Anisotropic Anomalous Diffusion Filtering Applied to Relaxation Time Estimation in Magnetic Resonance Imaging

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***Abstract*— Relaxometry mapping is a quantitative modality in magnetic resonance imaging (MRI) widely used in neuroscirence studies. Despite its relevance and utility, voxel measurement of relaxation time in relaxometry MRI is compromised by noise that is inherent to MRI modality and acquisition hardware. In order to enhance signal to noise ratio (SNR) and quality of relaxometry mapping we propose application of anisotropic anomalous diffusion (AAD) filter that is consistent with inhomogeneous complex media. Here we evaluated AAD filter in comparison to two usual spatial filters: Gaussian and non local means (NLM) filters applied to real and simulated T2 relaxometry image sequences. The results demonstrate that AAD filter is comparatively more efficient in noise reducing and maintaining the image structural edges. AAD shows to be a robust and reliable spatial filter for brain image relaxometry.**

I. INTRODUCTION

Magnetic resonance imaging provides several useful tools for studying the human body and has been of great importance to medicine. Among the different imaging techniques, the relaxometry technique has demonstrated to be a key factor for the study of iron distribution in the brain in some brain diseases such as Multiple Sclerosis [1], Parkinson's disease [2], and brain tumors [3]. However, limitations of this image technique, such as noise and long acquisition time, provide some barriers to increasing the precision.

Some approaches have been applied to the relaxometry imaging protocol using several different image enhancement techniques. Recently, some research has been conducted with different approaches to improve relaxometry precision that involves mathematical fitting [4] and spatial filtering such as Non Local Means (NLM) and Gaussian classical filtering [5]. These methods show particular solutions to enhance relaxometry images, and in particular the use of the NLM algorithm has been demonstrated to be more suitable than the classical Gaussian filtering method [5].

The use of diffusion equations to filter digital images is well know and applied in several studies with biomedical images [6]. In summary, the diffusion equation, with a classical approach, could be solved with isotropic assumption (classical Gaussian filtering) and anisotropic assumption [6]. These two types of filters have been a wide application in several biomedical image techniques. However, in recent studies it has been shown that AAD filtering (resulting by porous media equation [7]) has a better filtering performance than the classical diffusion approach when it is applied to MRI brain imaging [8].

Others facts that support the use of anomalous diffusion filtering could be provided by the natural complex structure found in the human body. In the literature research is found that supports the anomalous behavior in the MRI signal decay [9] and other anomalous highlights involved to brain images [10]. Furthermore, the AAD filter demonstrates potential to enhance MRI diffusion weighted image quality [8], and this applicability is hereby extended to relaxometry images.

Here we will study three methods for filtering images of MRI relaxometry aiming to reduce the noise and improve the accuracy of the estimate of T2 relaxation times. It will be used simulated and real images with controlled noise intensity in order to estimate the optimal parameters for each filtering approach.

II. MATERIALS AND METHODS

A. Anisotropic Anomalous Diffusion Filter

The AAD filter is know as a generalization of the classical diffusion process, in this case the anisotropic diffusion filter [6]. The numerical algorithm is shown in Equation (1) and it is basically the classical approach with some differences. The Equation (1) simulates the diffusion process in each site in the image in discrete form, regulated by the anomalous parameters such as the q value (a parametric curve adjust) and the diffusion coefficient (D_q).

$$I_{t+1,\beta} = I_{t,\beta} + \left[D(\overrightarrow{r})_q . \overrightarrow{\nabla} I^{2-q}_{t,\beta-1} + D(\overrightarrow{r})_q . \overrightarrow{\nabla} I^{2-q}_{t,\beta+1} \right] \quad (1)$$

For simplicity, $I^{2-q}_{t,\beta}$ represents the image at time t according to locating a 3×3 neighborhood of the center pixel for a defined anomalous parameter q. The $D(\overrightarrow{r})_q$ parameter add to the edge detection [6] function is used to regulate the iterative diffusion process and the discrete formulation is similar to the classical anisotropic approach, i.e. $D(\overrightarrow{r})_q = D_q . exp\left[\frac{|\overrightarrow{\nabla} I(\overrightarrow{r},t)|^2}{\kappa^2} \right]$. The parameter β informs the spatial position of the neighbor relative to the central pixel and with this orientation the central pixel has a weighted value based on the anomalous solution generated by Equation (1).

There are two main points to discuss about these differences: the anomalous *q-distribution* and the diffusion coefficient (D_q). What we call the *q-distribution* is in fact the

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probability distribution function that represents the solution of the Equation (1). The q-distribution in this case could be called the q-Gaussian probability distribution and it is well know in physics simulation problems [7]. The diffusion coefficient, D_q, is a parameter that regulate the diffusion intensity in determined site in the image and it is represented by Equation (2). It is close to the relationship between D_q and the q parameter, and both create a specific q-Gaussian distribution in each neighborhood on the image.

$$D_q = \begin{cases} \frac{\alpha}{2}.D.\left(\sqrt{\frac{(q-1)}{\pi}}.\frac{\Gamma(\frac{1}{q-1})}{\Gamma(\frac{1}{q-1}-\frac{1}{2})}\right)^{\frac{2-2q}{3-q}} & , 1 < q < 2 \\ \frac{\alpha}{2}.D & , q = 1 \\ \frac{\alpha}{2}.D.\left(\sqrt{\frac{(1-q)}{\pi}}.\frac{\Gamma(1+\frac{1}{1-q})}{\Gamma(\frac{3}{2}+\frac{1}{1-q})}\right)^{\frac{2-2q}{3-q}} & , q < 1 \end{cases} \quad (2)$$

Where $\alpha = (2-q)(3-q)]^{2/(3-q)}$ and $0 < q < 2$ are the range for the q parameter for numerical stability in Equation (1). In summary, the anomalous diffusion generates another probability distribution function that it is suppose to be more suitable for image with complex features.

As seen in Equation (1) we can define the two parameters responsible for the filtering intensity: the diffusion coefficient (D_q) and the number of iterations (t). However, we assume the number of iterations as the intensity variable and we fixed D_q because of the q value dependence, i.e. depending on the q value the parameter D_q change its range as show in Equation (2). The t parameter can be adjusted in comparison with the variance (σ^2) [7], with a relationship given by $\sigma = \sqrt{2.D_q.t^{2/(3-q)}}$. This variance dependence with time is well know as the generalized Einstein equation when the anomalous distributions and it describes the smoothing parameter behavior with the AAD filter.

B. Images

In order to study the behavior of the AAD filter we proposed, in a first approach, a simulated image generated by Matlab software with a 256×256 matrix size. A relaxometry sequence was generated with this simulated image using a mean value found in different brain tissues such as: White matter (WM), gray matter (GM), cerebro-spinal fluid (CSF) and in the globus pallidus region (GP). These values were assumed as the real value that should be measured in the relaxometry exam and represents the values in a healthy brain. Eleven time echos (TE) were generated to create the simulated relaxometry, and these values are usually used in real protocol relaxometry exams (TE = 24, 36, 48, 60, 72, 84, 96, 108, 120, 132 and 144 ms). In each image in the simulated relaxometry we added three different noise intensities related with the signal to noise ratio (SNR). The intensities were set with a SNR = 15, 30 and 60, respectively [5]. The Gaussian white noise was the only noise distribution used in this study.

C. Quality Metrics and Pixel-by-pixel relaxation estimation

Our approach to determine the $T2$ relaxometry from the data acquired was followed with a mono exponential adjustment as seen in Equation (3). This mono exponential fitting approach is a common assumption in several studies with relaxometry signals [11].

$$S(t) = S_0.e^{-\frac{t}{T2}} + C \quad (3)$$

Where $S(t)$ and S_0 are the pixel intensity values acquired at time t and $t = 0$, respectively. The $T2$ value is the relaxation constant that are tissue characteristics and the constant C is an offset parameter that is related with the noise and the acquisition system of the experiment.

The filtering analysis was followed with the root mean square error (RMSE) as seen in Equation (4). Each pixel within all of the four specific regions of interest (ROI), in the simulated and real image, was estimated by the curve fitting previously described.

$$RMSE = \sqrt{\frac{1}{m \times n}\sum_{i,j}|I_o(i,j) - I_f(i,j)|^2} \quad (4)$$

Where $m \times n$ are the total number of pixels in the image, $I_o(i,j)$ and $I_f(i,j)$ are the original and filtered pixel values at point (i,j), respectively. The RMSE is a cumulative response that represents the filtered pixel value recovered in comparison with the original image.

Two other filters were used in this study for comparison: Non local mean filter (NLM) and the Gaussian filter. Both approaches are well know in image processing and enhancement research and they are already applied to relaxometry problems [5].

III. RESULTS AND DISCUSSION

A. Filtering Parameter Set

Firstly, the number of iterations (t) was selected in order to determine the best filtering result with the simulated image

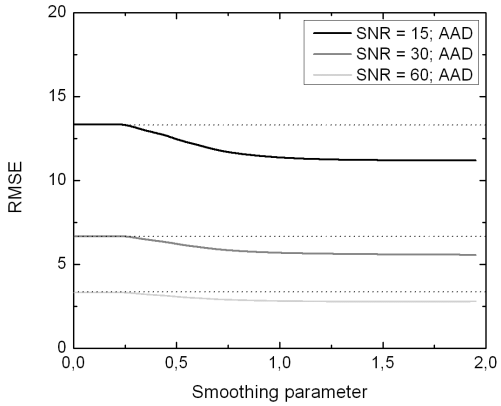

Fig. 1. RMSE results with different smoothing parameter values, that it is related with σ and the number of iteration, t. The smoothing parameter has a relationship with the number of iterations of AAD filter and it is related with the generalized Einstein equation as seen in the section II-A. The dotted line represents the RMSE value found with the noisy image before using the filters.

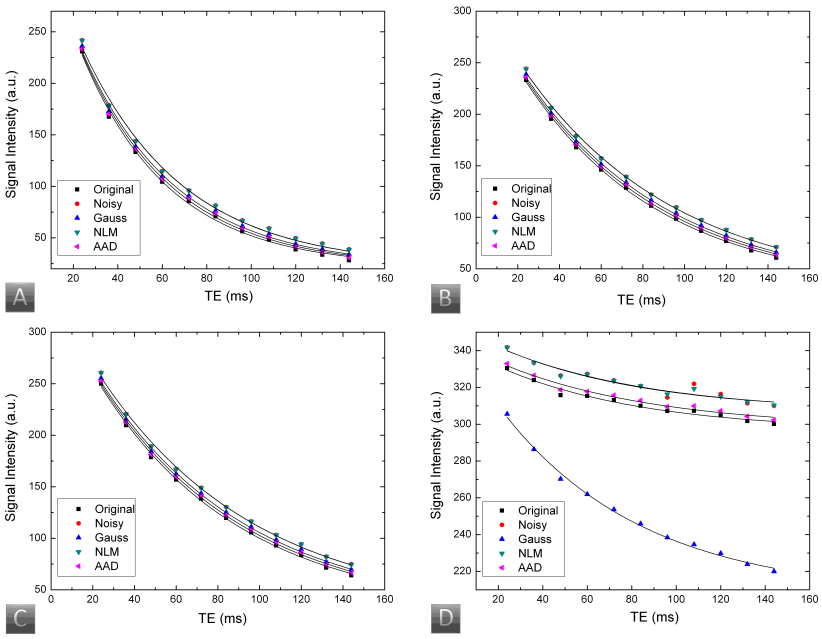

Fig. 2. Exponential fitting in order to determine the relaxometry constant T2 for each brain region. A) Globus Pallidus, B) White Matter, C) Gray Metter and D) cerebro-spinal fluid. The AAD filter showed a better image enhancement that resulted in more acuracy with the original image pixel values.

described in the section II-B. See Figure 1 that shows the best σ for each noise intensity chosen. The σ values chosen to filter each image with a determined noise add was: $\sigma_{15} = 1.5$, $\sigma_{30} = 1.0$ and $\sigma_{60} = 1.0$ respectively. For the AAD filter we used the q value setup as $q = 1.3$ which it is a optimized value for MRI images [8]. The filtering parameters values for NLM ($h = 1.5\sigma$) and Gaussian ($\sigma = 1.0$) filter were selected based on the literature [5].

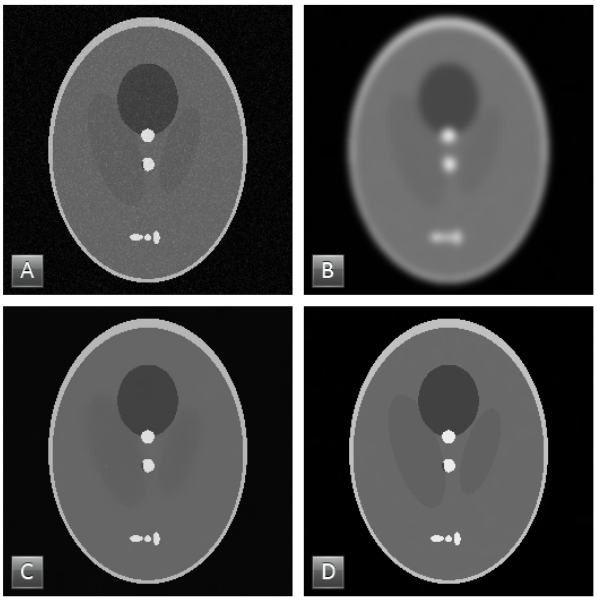

Fig. 3. Filter applied on the simulated T2 relaxometry image. A) Original noisy image with SNR = 15 noise intensity, B) Gaussian filter, C) NLM filter and D) AAD filter. Note the edge preservation resulting when the AAD filtering is applied.

B. Simulated Relaxometry

Images with additive noise were filtered with AAD, NLM and Gaussian filters. Figure 3 illustrates some filtered image examples. Almost all regions show a good noise attenuation. The only region that shows a non-robust denoising effect was the CSF region. The reason for not obtained a better smoothing response with in CSF region is mainly due to the small pixel quantity that is related with this tissue type in the simulated image. Only the small circles were selected with the CSF relaxometry values and it led to imprecision in the relaxometry estimation for the CSF area.

The relaxometry estimations were calculated by the exponential fitting as Equation (3). The results of the T2 numerical estimation found in each brain region is show in Table I and the fitting relaxometry curves are shown in Figure 2. AAD shows to be more reliable when compared to Gaussian and NLM filters by reducing noise without displacing exponential curve from original one, as can be observed in Figure 2.

TABLE I

T2 RELAXATION CONSTANT MEASURED IN EACH BRAIN TISSUE FOR BOTH SIMULATED AND REAL IMAGES.

Tissue	Original	Gaussian	NLM	AAD
GP_s	36.6 ±0.9	42.4 ±0.9	45.3 ±1.2	41.0 ±0.9
WM_s	65.6 ±0.7	69.2 ±0.7	73.0 ±0.9	67.5 ±0.7
GM_s	68.3 ±0.9	71.8 ±0.9	75.0 ±1.1	70.0 ±0.9
CSF_s	65.4 ±15.9	63.9 ±5.6	65.8 ±21.1	69.9 ±17.8
GP_r	51.5 ±0.5	49.7 ±1.4	48.0 ± 1.3	48.1 ±1.3
WM_r	72.6 ±0.7	69.2 ±1.1	69.8 ±1.2	70.0 ±1.3
GM_r	82.6 ±0.7	76.9 ±1.3	79.4 ±1.4	79.5 ±1.4
CSF_r	130.3 ±8.5	80.1 ±4.3	115.3 ±9.5	115.1 ±9.1

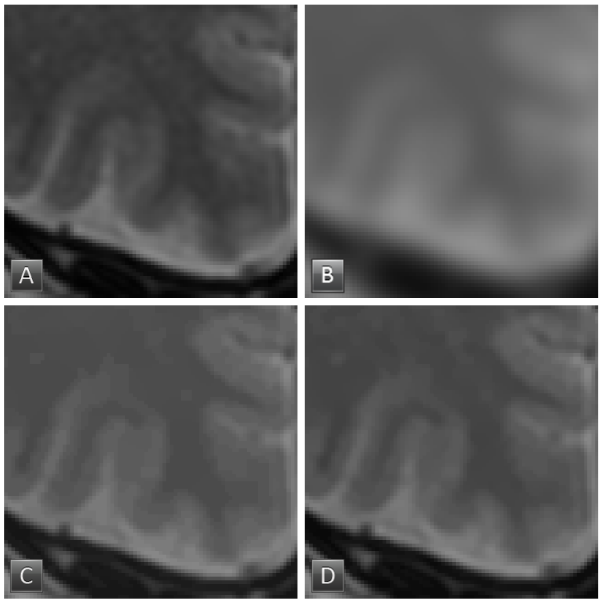

Fig. 4. Filter applied on the real T2 relaxometry image. All images were amplified at the frontal lobe cortex in order to see the local smoothing. A) Original image, B) Gaussian filter, C) NLM filter and D) AAD filter. The edge preservation is well performed with the AAD filtering.

Table I shows both the simulated and real T2 relaxometry estimation. We only illustrated the values found with SNR=15, the other noise intensities have the same response behavior.

C. Real Relaxometry

This study was performed with a real T2 relaxometry image and the results found with the real data are show in Table I. The results for all filters studied here can be seen in Figure 4. We can note that in addition to noise reduction, edges was also preserved when the AAD filter was applied. On the other hand, for NLM filter, the T2 estimation did not present the decrease in standard error as did the AAD filter.

IV. CONCLUSIONS

The non homogeneous media provide a spatial complexity that the anomalous diffusion (AAD) filter show better smoothing efficiency. In comparison with the non local means (NLM) and Gaussian filters, the AAD approach demonstrated local filtering preservation suitable for further processing such as brain tissue segmentation. The NLM filter has a good edge preservation performance, however it does not preserve, as seen with the AAD filter, the natural inhomogeneity observed in the human brain tissues as seen in the white and gray matter.

The AAD filter provides a robust and stable solution when compared with the other usual spatial filters. The use of anomalous distribution demonstrate to be more suitable for relaxometry estimation even with intense noise is applied. The T2 estimation enhancement by the AAD filter application could be applied to many applications of relaxometry in biomedical imaging and this should improve several medical diagnostic and neuroscience studies.

This study allows us to state toward the superiority of AAD in improve relaxometric measurements in MRI relaxometry images. The results with real images are promising, but a more extensive study will be necessary in order to fully characterize the AAD performance in relaxometry mapping enhancement. In a future study this approach to real MRI imagens protocols will be applied in order to investigate the AAD smoothing in a real situation.

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