Optimising the Windkessel Model for Cardiac Output Monitoring During Changes in Vascular Tone*

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***Abstract*— Algorithms for estimating cardiac output (CO) from the arterial blood pressure wave have been observed to be inaccurate during changes in vascular tone. Many such algorithms are based on the Windkessel model of the circulation. We investigated the optimal analytical approaches and assumptions that make up each algorithm during changes in vascular tone. Several analytical approaches and assumptions were evaluated on data from 15 critically ill patients by comparison with thermodilution measurements. We found that the most accurate algorithms assumed a constant compliance for the duration of the beat. They produced a percentage error of ±31% by maintaining the compliance and outflow terms in the Windkessel model. For any algorithm, the following assumptions gave highest accuracy: (i) outflow pressure into the microcirculation is zero; (ii) end of systole is identified using the second derivative of pressure. None of the tested algorithms reached the clinically acceptable accuracy of ±30%.**

I. INTRODUCTION

Cardiac output (CO) monitoring is used to assess the haemodynamic state of peri-operative and critically ill patients. It is a key indicator of oxygen delivery, and can guide fluid management and vasoactive drug use. Methods of measuring CO fall into two categories: intermittent, and continuous. Intermittent methods are usually more accurate than continuous methods but are often too invasive to perform at a high enough frequency to guide therapies. Therefore, in high-paced settings such as peri-operative care, a less accurate continuous method is commonly used to track changes in CO in real time. However, we have previously observed that continuous methods fail to track CO accurately during changes in vascular tone [1], which is often modified by fluid administration and vasoactive drugs.

Many devices estimate CO from the arterial blood pressure (ABP) wave using the Windkessel model of the circulation [2], producing a value of CO which is proportional to the true value. The constant of proportionality required to obtain an accurate value may be found by calibration with an intermittent measurement. Our observation of inaccuracies during changes in vascular tone after calibration suggests that the implementations of the Windkessel model are compromised by changes in tone. We sought to identify optimal analytical approaches and assumptions for CO monitoring using the Windkessel model during a change in vascular tone.

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II. THE WINDKESSEL MODEL

The two-element Windkessel model of the circulation is illustrated in Fig. 1. Conservation of mass allows us to write the time-dependent inflow, $Q_{\mathrm{in}}(t)$, as

$$Q_{\mathrm{in}} = Q_{\mathrm{dist}} + Q_{\mathrm{out}} \quad , \tag{1}$$

where $Q_{\mathrm{dist}} = \frac{\mathrm{d}V}{\mathrm{d}P}\frac{\mathrm{d}P}{\mathrm{d}t}$ is the distending flow, $\frac{\mathrm{d}V}{\mathrm{d}P} = C(P)$ is the compliance of the systemic circulation, and Q_{out} is the outflow which is modelled using Pouiseille's law: $Q_{\mathrm{out}} = \frac{1}{R}\left[P - P_{\mathrm{out}}\right]$. Substituting into (1) gives

$$Q_{\mathrm{in}} = C(P)\frac{\mathrm{d}P}{\mathrm{d}t} + \frac{P - P_{\mathrm{out}}}{R} \quad . \tag{2}$$

The inflow blood volume during one beat, $V_{\mathrm{in}}(t)$, or *stroke volume*, can be calculated as

$$V_{\mathrm{in}} = \int_{\mathrm{beat}} Q_{\mathrm{in}}\mathrm{d}t \tag{3}$$

$$= \int_{P(t_0)}^{P(t_{\mathrm{d}})} C(P)\mathrm{d}P + \frac{1}{R}\int_{t_0}^{t_{\mathrm{d}}} \left[P - P_{\mathrm{out}}\right]\mathrm{d}t \quad , \tag{4}$$

where t_0 and t_{d} are the times of onset and end of the beat, respectively. Equation 4 consists of a compliance and an outflow term. Beat-by-beat CO, $\mathrm{CO}_{\mathrm{est}} = V_{\mathrm{in}} \cdot$ HR, where $\mathrm{HR} = 1/\left[t_{\mathrm{d}} - t_0\right]$ is the heart rate.

III. ANALYTICAL APPROACHES AND ASSUMPTIONS

Several analytical approaches have been used to estimate V_{in} using the Windkessel model. Each approach simplifies (4) so that V_{in} is assumed to be proportional to a single

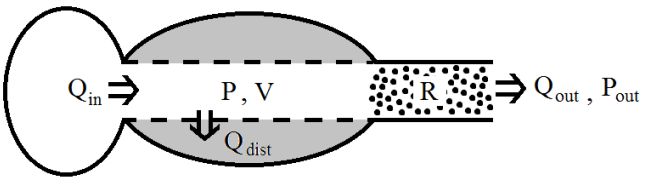

Fig. 1. The Windkessel model of the circulation. The heart is modelled as a flow source, $Q_{\mathrm{in}}(t)$, where t is time. The large arteries are modelled as an elastic tube containing a volume of blood, $V(t)$, at pressure $P(t)$, and the peripheral arteries as a constant resistance to flow, R, with an outflow pressure into the microcirculation, P_{out}, which can be time-dependent. A positive flow Q_{dist} distends the tube.

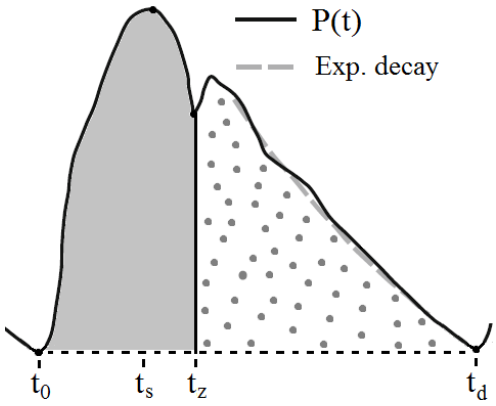

Fig. 2. Nomenclature used to describe the arterial blood pressure wave (modified from [3]).

TABLE I
ASSUMPTIONS TAKEN FOR THE WINDKESSEL MODEL

Variable	Expression	Ref	Abbr.
Resistance, R_i	k	[2]	R1
	0	[4]	OP1
Outflow	$P(t_0) - \frac{t-t_0}{t_d-t_s}\left[P(t_s) - P(t_d)\right]$	[5]	OP2
Pressure, P_{out}	$P(t_0)$	[6]	OP3
	$P(t_0) - e^{\frac{t-t_0}{\tau}}$	[6]	OP4
	k	[7]	C1
Compliance,	$\frac{k}{P(t_s)+P(t_d)}$	[8]	C2
$C(P)$	$\frac{k}{P(t_s)+2P(t_d)}$	[9]	C3
	$\frac{k}{\int_{t_0}^{t_d}[P(t)-P_{out}(t)]dt}$	[7]	C4
End of systole,	$0.3\sqrt{t_d - t_0}$	[10]	Z1
t_z	First zero-slope	[7]	Z2
	Zero-crossing 2nd derivative	[11]	Z3
Time constant,	$P(t_d) = P(t_s)e^{-\frac{t_d-t_0}{\tau}}$	[7]	D1
$\tau = RC$	$P(t_d) = P(t_s)e^{-\frac{t_d-t_s}{\tau}}$	[12]	D2

term which is a function of the ABP wave, $P(t)$, only. Unknown variables in expressions for V_{in} are evaluated using the assumptions in Table I. This reduces expressions for V_{in} to a function of $P(t)$ and a constant of proportionality, k, which can then be determined by calibration with a reference measurement. Each analytical approach is described below using the nomenclature illustrated in Fig. 2.

A. Elimination of either the compliance or outflow term

If either the compliance or outflow term is eliminated from (4), the expression for V_{in} is reduced to a single term. The compliance term can be eliminated by assuming periodic flow ($P(t_0) = P(t_d)$), giving

$$V_{in} = \frac{1}{R}\int_{t_0}^{t_d}\left[P - P_{out}\right]dt \quad . \tag{5}$$

Similarly, the outflow term can be eliminated by assuming infinite resistance, giving

$$V_{in} = \int_{P(t_0)}^{P(t_d)} C(P)dP \quad . \tag{6}$$

B. Zero flow in diastole and periodic flow

Separating (5) into integrals over systole and diastole, and rearranging, gives

$$V_{in} = \frac{1}{R}\int_{t_z}^{t_d}\left[P(t) - P_{out}\right]dt\left[1 + \frac{\int_{t_0}^{t_z}[P - P_{out}]dt}{\int_{t_z}^{t_d}[P - P_{out}]dt}\right] \quad . \tag{7}$$

If the inflow is assumed to be zero in diastole, then the outflow during diastole is solely due to a transfer of blood from the compliance term to the outflow term. Therefore,

$$\frac{1}{R}\int_{t_z}^{t_d}\left[P - P_{out}\right]dt = -\int_{P(t_z)}^{P(t_d)} C(P)dP \quad .$$

Substituting into (7) gives

$$V_{in} = -\int_{P(t_z)}^{P(t_d)} C(P)dP\left[1 + \frac{\int_{t_0}^{t_z}\left[P - P_{out}\right]dt}{\int_{t_z}^{t_d}\left[P - P_{out}\right]dt}\right] \quad . \tag{8}$$

C. Constant compliance for the duration of the beat

If compliance is assumed to be constant throughout a beat (C), then during diastole P can be modelled as an exponential decay with time constant $\tau = RC$. Equation (4) can then be manipulated to give

$$V_{in} = C\left[\left[P(t_d) - P(t_0)\right] + \frac{1}{\tau}\int_{t_0}^{t_d}\left[P - P_{out}\right]dt\right] \quad , \tag{9}$$

where τ can be calculated using either of assumptions D1-2.

D. Power analysis

In the Windkessel model, power is only dissipated in the resistance R. Therefore, the instantaneous power required to drive the flow can be written in terms of the inflow, Q_{in}, and the pressure gradient across the resistance, $P - P_{out}$, as $Q_{in}^2 R = [P - P_{out}]^2/R$, so that the root-mean-square average power, $\mathcal{P}_{avg}$, is given by

$$\mathcal{P}_{avg} = < Q_{in} > R = \frac{< P - P_{out} >}{R} \quad ,$$

where $< X(t) > = \sqrt{\frac{1}{t_d - t_0}\int_{t_0}^{t_d} X(t)^2 dt}$. Therefore,

$$< Q_{in} > = \frac{1}{R^2} < P - P_{out} > \quad .$$

V_{in} is assumed to be proportional to $< Q_{in} >$. Therefore,

$$V_{in} = \frac{k}{R^2} < P - P_{out} > \quad , \tag{10}$$

where k is a constant. Previously it has been suggested that the relevant power is a function of only the pulsatile component of the wave [7]. This is based on an electrical circuit analogy, in which AC power is a function of the oscillatory component of the signal. This gives

$$V_{in} = \frac{k}{R^2} < P_{pulsatile} > \quad , \tag{11}$$

where $P_{pulsatile} = P - P_{out} - \frac{1}{t_d - t_0}\int_{t_0}^{t_d}\left[P - P_{out}\right]dt$.

IV. METHODS

A. Clinical dataset

The dataset used in this study has been described previously [1] and is only summarised here. ABP signals were acquired from 15 critically ill patients with an age of 54 ± 28 years (med ± iqr), 9 of whom were male. Reference CO measurements, CO_{ref}, were obtained using triplicate transpulmonary thermodilutions. Throughout the recording each patient was receiving continuous infusion of norepinephrine, a vasoactive and inotropic drug which affects vascular tone and cardiac contractility. The dosage of norepinephrine was doubled during the recording for a median of 11 mins (minimum 4 mins). This gave a step-change in cardiovascular properties since the half-life of norepinephrine is 1-2 mins. CO_{ref} measurements were taken before and during the dosage increase (see Table II).

B. Testing algorithms using different assumptions

Algorithms for retrospective calculation of CO_{est} were derived using each expression for V_{in} (5, 6, 8, 9, 10, 11), and each set of compatible assumptions, as listed in Table I. Individual beats were identified from the ABP signal using the algorithm described in [13]. Poor quality beats were identified using the signal abnormality index described in [7], and excluded from the analysis.

To mimic calibration with an intermittent CO measurement at the bedside, each algorithm's CO_{est} values were scaled so that the mean CO_{est} during the period of CO_{ref} measurement before the dosage increase was equal to that CO_{ref} (as described in [7]). The precision of each algorithm during a change in vascular tone was assessed by comparing the CO_{ref} measurement with the mean CO_{est} evaluated during the period of CO_{ref} measurement. The periods of CO_{ref} measurement were long enough to ensure that mean CO_{est} values during these periods were robust to interbeat fluctuations in V_{in}.

A range of statistical parameters suggested in [14], [15] was used: coefficient of determination (R^2), root mean square error (RMSE, l min^{-1}), mean difference (bias, l min^{-1}), limits of agreement (LOA, l min^{-1}), and percentage error (PE, %). A clinically-acceptable algorithm may have a PE of up to ±30% [14].

V. RESULTS

The results for the best-performing algorithms based on each expression for V_{in} are shown in Table III. Each algorithm is described as its equation and assumptions from Table I. The most accurate algorithms used (8) or (9). Exemplary time profiles of CO_{est} are shown in Fig. 3.

Table IV gives the PE found using each expression for V_{in} with all possible assumptions as listed in Table III. Of all the outflow pressure assumptions, OP1 gave the highest accuracy. Assumptions for compliance gave similar accuracy, apart from C1. Using Z3 to identify the end of systole gave higher accuracy than other methods. The method for estimation of the diastolic decay constant (D1 or D2) had little effect.

TABLE II
INTERVENTION CHARACTERISTICS

Characteristic, med ± iqr	Before	During
CO_{ref} (l min^{-1})	6.6 ± 2.5	7.4 ± 3
CO_{ref} measurement duration (s)	195 ± 75	188 ± 132

TABLE III
BEST PERFORMING ALGORITHMS

Algorithm	R^2	RMSE	Bias [LOA]	PE
(9), OP1, C3, D1	0.62	1.0	0.1 [-2.1 to 2.2]	31
(8), C4, OP2, Z3	0.62	1.1	-0.2 [-2.3 to 2.0]	31
(5), R1, OP1	0.49	1.9	1.4 [-1.2 to 3.9]	37
(10), R1, OP4, D1	0.48	2.3	1.8 [-1.2 to 4.7]	43
(11), R1, OP1	0.43	2.4	1.9 [-1.2 to 4.9]	44
(6), C4	0.56	3.6	-1.4 [-8.1 to 5.4]	97

VI. DISCUSSION

We have systematically tested the accuracy of several algorithms for estimating CO from the ABP wave based on the Windkessel model during a change in vascular tone. The most accurate algorithms, based on (8) or (9), maintain the compliance and outflow terms in the Windkessel model.

The assumption that stroke volume is proportional to the power in the wave is not restricted to the Windkessel model and can be applied to more complex models of the circulation. Despite this, we found that algorithms based on this assumption (10, 11) were less accurate, in line with previous findings in [16]. We found no substantial difference in accuracy when using either the power contained in the entire wave (10), or only the pulsatile component (11).

Elimination of the outflow term by assuming infinite peripheral resistance is highly inconsistent with physiology. Algorithms based on this approach (6) were highly inaccurate. In contrast, algorithms based on elimination of the compliance term (5) by assuming periodic flow performed relatively well. Whilst it has been suggested that these algorithms should only consider the pressure during systole [6], doing so did not improve their performance substantially. Below we discuss the impact on algorithm performance of each method used to evaluate unknown variables.

Detection of the end of systole can be difficult in peripheral waves since the dicrotic notch, indicating aortic valve closure, is often not preserved. Despite this, an algorithm that requires detection of the end of systole, using (8), produced a relatively high accuracy. This shows that CO can be estimated in critically ill patients using this approach. The highest accuracy was achieved using the method suggested in [11], Z3, which identifies the end of systole as the time of the first zero crossing of the second derivative of pressure.

We found no benefit to using a more complex assumption for outflow pressure than that it is constant at 0 mmHg (OP1). In contrast, accuracy was reduced if compliance was assumed to be constant throughout the change in vascular tone (C1).

TABLE IV

PERCENTAGE ERRORS WITH DIFFERENT EXPRESSIONS FOR STROKE VOLUME AND ASSUMPTIONS LISTED IN TABLE I. EACH ASSUMPTION IS VARIED WHILST ALL OTHERS ARE HELD AT OPTIMAL PERFORMANCE.

Expression for	Outflow Pressure				Compliance				End of Systole			Time Constant	
Stroke Volume	OP1	OP2	OP3	OP4	C1	C2	C3	C4	Z1	Z2	Z3	D1	D2
(9)	31	37	37	34	44	31	31	31				31	31
(8)	31	31	35	31	40	32	32	31	46	154	31		
(5)	37	47	46	41									
(10)	44	43	43	43									
(11)	44	46	44	46									
(6)					260	103	98	97					

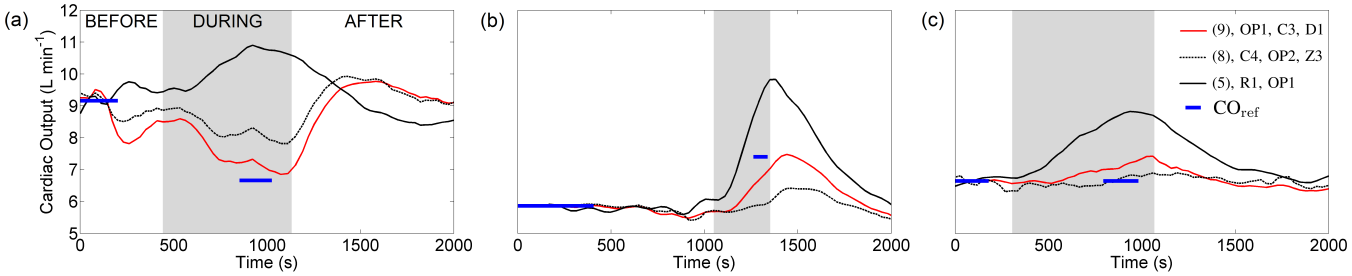

Fig. 3. Examples of reference cardiac output measurements (CO_{ref}) and time profiles of estimates of cardiac output (CO_{est}) before, during and after increase in norepinephrine dosage. Time periods of increased dosage are shaded. Plots of CO_{est} calculated using the three best performing algorithms are shown. In (a), algorithms (8) and (9) trend correctly as CO_{ref} decreases, whereas (5) does not. In (b) the magnitude of change in CO_{est} during dosage increase varies considerably between algorithms. In (c), algorithm (5) calculates an increased CO_{est} whilst CO_{ref} remains relatively constant.

We found that compliance should be re-calculated on at least a beat-by-beat basis, whilst acknowledging that this does not account for intrabeat changes which occur physiologically.

VII. CONCLUSION

The optimal approaches for estimating CO during a change in vascular tone, (8) and (9), were found to: (i) maintain the compliance and outflow terms in the Windkessel model; and (ii) re-calculate compliance on a beat-by-beat basis. Algorithms also had higher accuracy when the end of systole was identified using the zero crossing of the second derivative of pressure, and when outflow pressure was assumed to be zero. The choice of methods for estimating compliance and the systemic time constant had little effect on accuracy.

During this carefully controlled change in vascular tone, none of the considered algorithms tracked CO to within the clinically-acceptable ± 30 % of the reference measure.

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