot of Fig. 1. Determining this reoccurrence interval requires an additional pre-processing step that is usually done accomplished with a matched-filter and threshold.

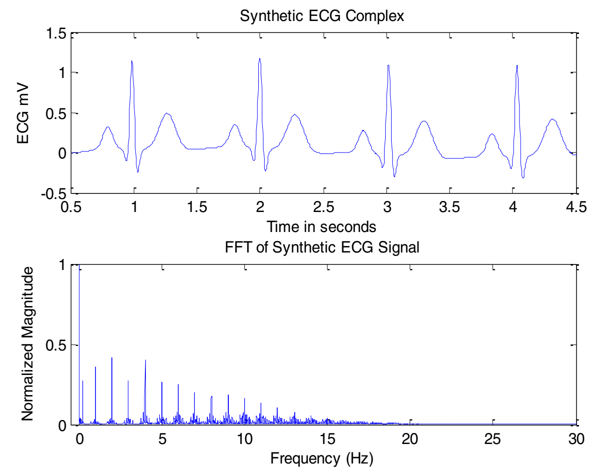

Fig. 1: Synthetic ECG signal

More recently, eigenanalysis techniques have been applied to remove the broad-band interference from the ECG as well. These include both principal component analysis and independent component analysis [10][11][12]. The two primary drawbacks of the eigenanalysis techniques are they require considerable more resources and computational time due to the increased computational complexity. This makes them less suitable for real-time processing.

In this paper, an adaptive coherent averaging structure is presented to remove contaminating broad-band interference while preserving the morphological features of the ECG signal. Consecutive ECG complexes resemble each other and are generally uncorrelated with the interference, however, the time varying delay between complexes creates a quasi-periodic nature. This nature is exploited by adaptively averaging multiple consecutive ECG complexes, thus improving the signal-to-noise ratio (SNR). The proposed structure requires no *a priori* knowledge of either the primary

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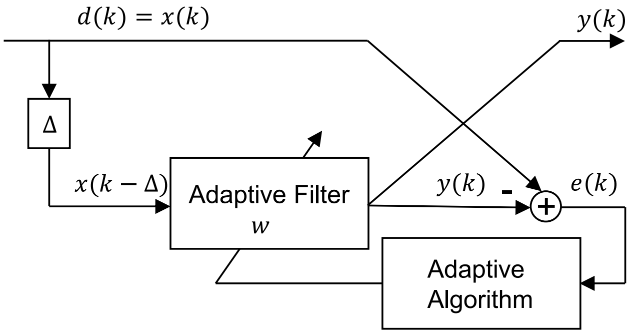

Fig. 2: The adaptive line enhancement structure

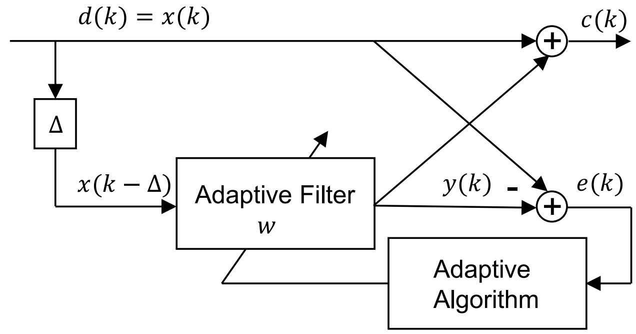

Fig. 3: The adaptive coherent averaging structure

signal or the interference. The cascade structure is scalable to a desired SNR and can be implemented in real-time hardware.

The remainder of the paper is organized as follows: Section II describes the adaptive coherent averaging architecture. Section III presents analysis of the LMS and RLS adaptive algorithms. Sections IV analyzes the performance of the two adaptive algorithms. Conclusions are presented in Section V.

II. Description of the Architecture

The adaptive line enhancement (ALE) structure, shown in Fig. 2, was introduced in 1975 by Widrow *et al.* [2] as a simplified form of the adaptive noise cancellation (ANC) structure. Both the ANC and ALE structures estimate narrow-band signals corrupted by additive broadband noise or interference. However, the ALE structure has a distinct advantage over the ANC structure in that it only requires a single input.

The primary input of the ALE structure, $d(k)$, is composed of the signal of interest and additive noise. The reference input is a time delayed version of the primary input. The derived reference input is processed with an adaptive transversal filter to form, $y(k)$, the adaptive filter output. The filter output is then subtracted from the primary input to produce the error signal, $e(k)$. For each sample, the adaptive algorithm recursively adjusts the weighting coefficients to minimize the expected error power.

Since the ALE structure operates on a single input, it does not require *a priori* knowledge of the characteristics of the signal of interest or the additive noise. The ALE is able to accomplish this by exploiting the difference in correlation lengths between the signal of interest and the additive noise. This is accomplished by appropriately choosing the size of the delay between the primary input and the derived reference. After convergence, the output, $y(k)$, is the optimum estimate of the signal of interest present in the primary input. When attenuating additive white Gaussian noise (AWGN), the ALE only requires one delay element, due to the fact white Gaussian noise is independent and identically distributed (i.i.d). This case is known as a one-step adaptive linear predictor.

The proposed structure, termed adaptive coherent averaging (ACA), is show in Fig. 3. The ACA structure is a modified version of the ALE structure designed to be cascaded in order to provide a coherent average as the final output. The primary input, $d(k)$, the derived reference input, the adaptive filter output, $y(k)$, and the error signal, $e(k)$, are all the same as the ALE structure. A cascade output, $c(k)$, is the sum of the primary input, $d(k)$, and the adaptive filter output, $y(k)$.This output serves as the primary input to the next stage.

The adaptive filter output, $y(k)$, of the ACA structure is defined as

$$y(k) = \sum_{i=1}^{p} w_i(k) x(k - \Delta - i), \tag{1}$$

$$y(k) = \boldsymbol{w}^T(k)\boldsymbol{x}(k-\Delta), \tag{2}$$

where $[\cdot]^T$denotes transpose, p, is the filter order, and $\boldsymbol{w}$(k) and $\mathbf{x}(k-\Delta)$ are vectors defined as

$$\boldsymbol{x}(k-\Delta) = [x(k-\Delta) \quad \dots \quad x(k-\Delta-p+1)]^T, \tag{3}$$

and

$$\boldsymbol{w}(k) = [w_1(k) \quad \dots \quad w_p(k)]^T. \tag{4}$$

The error signal, $e(k)$, is the difference between the desired input and the filter output defined as

$$e(k) = d(k) - y(k), \tag{5}$$

$$e(k) = x(k) - \boldsymbol{w}^T(k)\boldsymbol{x}(k-\Delta), \tag{6}$$

The cascade output, $c(k)$, is the scaled sum of the desired input and the filter output defined as

$$c(k) = \alpha_n d(k) + (1-\alpha_n) y(k), \tag{7}$$

$$c(k) = \alpha_n x(k) + (1-\alpha_n)\boldsymbol{w}^T(k)\boldsymbol{x}(k-\Delta), \tag{8}$$

where, α_n, is the scale factor at stage n. For the remainder of this paper we set, α_n, to

$$\alpha_n = \frac{n-1}{n}, \tag{9}$$

which provides equal weighting to each of the averaged values.

The cascade form of the adaptive coherent averaging structure is shown in Fig. 4. The cascade form is composed of two block element types: the ALE and the ACA. Each block element has a primary input (P), a reference input (R) and a cascaded output (C). It is important to observe that the cascade output of a stage is the primary input to the next stage and that

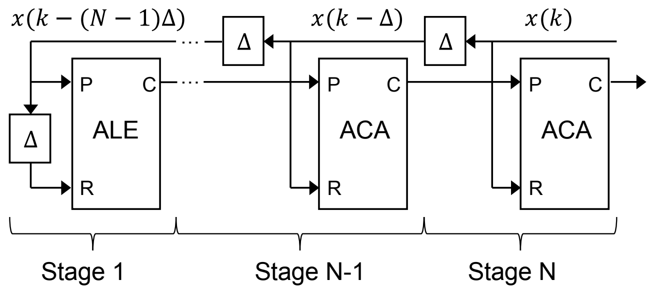

Fig. 4: Cascade form of the ACA structure

the input vector, $x(k)$, is used as the reference signal, with the greatest delay on the lowest stage.

The cascade output of a 4-stage ACA structure is defined as follows:

$$c_1(k) = \boldsymbol{w}_1^T(k)\boldsymbol{x}(k - N\Delta), \tag{10}$$

$$c_2(k) = \frac{1}{2}c_1(k) + \frac{1}{2}\boldsymbol{w}_2^T(k)\boldsymbol{x}(k - (N-2)\Delta), \tag{11}$$

$$c_3(k) = \frac{2}{3}c_2(k) + \frac{1}{3}\boldsymbol{w}_3^T(k)\boldsymbol{x}(k - (N-3)\Delta), \tag{12}$$

$$c_4(k) = \frac{3}{4}c_3(k) + \frac{1}{4}\boldsymbol{w}_4^T(k)\boldsymbol{x}(k - (N-4)\Delta). \tag{13}$$

For an *N*-stage ACA structure, the following recursive eqaution can be used:

$$c_n(k) = \begin{cases} \boldsymbol{w}_n^T(k)\boldsymbol{x}(k - N\Delta), n = 1 \\ \frac{n-1}{n}c_{n-1}(k) + \frac{1}{n}\boldsymbol{w}_n^T(k)\boldsymbol{x}(k - (N-n)\Delta), n = 2, \dots, N \end{cases} \tag{14}$$

where, $n \leq N$, denotes the output at a given stage.

Using (14), it can be observed that the output of the final N-stage ACA structure is an average of N, time delayed, and filtered input vectors shown below

$$c_N(k) = \frac{1}{N}\left[\sum_{i=1}^{N}\left(\boldsymbol{w}_i^T(k)\boldsymbol{x}(k - (N-i)\Delta)\middle| c_{i-1}(k)\right)\right]. \tag{15}$$

It is important to recall from Fig. 4 that the primary input to all stages subsequent to stage 1 is an enhanced version of the signal. Therefore, all subsequent stages have the capability to further enhance the input signal based on adaptive filter theory.

III. Analysis of Adaptive Algorithms

For our analysis we implement the least mean-square (LMS) and the recursive least square (RLS) adaptive algorithms.

Although consecutive ECG complexes will resemble each other, the quasi-periodic nature of the ECG introduces a varying delay between complexes. This requires the adaptive algorithm to continuously adjust the filter weights, otherwise known as tracking.

A. Least Mean-Squares (LMS)

The LMS algorithm originally introduced by Widrow and Hoff in 1960 [13] is highly considered for its simple and robust gradient descent method. The algorithm minimizes the mean-square error (MSE) as follows [14]:

$$E[e^2(k)] = E[(d(k) - y(k))^2], \tag{16}$$

$$E[e^2(k)] = E[(x(k) - \boldsymbol{w}^T(k)\boldsymbol{x}(k - \Delta))^2], \tag{17}$$

$$\begin{aligned} E[e^2(k)] &= E[x^2(k)] \\ &- 2E[x(k)\boldsymbol{x}^T(k-\Delta)]\boldsymbol{w}(k) \\ &+ \boldsymbol{w}^T(k)E[\boldsymbol{x}(k-\Delta)\boldsymbol{x}^T(k-\Delta)]\boldsymbol{w}(k), \end{aligned} \tag{18}$$

$$\begin{aligned} E[e^2(k)] &= E[x^2(k)] \\ &- 2\boldsymbol{P}^T\boldsymbol{w}(k) + \boldsymbol{w}^T(k)\boldsymbol{R}\boldsymbol{w}(k), \end{aligned} \tag{19}$$

where, **P**, is the cross-correlation vector and, **R**, is the input correlation matrix defined as

$$\boldsymbol{P} = E[x(k)\boldsymbol{x}^T(k-\Delta)], \tag{20}$$

$$\boldsymbol{R} = E[\boldsymbol{x}(k-\Delta)\boldsymbol{x}^T(k-\Delta)]. \tag{21}$$

Widrow *et al.* have shown that by utilizing gradient descent to minimize the MSE of the quadratic function shown in (19), the optimum weight vector also known as the Wiener-Hopf weight vector shown below is achieved. [2]

$$\boldsymbol{w}_{opt} = \boldsymbol{R}^{-1}\boldsymbol{P}. \tag{22}$$

The LMS algorithm implements gradient descent through an iterative weight update given by

$$\boldsymbol{w}(k) = \boldsymbol{w}(k-1) - 2\mu e(k)\,\boldsymbol{x}(k-\Delta), \tag{23}$$

where, $1/\lambda_{max} > \mu > 0$, is the step size.

B. Recursive Least-Squares (RLS)

The RLS algorithm minimizes the weighted sum of squared errors as follows:

$$\sum_{k=0}^{n}\lambda^{n-k}\,e^2(k) = \sum_{k=0}^{n}\lambda^{n-k}\,(d(k) - y(k))^2. \tag{24}$$

The expansion of this cost function is shown in [14]. Similar to LMS, the RLS algorithm implements an iterative weight update given by

$$\boldsymbol{w}(k) = \boldsymbol{w}(k-1) - \boldsymbol{K}(k)\,e(k), \tag{25}$$

where, λ, is the forgetting factor and, $\mathbf{K}(k)$, is defined as

$$\boldsymbol{K}(k) = \frac{\boldsymbol{P}(k-1)\boldsymbol{x}(k-\Delta)}{\lambda + \boldsymbol{x}^T(k-\Delta)\boldsymbol{P}(k-1)\boldsymbol{x}(k-\Delta)}, \tag{26}$$

and, $\mathbf{P}(k)$, is defined as

$$\boldsymbol{P}(k) = \frac{1}{\lambda}[\boldsymbol{P}(k-1) - \boldsymbol{K}(k)\boldsymbol{x}^T(k-\Delta)\boldsymbol{P}(k-1)]. \tag{27}$$

Table 1: Output SNR for different input SNR of synthetically generated ECG with AWGN

Input SNR (dB)	Output SNR (dB)							
	Original ALE		2-Stage ACA		4-Stage ACA		8-Stage ACA	
	LMS	RLS	LMS	RLS	LMS	RLS	LMS	RLS
-10	1.79	2.33	2.02	2.25	2.04	2.21	2.36	2.52
-5	5.22	5.28	5.23	5.21	5.23	5.20	5.70	5.66
0	8.76	8.75	8.95	8.72	9.35	8.72	10.23	9.25
5	11.94	12.17	12.44	12.20	13.39	12.22	14.96	12.57
10	14.17	15.68	14.84	15.93	16.39	16.17	19.16	16.22

Table 2: Output SNR for Different Input SNR of Synthetically Generated ECG with AWGN

	Output SNR (dB) for RLS-LMS Hybrid			
Input SNR (dB)	Original ALE	2-Stage ACA	4-Stage ACA	8-Stage ACA
-10	2.33	2.28	2.25	2.56
-5	5.28	5.23	5.28	5.83
0	8.75	8.91	9.29	10.23
5	12.17	12.74	13.55	15.01
10	15.68	16.71	17.75	19.80

IV. PERFORMANCE ANALYSIS

The output signal-to-noise ratio (SNR) was the criteria chosen for analyzing the performance of the ACA structure. The output SNR is defined as

$$SNR = 10log_{10}\left(\frac{s^2(k)}{\left(s(k)-\hat{s}(k)\right)^2}\right), \quad (28)$$

where, $s(k)$, is the uncorrupted ECG and, $\hat{s}(k)$, is the enhanced ECG given by, $y(k)$, in (1).

We used a synthetic ECG model introduced by McSharry *et al.* [15] in order to obtain an uncorrupted ECG signal. This model produces realistic, quasi-periodic ECG complexes based on three coupled ordinary differential equations.

A comparison of the cascaded ACA results for both LMS and RLS are shown in Table 1. It is clear that RLS provides a greater initial gain for the original ALE, which becomes more noticeable as the SNR increases. However, the LMS provides the better multi-stage ACA gain, where RLS appears to remain relatively flat.

The results of Table 1 lead us to modify the cascade structure from a homogenous LMS or RLS structure to one that takes advantage of the strength of each algorithm observed in the prior paragraph. The modified hybrid structure uses RLS in the ALE block of Fig. 4 and LMS in the remaining ACA blocks. Results of this hybrid configuration are shown in Table 2.

V. CONCLUSION

An adaptive coherent averaging structure capable of removing broad-band interference from the ECG while preserving the morphological features of the signal has been proposed. The cascade form of the ACA shows between 0.5 and 5.0 dB SNR gain over the original ALE, dependent upon the input SNR.

An optimized cascaded ACA structure was implemented using both RLS and LMS, achieving approximately 10dB gain over the input SNR. The cascade nature of the proposed structure is both scalable and suitable for implementation in real-time hardware.

The proposed structure shows potential for enhancement of other biomedical recurrent signals as well as those found in Sonar and Radar.

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