Electrodermal Activity Processing: a Convex Optimization Approach

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Abstract— **This paper reports on a novel model based on convex optimization methods for the analysis of the skin conductance (SC) as response of the electrodermal activity (EDA) to affective stimuli. Starting from previous assessed methodological approaches, this new model proposes a decomposition of SC into tonic and phasic components through the solution of a convex optimization problem. Previous knowledge about the physiology of the EDA is accounted for by means of an appropriate choice of constraints and regularizers. In order to test the effectiveness of the new approach, an experimental session in which 9 healthy subjects were stimulated using affective pictures gathered from the IAPS database was designed and carried out. The experimental session included series of negative-valence high-arousal images and series of neutral images, with an inter-stimulus interval of about 2 seconds for both neutral and high arousal pictures. Next, a statistical analysis was performed on a set of features extracted from the phasic driver and the tonic signal estimated by the model. Results showed that the phasic driver extracted from the model was able to strongly distinguish arousal sessions from neutral ones. Conversely, no significant difference was found for the tonic components. This experimental findings are consistent with the literature and confirm that the phasic component is strictly related to changes in the sympathetic activity of the autonomic nervous system. Although preliminary, these results are very encouraging and future work will progress to further validate the model through specific and controlled experiments.**

I. INTRODUCTION

The electrodermal activity (EDA) represents alterations in the electrical properties of the skin, e.g. skin conductance (SC), related to the level of psychologically-induced sweating. Sweat glands, and in particular the eccrine sweat glands responsible for the psychological response, are innervated by the sympathetic branch of the autonomic nervous system (ANS). Therefore, the EDA signal is considered a good and viable way to monitor the ANS.

There are two main components to the overall complex referred to as EDA, with different time scales and relationships with the triggering stimuli: tonic and phasic. The tonic EDA is given by the skin conductance level (SCL) which represents the slow-varying baseline level of the SC. Variations in the SCL are thought to reflect general and slow changes in the ANS dynamics. The phasic EDA is represented by a fast changing component, called skin conductance response (SCR), reflecting the evoked response of the eccrine sweat glands to an external stimulus. The SCR is defined as the SC transient arising within a predefined window (1–5 s) after the stimulus onset and satisfying a minimum amplitude criterion (0.05 μS) [1]. Recent evidences suggest that these two components rely on different neural mechanisms [2] and, consequently, that both convey relevant and non-redundant information about the ANS activity.

The SC can be easily measured by applying a constant 0.5 V potential across two points of skin contact, usually on the surface of the hands or more specifically of the fingers, where there is a high concentration of eccrine sweat glands [3], [4]. EDA is used in a wide range of experimental setups because it is a relatively straightforward measure providing valuable information about the ANS response to a broad range of external stimuli. In particular, SC analysis is commonly used to quantify the levels of arousal associated to emotional and cognitive processes [2], [5], [6].

A main limitation of EDA is that it imposes a minimum inter-stimulus interval (ISI), i.e. the temporal distance between consecutive stimuli. This is due to the fact that, in a typical SCR, the conductance level initially increases and then asymptotically decreases back to the baseline level following a slow exponential decay. Most authors recommend a minimum ISI of 10-20 s [2], [7] because for shorter intervals —like those employed in many experimental paradigms in cognitive neuroscience (1–2 s)— consecutive SCRs overlap. To overcome this limitation, several mathematical solutions have been developed to decompose the phasic EDA into individual SCRs associated to each stimulus. One of the first EDA model was proposed by Lim *et al.* [8]. They used a four-parameter model (extendible to eight parameters) in order to fit SC data and perform a decomposition in its components. The method requires visual inspection to establish the best model for the selected segment. Alexander *et al.* [9] introduced a method based on a deconvolution process in order to overcome the need for visual inspection. Their model assumes that the SC is the output of the convolution between a biexponential impulse response function (IRF) and discrete bursting episodes representing the sudomotor nerve activity (SMNA). Assuming that the IRF is known and time-invariant, their method estimates the SMNA from the measured SC. More recently, Bach *et al.* [10] adopted a similar linear time-invariant (LTI) convolution model and extracted the IRF by means of PCA (principal component analysis) across all available partic-

The research leading to these results has received partial funding from the European Union Seventh Framework Programme FP7/2007-2013 under grant agreement n 601165 of the project WEARHAP and by the School of computer science and electronic engineering, University of Essex.

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978-1-4244-7929-0/14/$26.00 ©2014 IEEE

ipants and trials. The canonical IRF was then fitted with a gamma distribution. Benedek and Kaernbach [1] applied a nonnegative deconvolution procedure based on the long division algorithm to enforce the estimated SMNA to be compact and nonnegative. Assuming the Edelberg model of the dynamic law of diffusion of the sweat, they adopted a biexponential IRF called Bateman function. In this paper we propose to decompose the EDA using a novel method based on convex optimization, a tool that has been applied to a steadily increasing number of applications [11]. This approach allowed the development of a new model that can be fitted efficiently and whose solution —thanks to the use of constraints and sparsifying regularizers— incorporates the previous knowledge about the problem.

II. ALGORITHM

A. Convex Optimization

A set $K \in \mathbb{R}^n$ is convex if:

$$\lambda x + (1-\lambda) y \ \in \ K \tag{1}$$

$\forall\ x, y \ \in \ K$ and $\lambda \ \in \ [0;1]$. A function is convex if:

$$f(\lambda x + (1-\lambda) y) \ \leq \ \lambda f(x) + (1-\lambda) f(y) \tag{2}$$

$\forall\ x, y \ \in \ K$ and $\lambda \ \in \ [0;1]$. The meaning of the inequality (2) is that, for any two points x and y in the domain of the function, the segment between $(x; f(x))$ and $(y; f(y))$ lies above the graph of the function. Equivalently, we can define a convex function as a function whose epigraph is a convex set [11].

Considering a standard optimization problem:

$$\begin{aligned} &\text{minimize } f_0(x) \\ &\quad \text{subj. to } f_i(x) \leq 0 \quad i = 1, \ldots, m\ , \end{aligned} \tag{3}$$

the optimal choice is the one minimizing the objective function $f_0(x)$, which represents the cost of choosing x, while simultaneously satisfying the constraints $f_i(x) \leq 0$. An optimization problem is convex when both the objective and the constraint functions are convex. In the context of mathematical optimization, the most important consequence of convexity is that necessary conditions for local optimality are also sufficient for global optimality. Moreover, important categories of convex optimization problems can be solved efficiently (this is rarely the case for general nonconvex problems).

A special subclass of convex optimization problems is represented by least-square problems where the goal is the unconstrained minimization of a quadratic objective function $\|Ax - b\|_2^2$. For this class of problems —frequently arising in regression analysis, parameter estimation and data fitting methods [11]— an analytical solution exists. An important statistical interpretation is that the least-square solution coincides with the maximum likelihood estimation in the case of a linear model corrupted by additive Gaussian noise. Regularization, e.g. adding a norm of the optimization variable x as an extra term to the cost function, can be applied to least-squares problems to prevent overfitting (L_2-norm) or to favour sparse solutions (L_1-norm). While in the former case an analytical solution exists, in the case of the L_1-regularization the problem can be cast as a quadratic program (QP), i.e. a convex problem with quadratic cost function and affine constraints:

$$\begin{aligned} &\text{minimize } \frac{1}{2} x^{\mathrm{T}} P\, x + q^{\mathrm{T}} x + r \\ &\quad \text{subj. to } Hx - g \leq 0 \text{ and } Ux - v = 0\ . \end{aligned} \tag{4}$$

B. EDA Model as QP Problem

Our model assumes that the observed SC is the sum of a slow tonic component, the phasic activity, and an additive noise term. We parametrize the tonic component by means of a cubic spline with equally-spaced knots every 10 s. The phasic activity, i.e. the SCR component, is modelled as the convolution between the sudomotor nerve activity and an impulse response $h(t)$ shaped like a biexponential Bateman function [12]:

$$h(t) = (e^{-\frac{t}{\tau_1}} - e^{-\frac{t}{\tau_2}})\ u(t) \tag{5}$$

where $u(t)$ is the unitary step function. As mentioned before, a biexponential function was previously proposed by Alexander *et al.* [9] as a model for the SCR shape and has a simple interpretation in terms of a bicompartmental pharmacokinetic diffusion model [12].

In mathematical terms, we decompose the vector y of measured SC data as:

$$y = Ap + B\ell + Cd + \epsilon\ , \tag{6}$$

where:

- y is the sampled SC raw data $y = [y_1, y_2, \ldots, y_N]^{\mathrm{T}}$;
- A is a sparse Toeplitz matrix with time-reversed, sampled, truncated copies of $h(t)$ along the diagonal band;
- p is a sparse vector with positive entries representing the sampled SMNA;
- Ap is the phasic component obtained as the result of the convolution between the sudomotor nerve activity p and the biexponential IRF;
- B is a tall matrix whose columns are the cubic B-spline basis functions;
- ℓ is the vector of spline coefficients;
- C is an $N \times 2$ matrix with $C_{i,1} = 1$ and $C_{i,2} = i/N$;
- d is a vector made of an offset term and a slope coefficient for the linear trend;
- $t = B\ell + Cd$ is the tonic component;
- ϵ is a vector of i.i.d. Gaussian noise terms incorporating model prediction errors as well as measurement errors and artifacts.

In order to estimate the unknown vectors p, ℓ, and d from the available data y, we consider the optimization problem where the objective function to be minimized is a quadratic measure of misfit or prediction error (ϵ) between the observed data and the values predicted by the model. Moreover, to account for the prior knowledge —including assumptions about the spiking nature and nonnegativity of the driving input (p)— constraints as well as sparsifying and regularizing terms were added. A successful approach to sparse deconvolution involves the regularization of the input variable by means of an L_1-norm cost function [13]–[15]. We therefore obtain the optimization problem:

$$\begin{aligned} &\text{minimize } \frac{1}{2}\left\|Ap + B\ell + Cd - y\right\|_2^2 + \lambda\left\|p\right\|_1 + \frac{\gamma}{2}\left\|\ell\right\|_2^2 \\ &\quad \text{subj. to } p \geq 0 \; . \end{aligned} \tag{7}$$

Please note that the model has two parameters, λ and γ, acting as regularizers for the phasic and tonic components, respectively. A large λ (stronger L_1 regularization of p) yields a sparser estimate with most noise-induced spurious spikes suppressed but also more signal distortion (i.e. attenuation of genuine activations). Conversely, a small λ produces a less distorted but noisier solution. Concerning γ, higher values mean a stronger penalization of ℓ, i.e. a smoother tonic curve. After some matrix algebra, the optimization problem (7) can be re-written in the standard QP form of (4) and solved efficiently using one of the many solvers available.

III. MATERIALS AND METHOD

This section describes the experimental protocol and the optimization process applied to the acquired SC dataset.

A. Experimental Protocol

In this study, we enrolled nine healthy subjects (aged 20–32; 4 females) who voluntarily underwent an experiment of induced emotions. All subjects signed a written informed consent prior to taking part to the study, and the protocol was approved by the local Ethics Committee. The experimental protocol started with a 5-minute rest period with eye closed, followed by 6 minutes of visual emotional stimuli selected from the international affective picture system (IAPS) [16]. The slideshow alternated 30 negative-valence high-arousal images and 30 neutral images with an ISI of 2 s. The negative emotional images were chosen according to the following criteria: arousal score > 6.7; valence < 4.3. During the whole experimental session the SC signal was acquired using a BIOPAC MP150 physiological acquisition system.

B. EDA Processing

The convex-optimization-based EDA model (cvxEDA) described in II-B was applied to each participant's SC signal. Rather than assuming a canonical IRF for all subjects, we assumed $\tau_1 = 0.7$ and adapted the parameter τ_2 on a per-subject basis. After fitting cvxEDA models for several values of $\tau_2 \in [2.0, 4.0]$, we chose the one minimizing the L_0-"norm", i.e. leading to the sparsest solution. The values $\lambda = 0.01$ and $\gamma = 1$ were employed for all subjects. These values were chosen through an exploratory analysis on simulated data and real signals from a different dataset.

To validate the model and verify that the recovered components carry meaningful information about the ANS activity, we extracted a set of features from the phasic driver p an the tonic signal t. For the phasic activity we computed the features over the 2-second non-overlapping windows related to each image. For the slower tonic component we adopted longer (20 s) windows assuming that its upper cut-off frequency can be considered 0.05 Hz [17]. In Table I, the feature set is summarized along with the corresponding description.

TABLE I
FEATURES EXTRACTED FROM EDA PHASIC AND TONIC COMPONENTS

Feature	Description
MAX-T	Maximum value of the tonic curve t
MAX-P	Maximum value of the SMNA p
Mean-T	Mean value of the tonic curve t
Mean-P	Mean value of the SMNA p
STD-T	Standard deviation of the tonic curve t
STD-P	Standard deviation of the SMNA curve p

C. Statistical Analysis

After rejecting the hypothesis of Gaussianity of the features ($p < 0.05$ given by Kolmogor-Smirnov test), we conducted a non non-parametric statistical inference analysis. An inter-subject analysis was performed to compare the features related to the arousal sessions and those related to neutral sessions. Statistical significance was assessed by means of a Mann-Whitney U test.

IV. EXPERIMENTAL RESULTS

For all subjects, the cvxEDA model produced the expected results: the SC data (Fig. 1A) was decomposed into two signals, the sparse component p and the smooth one t. Specifically, we interpret the former (Fig. 1B) as the activity of the sudomotor nerve and the latter as the activity of the tonic component (Fig. 1C).

The features extracted from the fitted phasic driver showed a strong significant statistical difference between the two types of visual stimulation (see Tab. II). As expected, an increased phasic activity is present during the presentation of the high-arousal images. Instead, the tonic features were unable to distinguish the neutral session from the arousal elicitation.

V. DISCUSSION AND CONCLUSIONS

In this study we propose a novel approach for the analysis of the EDA based on convex optimization methods. The model builds upon previous methodological approaches and addresses the problem of overlapping responses encountered in experimental paradigms involving short ISIs. Our

TABLE II
MEDIAN ± MAD (MEDIAN ABSOLUTE DEVIATION FROM THE MEDIAN) OF EACH FEATURE IN AROUSAL AND NEUTRAL SESSIONS

Feature	Neutral	Arousal
MAX-T	1.66 ± 0.44	1.99 ± 0.52
MAX-P***	$(9.99 \pm 9.96) \cdot 10^{-8}$	$(6.05 \pm 6.04) \cdot 10^{-7}$
Mean-T	1.60 ± 0.40	1.89 ± 0.47
Mean-P***	$(1.74 \pm 1.73) \cdot 10^{-8}$	$(6.50 \pm 6.49) \cdot 10^{-8}$
STD-T*	$(2.49 \pm 1.73) \cdot 10^{-2}$	$(6.23 \pm 3.48) \cdot 10^{-2}$
STD-P***	$(2.22 \pm 2.21) \cdot 10^{-8}$	$(1.04 \pm 1.02) \cdot 10^{-7}$

Statistically significant differences are indicated by $*$ ($p < 0.05$) and $***$ ($p < 0.001$).

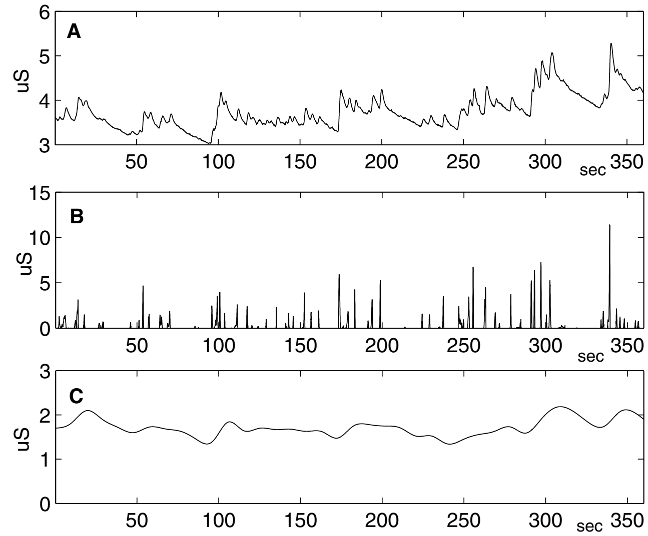

Fig. 1. Example of application of the cvxEDA decomposition procedure to the SC signal recorded during the visual emotional stimulation for a representative subject. Panel A: raw SC signal; panel B: sparse phasic driver component p; panel C: the slow tonic component t.

model decomposes the SC signal into a tonic component, parametrized by a cubic B-spline, and a phasic component that is the convolution of the sudomotor nerve activity with a Bateman IRF. The nonnegativity and sparsity of the SMNA driver are enforced by means of an appropriate choice of the constraints and through the use of an L_1-norm regularizer. Physiologically-plausible temporal scale and smoothness of the tonic input signal are accomplished through an adequate choice of the spacing between the knots of the spline and through an L_2 regularization of the spline's coefficients. Unlike other methods (for example those relying on the long division algorithm), our approach includes an additive noise term which incorporates modelling errors as well as measurement noise and artifacts. The unknown inputs (tonic and phasic EDA) can be obtained by minimizing a regularized version of the variance of this noise term.

This new approach was tested on recordings from an experimental paradigm in which 9 subjects were stimulated using affective pictures gathered from the IAPS database. The experimental session included series of negative-valence high-arousal images and series of neutral images. The ISI was set to 2 s for both neutral and high arousal pictures. As expected, the cvxEDA decomposition procedure retrieves a sparse component and a smooth one. In order to verify the effectiveness of the model and if the recovered components carry meaningful information about the ANS activity, a set of features were extracted from the phasic driver and the tonic signal. The statistical analysis of these features showed that the phasic driver estimated through the proposed methodology was able to significantly discern the two different kinds of stimuli. No significant difference was found for the tonic component. This finding may be explained by the fact that the tonic component is not stimulus-related and the short duration of the emotional stimulation (the arousal session was interrupted each minute by a minute of neutral images) may not be sufficient to induce significant changes in sympathetic tone which is characterized by slow dynamics (< 0.05 Hz).

Although preliminary, these encouraging results confirm that our EDA model based on convex optimization is able to produce a decomposition of the EDA which overcomes the issue of overlapping SCRs. The solution incorporates the previous knowledge about the phasic and tonic components without having to resort to heuristics and ad-hoc solutions. An additional advantage of casting our model as a convex optimization problem is that, once the problem is formalized, a globally optimal solution can be efficiently found using existing solvers.

REFERENCES

[1] M. Benedek and C. Kaernbach, "Decomposition of skin conductance data by means of nonnegative deconvolution," *Psychophysiology*, vol. 47, no. 4, pp. 647–658, 2010.

[2] M. E. Dawson, A. M. Schell, and D. L. Filion, "The electrodermal system," in *Handbook of psychophysiology*, J. T. Cacioppo, L. G. Tassinary, and G. G. Berntson, Eds. Cambridge University Press, 2007, ch. 7.

[3] S. Schmidt and H. Walach, "Electrodermal activity (EDA)–state-of-the-art measurements and techniques for parapsychological purposes," *Journal of Parapsychology*, vol. 64, no. 2, 2000.

[4] W. T. Roth, M. E. Dawson, and D. L. Filion, "Publication recommendations for electrodermal measurements," *Psychophysiology*, vol. 49, pp. 1017–1034, 2012.

[5] A. Greco, A. Lanata, G. Valenza, G. Rota, N. Vanello, and E. P. Scilingo, "On the deconvolution analysis of electrodermal activity in bipolar patients." *Proceedings of the 34th IEEE EMBS Conference*, vol. 2012, pp. 6691–6694, 2012.

[6] A. Greco, G. Valenza, A. Lanata, G. Rota, and E. Scilingo, "Electrodermal activity in bipolar patients during affective elicitation," *IEEE Journal of Biomedical and Health Informatics*, 2014.

[7] A. Breska, K. Maoz, and G. Ben-Shakhar, "Interstimulus intervals for skin conductance response measurement," *Psychophysiology*, vol. 48, no. 4, pp. 437–440, 2011.

[8] C. L. Lim, C. Rennie, R. J. Barry, H. Bahramali, I. Lazzaro, B. Manor, and E. Gordon, "Decomposing skin conductance into tonic and phasic components," *International Journal of Psychophysiology*, vol. 25, no. 2, pp. 97–109, 1997.

[9] D. Alexander, C. Trengove, P. Johnston, T. Cooper, J. August, and E. Gordon, "Separating individual skin conductance responses in a short interstimulus-interval paradigm," *Journal of neuroscience methods*, vol. 146, no. 1, pp. 116–123, 2005.

[10] D. R. Bach, G. Flandin, K. J. Friston, and R. J. Dolan, "Time-series analysis for rapid event-related skin conductance responses," *Journal of neuroscience methods*, vol. 184, no. 2, pp. 224–234, 2009.

[11] S. P. Boyd and L. Vandenberghe, *Convex optimization*. Cambridge university press, 2004.

[12] E. Garrett, "The Bateman function revisited: a critical reevaluation of the quantitative expressions to characterize concentrations in the one compartment body model as a function of time with first-order invasion and first-order elimination," *Journal of Pharmacokinetics and Pharmacodynamics*, vol. 22, no. 2, pp. 103–128, 1994.

[13] M. S. O'Brien, A. N. Sinclair, and S. M. Kramer, "Recovery of a sparse spike time series by L1 norm deconvolution," *IEEE Transactions on Signal Processing*, vol. 42, no. 12, pp. 3353–3365, 1994.

[14] R. Tibshirani, "Regression shrinkage and selection via the LASSO," *Journal of the Royal Statistical Society. Series B (Methodological)*, pp. 267–288, 1996.

[15] J. de Rooi and P. Eilers, "Deconvolution of pulse trains with the L0 penalty," *Analytica chimica acta*, vol. 705, no. 1, pp. 218–226, 2011.

[16] P. Lang, M. Bradley, and B. Cuthbert, "International affective picture system IAPS): Digitized photographs, instruction manual and affective ratings," *Technical report A-6. University of Florida*, 2005.

[17] A. Ishchenko and P. Shev'ev, "Automated complex for multiparameter analysis of the galvanic skin response signal," *Biomedical Engineering*, vol. 23, no. 3, pp. 113–117, 1989.