Identification of Ankle Joint Stiffness from Short Segments of Data: Application to Passive Dynamics during Movement

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***Abstract*— This paper presents a state-space (subspace) method for identification of parallel-cascade joint stiffness from short segments of data. It provides unbiased estimates of stiffness by accounting for the contributions of initial conditions of each segment. The method is important in situations where it is not possible to acquire a long stationary data due to switching or time-varying behavior. The power of the method was demonstrated by using it to efficiently characterize ankle joint stiffness through the joint's range of motion.**

I. INTRODUCTION

Joint stiffness defines the dynamic relation between the position of a joint and the torque acting about it [1]. Joint stiffness plays a critical role in control of posture and locomotion since it determines the load that the central nervous system must control. Consequently, its identification is important and accurate methods to estimate stiffness are required.

Joint stiffness at the ankle has a *parallel-cascade* (PC) structure consisting of two parallel pathways: (i) intrinsic stiffness which is due to mechanical properties of the joint, active muscles and passive tissues, and (ii) reflex stiffness arising from changes in muscle activation due to stretch reflex feedback [1].

The key assumption for identification methods is that the system is *time-invariant* (TI), i.e., the underlying dynamics are the same throughout the data set used for identification. However, this is often not the case for biomedical systems. For example, joint stiffness will change continuously with postural sway during upright stance, or with muscle fatigue [2], [3].

It may be possible to obtain many short TI data segments. For example, it is difficult to estimate joint stiffness at high activation levels because muscle fatigue occurs rapidly. However, subjects can perform multiple, short, large contractions without muscle fatigue. Similarly, it may be possible to segment non-stationary data from stance into a number of shorter stationary segments. Thus, it would be useful to have a method identify the system dynamics from short segments of stationary data.

Both non-parametric and parametric methods have been developed to identify joint stiffness from a single long data record. We recently developed a parametric subspace method that identifies a state-space representation of joint stiffness [4]. Subspace methods are advantageous since they require little a priori information, and are more robust than other methods.

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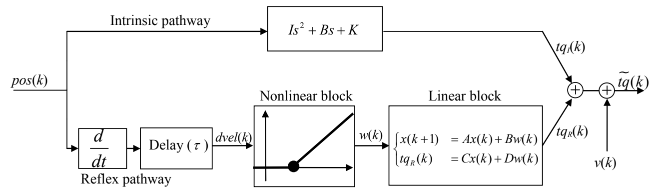

Fig. 1. Parallel-cascade model of ankle joint stiffness. Input is position $(pos(k))$ and output is the sum of intrinsic, reflex and voluntary torques plus measurement noise.

This paper extends our previous method [4] to work with short segments. The primary difficulty in doing so was related to initial conditions. The contribution of initial conditions decays at a rate determined by the system time constants. It is a common practice in identification of biological systems to ignore this transient and consider the system to be in steady-state. However, when only short segments of data are available, initial conditions must be considered and estimated, otherwise they will bias the estimates [5]. Consequently, we modified our state-space method to estimate the initial conditions for each segment and account for their contribution.

Section II presents the theory underlying the method. Section III shows experimental results for a time-varying task where ankle joint is moved passively through its range of motion. Section IV provides concluding remarks and a short discussion.

II. THEORY

Fig. 1 shows the PC structure. Intrinsic stiffness relates intrinsic torque $(tq_I(k))$ to angular joint position $(pos(k))$, velocity $(vel(k))$, and acceleration $(acc(k))$ by elastic (K), viscous (B), and inertia (I) parameters. Reflex stiffness has a block oriented nonlinear structure comprising a differentiator, a delay and a Hammerstein structure (static nonlinearity followed by a linear system).

Assume that p segments of input position and noisy output torque are available where the i-th segment has N^i data points, $i \in \{1, \cdots, p\}$.

$$
\begin{aligned}
pos^i &= \left[pos^i(0) \ \cdots \ pos^i(N^i-1)\right]^T \\
\tilde{tq}^i &= \left[\tilde{tq}^i(0) \ \cdots \ \tilde{tq}^i(N^i-1))\right]^T
\end{aligned} \tag{1}
$$

The superscript $(\cdot)^i$ indicates the i-th segment data .

A state-space model of the PC structure for the i-th segment is [4]:

$$
\begin{cases}
x^i(k+1) &= Ax^i(k) + B_\Omega U^i(k) \\
\tilde{tq}^i(k) &= Cx^i(k) + D_\Omega U^i(k) + v^i(k)
\end{cases} \tag{2}
$$

where the constructed input $U^i(k)$ is:

$$\begin{aligned} U^i(k) &= \left[U_R^i(k)\ U_I^i(k)\right] \\ U_R^i(k) &\triangleq \left[g_1(dvel^i(k))\ \cdots\ g_n(dvel^i(k))\right] \\ U_I^i(k) &\triangleq \left[pos^i(k)\ vel^i(k)\ acc^i(k)\right] \end{aligned} \tag{3}$$

where $vel^i(k)$ and $acc^i(k)$ are the velocity and acceleration computed from $(pos^i(k))$. $v^i(k)$ is an arbitrary colored output noise. A and C are the state-space matrices of the linear element in the reflex Hammerstein pathway. The static nonlinear element in the reflex pathway is approximated by a basis expansion $g(\cdot)$ of the delayed velocity $(dvel(k))$ with order n. Thus, the output of the nonlinear element is $w^i(k) = \sum_{i=1}^{n} \omega_i g_i(dvel^i(k))$. The state-space matrices B_Ω and D_Ω are:

$$\begin{aligned} B_\Omega &= \begin{bmatrix} b_1\omega_1 & \cdots & b_1\omega_n & 0 & 0 & 0 \\ \vdots & & \vdots & \vdots & \vdots & \vdots \\ b_m\omega_1 & \cdots & b_m\omega_n & 0 & 0 & 0 \end{bmatrix} \\ D_\Omega &= \begin{bmatrix} d\omega_1 & \cdots & d\omega_n & K & B & I \end{bmatrix} \end{aligned} \tag{4}$$

where $B = [b_1, \cdots, b_m]^T$ and $D = [d]$ are the state-space matrices of the linear element of the reflex stiffness pathway. $\Omega = [\omega_1, \cdots, \omega_n]^T$ contains the coefficients of the nonlinear expansion and $\{K, B, I\}$ are the intrinsic pathway parameters [4].

Use the extended Hankel matrix to merge data from all segments. The extended Hankel matrix $O_{i,j,t^1,\cdots,t^p}$ is defined by concatenating the Hankel matrices of all segments $\{1, \cdots, p\}$:

$$O_{i,j,t^1,\cdots,t^p} = \left[O_{i,j,t^1}, \cdots, O_{i,j,t^p}\right] \tag{5}$$

where t^i spans all of the recorded data $t^i = N^i - 2h + 2$ and $O_{i,j,j}$ is the Hankel matrix of the discrete signal $o(k)$ [6]:

$$O_{i,j,j} = \begin{bmatrix} o(i) & \cdots & o(i+k-1) \\ \vdots & \ddots & \vdots \\ o(i+j-1) & \vdots & o(i+j+k-2) \end{bmatrix} \tag{6}$$

Use *Multivariable Output Error State-sPace* (MOESP) [6] with past input as *instrumental variable* (IV) as the first step to estimate A, C. Apply it to the input $(U_{h,h,t^1,\cdots,t^p})$, past input $(U_{0,h,t^1,\cdots,t^p})$ and noisy output $(\tilde{T}Q_{h,h,t^1,\cdots,t^p})$ extended Hankel matrices. The IV $U_{0,h,t^1,\cdots,t^p}$ is not correlated with noise $V_{h,h,t^1,\cdots,t^p}$ but is correlated with output $\tilde{T}Q_{h,h,t^1,\cdots,t^p}$. Thus, unbiased estimates are guaranteed in the presence of an arbitrarily colored noise $(v(k))$ [6].

Use the estimates of A and C to construct the data equation needed to estimate the remaining parameters. Express the output of the i-th segment as [6]:

$$\begin{aligned} \tilde{tq}^i(k) = &\left[\sum_{\tau=0}^{k-1} {U^i}^T(\tau) \otimes CA^{k-1-\tau}\right] \text{vec}(B_\Omega) \\ &+ {U^i}^T(k)\text{vec}(D_\Omega) + CA^k x^i(0) + v^i(k) \end{aligned} \tag{7}$$

where $\text{vec}(\cdot)$ generates a vector by stacking the columns of the matrix $(\cdot)$:

$$\begin{aligned} \text{vec}(B_\Omega) &= [b_1\omega_1, \cdots, b_m\omega_1, \cdots, b_1\omega_n, \cdots, b_m\omega_n]^T \\ \text{vec}(D_\Omega) &= [d_1\omega_1\ \cdots\ d\omega_n]^T \end{aligned} \tag{8}$$

Replace A and C in (7) with their estimates $\hat{A}$ and $\hat{C}$ and define the matrix $\Gamma = [\Gamma_1, \cdots, \Gamma_p]$, the regressor for estimating the initial conditions:

$$\Gamma_1 = \begin{bmatrix} \hat{C} \\ \vdots \\ \hat{C}\hat{A}^{t^1-1} \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \ \cdots, \Gamma_p = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ \hat{C} \\ \vdots \\ \hat{C}\hat{A}^{t^p-1} \end{bmatrix} \tag{9}$$

Define Λ, the regressor for estimating the reflex parameters from (7):

$$\Lambda = \begin{bmatrix} 0 \\ U_R^1(0)\hat{C} \\ \vdots \\ \sum_{\tau=0}^{N^1-2} U_R^1(\tau)\hat{C}\hat{A}^{N^1-2-\tau} \\ \vdots \\ 0 \\ U_R^p(0)\hat{C} \\ \vdots \\ \sum_{\tau=0}^{N^p-2} U_R^p(\tau)\hat{C}\hat{A}^{N^p-2-\tau} \end{bmatrix} \tag{10}$$

Concatenate the segments to form:

$$\begin{aligned} U_I &= [U_I^1(0) \cdots U_I^1(N^1-1) \cdots U_I^p(0) \cdots U_I^p(N^p-1)] \\ \tilde{TQ} &= [\tilde{tq}^1(0) \cdots \tilde{tq}^1(N^1-1) \cdots \tilde{tq}^p(0) \cdots \tilde{tq}^p(N^p-1)] \end{aligned}$$

Now, express the data equation (7) in matrix form for all segments by separating the parameters of intrinsic and reflex pathways:

$$\tilde{TQ} = [\Phi_R \Phi_I] \begin{bmatrix} \theta_R \\ \theta_I \end{bmatrix} \tag{11}$$

where the reflex regressor Φ_R is constructed from Γ and Λ:

$$\Phi_R = [\Gamma_1\ \Gamma_2\ \cdots\ \Gamma_p\ \Lambda] \tag{12}$$

and θ_R contains the reflex parameters:

$$\begin{aligned} \theta_R = [\ &\zeta^1\omega_1 \quad \cdots \quad \zeta^p\omega_1 \quad b_1\omega_1 \quad \cdots \quad b_m\omega_1 \quad \cdots \quad b_1\omega_n \\ &\cdots \quad b_m\omega_n \quad d\omega_1 \quad \cdots \quad d\omega_n\ ]^T \end{aligned} \tag{13}$$

where ζ^i is the scaled version of the initial conditions:

$$\zeta^i = \frac{x^i(0)}{\omega_1} \tag{14}$$

The intrinsic regressor Φ_I is:

$$\Phi_I = U_I^T \tag{15}$$

and θ_I contains the intrinsic unknown parameters:

$$\theta_I = [K\ B\ I]^T \tag{16}$$

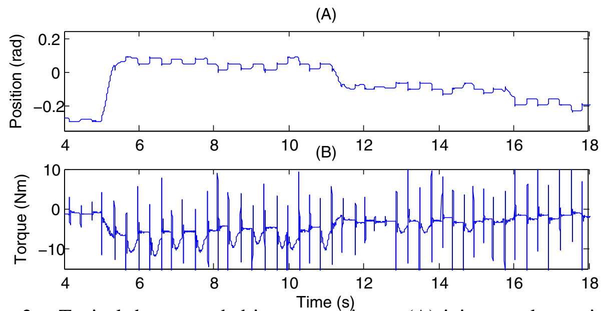

Fig. 2. Typical data recorded in an experiment (A) joint angular position; (B) joint torque.

The algorithm for identification of θ_I and θ_R from (11) is identical to [4]. Thus, proper orthogonal projection on the column space of reflex and intrinsic regressor is used to eliminate the effects of noise and reflex torques. This gives an estimate of the intrinsic parameter set θ_I. Then, an iterative method is used to estimate the static nonlinearity Ω, linear state-space parameters $\{b1, \cdots, b_m, d\}$, and initial condition set $\{\zeta^1, \cdots, \zeta^p\}$.

III. Experimental Results

The application of the subspace short segment identification method was demonstrated with data from an experiment in which the ankle was moved passively through its range of motion.

A. Methods

We recruited five healthy subject who gave informed consent to the experimental procedures, which had been reviewed and approved by McGill University Institutional Review Board. The experimental apparatus was similar to that described in [7]. The subject's left foot was attached to a hydraulic actuator using a custom made low-inertia boot. The actuator was operated as a position-servo mode that controlled ankle angular position. The neutral position (90 degrees angle between shank and foot) was taken as 0rad, a plantarflexing movement considered as negative and a dorsiflexing movement considered as positive. Subjects were instructed to remain relaxed and not to contract their ankle muscles voluntarily.

The angular position command sent to the actuator was the sum of an operating point trajectory (large displacement) and small perturbations. The operating point trajectory was a piecewise constant signal that spanned the subject's range of motion. This trajectory was generated by switching randomly between 10 levels spanning the range of motion at time intervals drawn from a uniform random variable with minimum and maximum of 4 and 7s. This was low-pass filtered with a second-order Butterworth filter with a cut-off of 2.5Hz to avoid sharp transients.

The position operating point trajectory had a very low frequency content and therefore was not suitable for identification. Consequently, a *pseudo random arbitrary level distribution sequence* (PRALDS) perturbation was added to the actuator input [4]. The switching rate of PRALDS was a uniform random variable with minimum and maximum of 250 and 350ms and its peak amplitude was 0.04rad. Fig. 2A shows a segment of the position recorded from a typical trial.

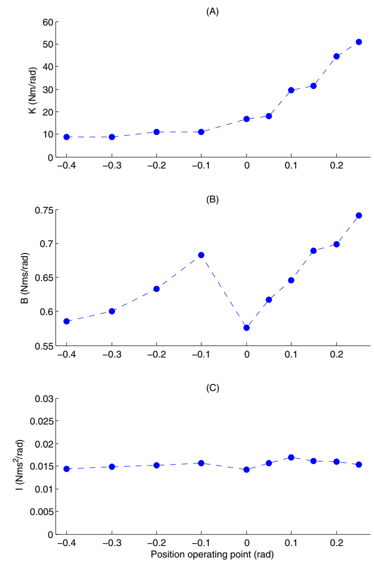

Fig. 3. Intrinsic stiffness parameters as a function of joint position: (A) elastic (K); (B) viscous (B); (C) inertia (I).

Angular joint position was measured using a precision potentiometer and the torque about the ankle using a torque transducer. 10 trials of 120s were recorded. Data were sampled at 1kHz and then decimated to 100Hz for analysis. EMG signals from triceps surae and tibialis anterior muscles were examined to confirm that subjects had no voluntary activity. The output torque (Fig. 2B) characteristics changed considerably with the position operating point (Fig. 2A); at a dorsiflexed position (+0.05), reflex activity was present and large; while at a more plantarfelxed position (-0.2rad), reflex activity was small altogether absent.

B. Results

Results are shown for a typical subject. The other four subjects behaved similarly.

Multiple stationary data segments were assembled from data recorded at each position level. This was achieved by segmenting the data according to the position operating point level. The first 1.5s of each segment was removed to avoid any transition effects (nonstationary condition). This yielded an average of 22 segments for each level with minimum and maximum of 18 and 27. The average segment length was $3.92 \pm 1.41s$.

The models estimated using the short segment method for all 10 operating positions predicted the torque very accurately and the residuals were small. The average identification VAF was high (90.2%± 2.5%).

Fig. 3 presents the intrinsic stiffness parameters as a function of position. The elastic parameter (K) increased as position moved from plantarflexion toward dorsiflexion (Fig. 3A). The viscous parameter (B) increased from -0.4 rad to -0.1 rad and, dropped at 0 rad and increased again with dorsiflexion (Fig. 3B). The inertia (I) parameter was almost constant (Fig. 3C).

The reflex pathway was only identified at positions larger than 0.05rad; elsewhere reflexes did not contribute significantly to ankle torque. Fig. 4(A) shows the static nonlinearities estimated at five positions. Systematic changes in both the slope and threshold of the nonlinearity are evident. At 0.25rad, the threshold was small and the slope was large and at 0.05rad, the threshold was larger and the slope smaller. Fig. 4(B) shows the linear dynamics which resembled a second-order, low-pass filter that became progressively more damped as the ankle moved toward plantarflexion.

IV. Discussion

This paper presents a method for the identification of ankle joint stiffness parallel-cascade structure from short segments of stationary data. The identification method estimates the parameters of the intrinsic stiffness directly. It fits a polynomial to the static nonlinearity and a state-space model to the linear dynamics of the reflex pathway. Initial conditions play an important role when using short data segments. The method accounts for them in the predicted output.

The method has similarities to some other identification methods. Kukreja et al. considered parametric identification of a Hammerstein structure from short segments of data [8]. They accounted for initial conditions but their model structure only allowed white Gaussian noise. Thus, it gives biased estimates from experimental data where the noise is not white. Zhao et al. used subspace methods to address this problem [5]. However, their method was ensemble-based and required all segments to have the same length. Furthermore, it did not estimate individual elements of the model but rather a state-space representation of the entire system with no connection to physiologically relevant parameters. Ludvig et al. recently presented a method for time-varying, non-parametric, identification of linear systems from short segments of data [9]. However, they did not account for initial conditions, so their method is only applicable for systems where initial conditions do not contribute significantly, such as systems with little memory, e.g. only the intrinsic pathway. Thus, it gives biased results when there is a reflex activity.

We successfully identified joint stiffness in a time-varying task where the ankle position operating point followed a piecewise constant trajectory. This paradigm was chosen because modulation of stiffness with joint position is well understood for time-invariant conditions. Moreover, in this paradigm, TI conditions were easily obtained, so extracting stationary data was straightforward. TI identification methods could not be used because none of the segments were long enough for a reliable identification. The segment length was 3.92s on average while at least 10s of data is required for a reliable TI identification.

The modulation of intrinsic stiffness with joint position (Fig. 3) was consistent with results previously reported for time-invariant studies [7]. The elastic and viscous parameters (K, B) increased from plantarflexion toward dorsiflexion while the inertia parameter was constant. Our results demonstrated a slope change at 0rad in the rate of increase of K and B. This is probably due to muscle activation as a result of reflex activity when the ankle was dorsiflexed.

Reflex stiffness was significantly modulated during the experiment in a manner consistent with the reported TI experiments [7]. In general, the reflex contribution decreased with joint plantarflexion. The identified system in Fig. 4

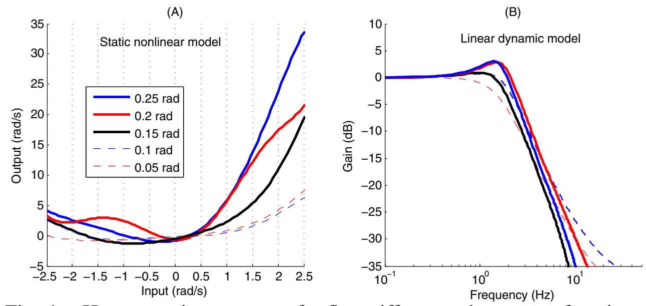

Fig. 4. Hammerstein structure of reflex stiffness pathway as a function of joint position: (A) static nonlinearity; (B) linear element gain.

further reveals that this decrease is resulted by both an increase in the threshold and a decrease in the slope of the static nonlinearity.

An important clinical application of the method would be in the assessment of the severity of the stiffness in spastic stroke patients. The most widely used clinical test is the Ashworth test, or its modified version, that involves imposing slow, passive movement of the ankle joint over its range of motion and the scoring of the resistance to the movement by a trained physician. This test is subjective and does not distinguish the intrinsic and reflex stiffnesses [10]. Our new method could be be a useful adjunct since it quantifies the intrinsic/reflex stiffness to passive movement and so provides for a more objective assessment.

The new method is a powerful tool that can be applied to characterize joint stiffness in tasks where either only short segments of data can be recorded (e.g. high contraction levels) or when the underlying dynamics are slow time-varying (e.g. stance). In future, it is of interest to validate the algorithm at different experimental conditions.

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