Three-Layer-Isotropic Skull Conductivity Representation in the EEG Forward Problem using Spherical Head Models

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Abstract— **We study the influence of different conductivity models within the framework of electroencephalogram (EEG) source localization on the white matter and skull areas. Particularly, we investigate five different spherical models having either isotropic or anisotropic conductivity for both considered areas. To this end, the anisotropic finite difference reciprocity method is used for solving the EEG forward problem. We evaluate a model of a numeric skull conductivity in terms of the minimum dipole localization/orientation error. As a result, both considered models of the skull reach the lowest dipole localization error (less than $6\,mm$), namely: *i*) single anisotropic layer and *ii*) three isotropic layers (hard bone/spongy bone/hard bone). Additionally, two different electrode configurations ($10-20$ and $10-10$ systems) are tested showing that the error decreases almost as much as twice for the latter one though the computational burden significantly increases.**

I. Introduction

Nowadays, several methods have been proposed to analyze brain structures with high precision for efficient surgery planning (mostly, in Epilepsy or Parkinson diseases) or to perform general brain studies. Mainly, these computer-based methods that are supported on noninvasive measurements (e.g., electroencephalogram - EEG, magnetic resonance imaging - MRI, or computed tomography) are used for diagnosis and preoperative brain surgery stages. Moreover, they are, in most of the cases, the only suitable analysis tools due to the high risk of alternative invasive interventions [9].

Meanwhile, noninvasive methods are commonly focused on location of neural activity sources inducing electrical potentials in the head. Those potentials can be measured by electrodes placed directly on the scalp (i.e., EEG). In this regard, the source localization EEG problem is divided into the following two subsequent tasks: *i*) The forward problem calculating electrode potentials on the scalp for a provided source configuration, *ii*) The inverse problem estimating source parameters from electrode potentials [5]. The latter problem solution usually results in an iterative task. The solution is accomplished, assuming that the electrode potentials measured on the scalp are similar to those calculated by the reference model.

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On the other hand, both the skull and white matter have strong anisotropic conductivity profoundly affecting performance of source localization tasks [7]. From the clinical point of view, the skull composition is assumed to have three embedded stratums (one spongy layer between other two hard layers), each one having a different conductivity. Moreover, the anisotropic skull conductivity can be described by a three-layers-isotropic conducting model, namely, two separate compact zones plus a soft one [1]. So far, this model has been only tested against most widely known numerical methods (based on either finite difference [6] or finite element approximations [10]). However, both methods may induce some error in the reconstructed head source parameters due to their numerical approximations [2].

To cope with this issue, we develop a 3-layer-isotropic spherical model that is compared further against the baseline analytical representation proposed in [3], when the conductivity of white matter and skull is modeled as anisotropic. Particularly, five different skull conductivity modelings are compared in terms of the minimum dipole localization/orientation error, using the *anisotropic finite difference reciprocity method* (AFDRM) to calculate the numeric potentials against the analytical solution. We use three different skull conductivity models, isotropic, anisotropic, and the suggested 3-layer isotropic, with anisotropic/isotropic white matter in order to analyze the influence of deep sources, and the different skull models. To consider the influence of the used number of electrodes, we also carry out testing of both the baseline anisotropic analytical and 3-layer isotropic skull models using the commonly known *10-10* and *10-20* EEG systems.

II. Methods

A. Forward Problem

In the EEG source location task, the forward problem estimates the electrode potential field, $\boldsymbol{V}$, placed at a specific point, (x,y,z), on the scalp that is generated due to current sources in the brain. Potential sources are modeled as current dipoles placed at position $\boldsymbol{r}\in\mathbb{R}^3$ with orientation $\boldsymbol{d}\in\mathbb{R}^3$. The scalar-valued potential $V(x,y,z)\subset\boldsymbol{V}$ on the surface of a conductive volume x,y,z is defined by the Poisson equation as follows:

$$\nabla\left(\Sigma(x,y,z)\nabla V(x,y,z)\right) = I\delta(\boldsymbol{r}-r_1) - I\delta(\boldsymbol{r}-\boldsymbol{r}_2) \quad (1)$$

where $I\in\mathbb{R}$ represents the current dipole magnitude, $\boldsymbol{\Sigma}\in\mathbb{R}^{3\times3}$ is the conductivity tensor, and $\boldsymbol{r}_1$ and $\boldsymbol{r}_2$ are the two concrete coordinates determining the dipole direction. Notation $\delta(\cdot)$ stands for the delta function.

In the case of isotropic volumes, the conductivity $\Sigma(x,y,z)$ is scalar-valued, while in anisotropic case, it becomes a tensor taking the following form:

$$\boldsymbol{\Sigma}_h^{(j)} = \boldsymbol{T}^{(j)\top}\boldsymbol{\Sigma}_s^{(j)}\boldsymbol{T}^{(j)} \tag{2}$$

where $\boldsymbol{\Sigma}_h^j$ is the conductivity head matrix defined in the uniform Cartesian coordinate system at the element j; $\boldsymbol{T}\in\mathbb{R}^{3\times 3}$ is the orthogonal matrix of unit length eigenvectors that is a rotation transfer matrix from the local to the global coordinate system; $\boldsymbol{\Sigma}_s^{(j)}=\mathrm{diag}(\sigma_{rad}^{(j)}, \sigma_{tan}^{(j)}, \sigma_{tan}^{(j)})$ is a diagonal matrix holding the local conductivity values in the tangential, $\sigma_{tan}^{(j)}$, and radial directions, $\sigma_{rad}^{(j)}$, respectively.

It must be noted that for modelling the anisotropic conductivity of the skull and white matter, we calculate normal vectors to the sphere reconstruction at every spatial point representing the values of the local tangential, $\sigma_{tan}^{(j)}$, and radial conductivity, $\sigma_{rad}^{(j)}$.

Additionally, for modeling anisotropic white matter conductivity, we also use the volume constrain [10]:

$$\sigma_{iso}^3 = \sigma_{rad}(\sigma_{tan})^2 \tag{3}$$

where σ_{iso} is the isotropic conductivity of the white matter.

B. Forward Solution

For the numeric case, Eq. (1) is solved using the anisotropic finite difference methodology in a 18-neighborhood representation, as proposed in [6]:

$$\sum_{i=1}^{18} a_i\boldsymbol{\phi}_i - \left(\sum_{i=1}^{18} a_i\right)\boldsymbol{\phi}_0 = I\delta(\boldsymbol{r}-\boldsymbol{r}_1) - I\delta(\boldsymbol{r}-\boldsymbol{r}_2) \tag{4}$$

where the $a_i\in\mathbb{R}$ coefficients holds the conductivity values and ensure the Dirichlet and Neumann boundary conditions [8], $\boldsymbol{\phi}_i\in\mathbb{R}^{1\times N_Z}$ is each discrete potential, being N_Z the non-zero voxels where head tissues are present, $\boldsymbol{\phi}_0$ is the potential in the neighborhood origin.

Generally speaking, Eq. (4) results in a linear system $\boldsymbol{A\phi}=\boldsymbol{I}$ with unknown terms, $\boldsymbol{\phi}$, that is solved using *successive over relaxation*. However, its implementation requires a high computational burden. Therefore, precalculated reciprocity potentials are employed to speed up the computation of the inverse solution.

C. EEG dipole source estimation

Within the inverse problem framework, we estimate the pairwise dipole parameters $(\boldsymbol{r},\boldsymbol{d})$ by calculating the best electrode potentials, in terms of the lowest relative residual energy, $e\in\mathbb{R}^+$, that we minimize as follows [6]:

$$e = \frac{\|\boldsymbol{v}_e(\boldsymbol{r},\boldsymbol{d}) - \boldsymbol{v}_m(\boldsymbol{r},\boldsymbol{d})\|_2^2}{\|\boldsymbol{v}_e(\boldsymbol{r},\boldsymbol{d})\|_2^2} + c(\boldsymbol{r}) \tag{5}$$

where the values $\boldsymbol{v}_e\in\mathbb{R}^{N_d\times 1}$ are the vector of electrode potentials of the analytical reference model; $\boldsymbol{v}_m\in\mathbb{R}^{N_d\times 1}$ are the electrode potential vector estimated by the numerical test models, being N_d the number of considered dipoles; and the term $c(\boldsymbol{r})\in\mathbb{R}^+$ is a penalization parameter that is set to zero for dipole positions inside the gray matter, otherwise they are very large. Notation $\|\cdot\|_2$ stands for the Euclidean norm.

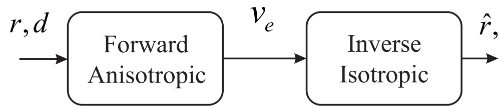

Fig. 1. Spherical head model and layer configuration used during testing.

As seen in Fig. 1 that shows the procedure that includes both the *reference* and *test* models to estimate the dipole error, we initially compute the electrode potentials $\boldsymbol{v}_e$ and then the dipole parameters, $(\hat{\boldsymbol{r}},\hat{\boldsymbol{d}})$. Namely, we introduce the following two error measures:

– the dipole localization error (DLE),

$$\varepsilon_L = \|\hat{\boldsymbol{r}} - \boldsymbol{r}\|_2$$

– the dipole orientation error (DOE),

$$\varepsilon_O = \arccos\left(\frac{\hat{\boldsymbol{d}}^\top\boldsymbol{d}}{\|\boldsymbol{d}\|_2\,\|\hat{\boldsymbol{d}}\|_2}\right)$$

III. Experimental Set-Up

We test the proposed 3-layer-model of skull conductivity within the inverse problem framework that is above explained, also including the white matter conductivity model. The 3-layer-model model is compared against both, the isotropic and anisotropic, conductivity models of Skull and white matter tissues. Therefore, each tested head model holds, at least, five different tissue layers (scalp, skull, gray matter, white matter, and thalamic inner sphere), as shown in the Fig. 2 displaying the concrete spherical disposition used in this work.

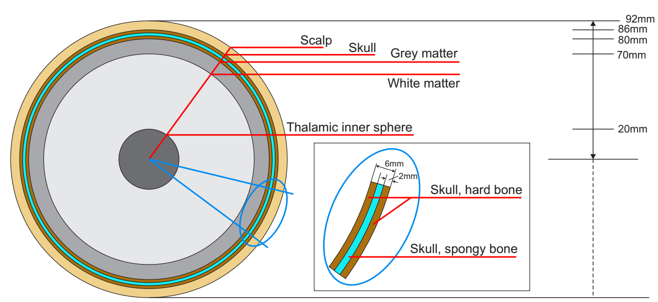

Fig. 2. Spherical head model and layer configuration used during testing.

Therefore, we compare the five spherical head models shown in Table I where the proposed 3-layer-model of skull conductivity are marked in bold. All tested models are generated using the numerical approximation AFDRM method assuming the following set-up values: a 1-*mm*-voxel size resulting in a $186\times 186\times 186$ data set, the anisotropic ratio in the skull is fixed as $1:1.82$ ((radial: tangential), as used in [7]). Besides, we assume during solution the volume constraint, as defined in [10].

Table II shows all considered values of tissue conductivity as well as the assumed anisotropic ratio (radial:tangential),

TABLE I

HEAD MODELS INCLUDING CONDUCTIVITY REPRESENTATION OF THE WHITE MATTER AND SKULL

Model	*White Matter*	*Skull*
A [6]	Isotropic	Isotropic
B [6]		Anisotropic
C		**3-layer-Isotropic**
D [6]	Anisotropic	Anisotropic
E		**3-layer-Isotropic**

as suggested in [7]. Testing is carried out using the EEG $10-20$ system (i.e., 19 electrodes and 18 leadpairs) in the above explained reciprocity approach providing 3262312 non-zero potentials and $1\,mm$ voxel size. For every single leadpair calculation, the AFDRM algorithm lasts about 40 minutes using the `Intel core i7` processor with 8Gb RAM (not mentioning that the solution must be calculated for every leadpair). In turn, to implement the inverse solution, we assume a set of 6000 dipole sources where the distance between the test dipoles is $5\,mm$. Testing is carried out in three different dipole orientations (x, y, and z), resulting in 18000 calculations.

To get a better idea about the feasibility of the proposed approach, we employ the EEG *10-10* system with 30 electrodes and 29 lead pairs, but just for the *C* and *D* models, as the most complex ones.

TABLE II

USED TISSUE VALUES FOR CONDUCTIVITY AND ANISOTROPIC RATIO. RATIO VALUE 1:1 IMPLIES ISOTROPIC TISSUE

Tissue	*Conductivity* [*S/m*]	*Anisotropic Ratio*
Scalp	0.33	1:1
Skull (one-layer)	0.02	1:1.82
Hard bone	0.0064	1:1
Spongy bone	0.02865	1:1
Grey matter	0.33	1:1
White matter	0.14	9:1
Thalamic area	0.33	1:1

IV. RESULTS AND DISCUSSION

Performed values of dipole localization error (*DLE*) are shown in Fig. 3, where each row represents every considered simulation model, while the columns stand for the dipole orientation. The views are the axial cuts of the spherical models and the dots display the 19 electrodes projected on each actual cut. Table III summarizes the computed mean and standard deviation of the *DLE* and *DOE* values estimated for the gray matter (GM) and the thalamic inner sphere (TL) (models including the proposed 3-layer-isotropic representation are marked in bold). As a result, both models *D* and *E* reach the smallest values of *DLE* and *DOE*. Namely, the *E* model has a maximum *DLE* of $5.74\,mm$ in the gray matter and a maximum *DOE* of $19.14\,deg$ in the thalamic inner sphere. Also, the *model D* has a maximum *DLE* of $7.97\,mm$ in the gray matter and a maximum *DOE* of $18.62\,deg$ in the gray matter, as shown in Fig. 4.

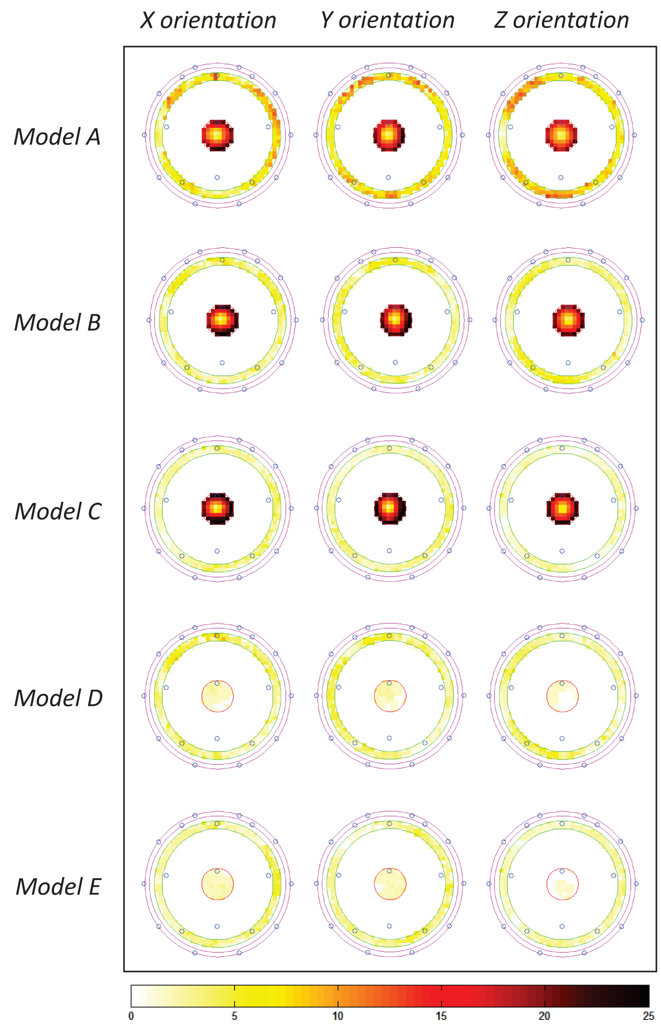

Fig. 3. Spatially distributed *DLE* values for all considered simulation models, estimated for *10-20* EEG system and 19 electrodes.

TABLE III

ESTIMATED LOCALIZATION AND ORIENTATION ERROR VALUES, $\varepsilon_L, \varepsilon_O$, IN THE GRAY MATTER AND THE THALAMIC INNER SPHERE.

model	*GM* [*mm*]	*GM* [*deg*]	*TL* [*mm*]	*TL* [*deg*]
A	6.52 ± 2.83	9.18 ± 7.95	14.85 ± 5.03	2.76 ± 2.97
B	3.36 ± 1.63	5.24 ± 5.98	16.66 ± 5.60	2.81 ± 2.91
C	3.17 ± 1.72	3.94 ± 3.96	18.09 ± 6.06	2.84 ± 3.05
D	2.96 ± 1.58	3.80 ± 4.01	1.34 ± 0.82	1.47 ± 0.79
E	2.41 ± 1.20	3.42 ± 3.44	1.37 ± 0.62	1.70 ± 1.73

Therefore, based on the obtained results shown in Fig. 4 and Table III, we select the models *E* and *D* as having the best skull conductivity representation for further testing. Particularly, we test both models on the EEG $10-10$ system with 30 electrodes. Table IV shows that the *D* model reaches significant diminution of *DLE* value, while *DOE* value gets a bit better for both *D* and *E* models. As seen in Fig. IV and Fig. 3 showing performed *DLE* values for 30 and 19 electrodes, respectively, we can infer that adding more electrodes allows reducing localization error.

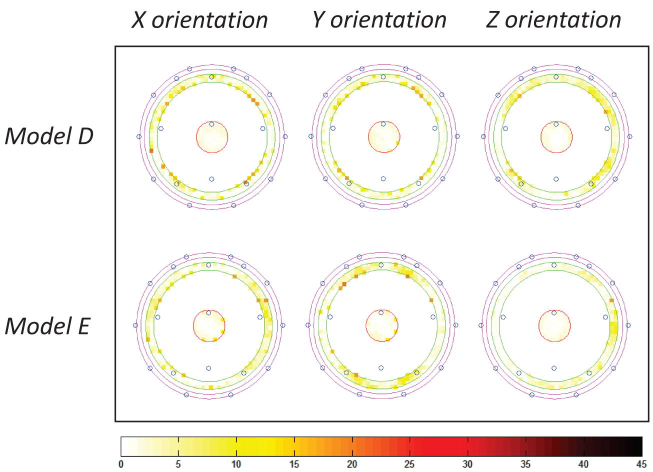

Fig. 4. Performed values of *DOE* computed for the *10-20* EEG system and 19 electrodes.

model	GM [*mm*]	GM [*deg*]	TL [*mm*]	TL [*deg*]
D	1.79 ± 1.16	3.12 ± 3.37	0.88 ± 0.70	1.58 ± 1.46
E	2.39 ± 1.27	3.06 ± 3.71	0.85 ± 0.84	1.45 ± 0.68

TABLE IV
SUMMARIZED VALUES OF *DLE* AND *DOE* FOR *10-10* EEG SYSTEM AND 30 ELECTRODES

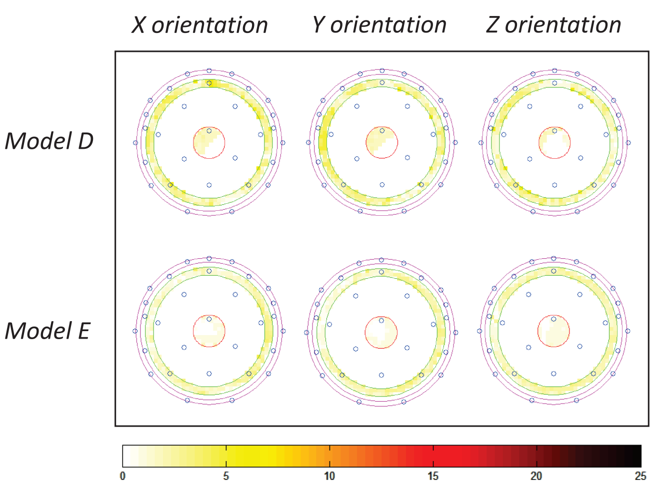

Fig. 5. Performed *DLE* values computed for the *10-10* EEG system and 30 electrodes.

V. CONCLUSIONS

To deal with anisotropic Skull Conductivity, we develop a 3-layer Isotropic tissue representation within the EEG Forward Problem framework using Spherical Head Models. We carry out comparison, in terms of the dipole localization and orientation errors, with other baseline Skull Conductivity models using the numerical approximation AFDRM method. Particularly, we propose two head models assuming either isotropic (model *C*) or anisotropic (model *E*) conductivity of the white matter. Obtained results on simulated EEG data show that deep sources placed in the thalamic inner sphere have very large DLE and DOE errors (as much as 26 *mm*) using the former model pointing out that the white matter anisotropy should be strongly considered.

In contrast, the latter model turns to be a suitable conductivity representation that performs the lowest error values that are close to the baseline E model. However, the proposed threes-layer-isotropic model requires an additional image segmentation step for realistic, patient dependent head models that is far from being an easy task [7].

Another finding through this work is that adding more electrodes (and lead pairs calculations) allows considerably reducing the dipole localization/orientation error, but it implies calculation of more lead-pairs, which increases the computational burden of the precalculated potentials in the reciprocal approach.

As a future research, we plan to analyze the EEG source localization errors in realistic head models using state of the art inverse solution such as *multiple sparse priors* approach [4] employing the skull conductivity models of this work. We also want to analyze different anisotropic ratios for the skull and the white matter in order to find the best possible head model.

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