# Automatic Gesture Analysis Using Constant Affine Velocity

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*Abstract*— Hand human gesture recognition has been an important research topic widely studied around the world, as this field offers the ability to identify, recognize, and analyze human gestures in order to control devices or to interact with computer interfaces. In particular, in medical training, this approach is an important tool that can be used to obtain an objective evaluation of a procedure performance. In this paper, some obstetrical gestures, acquired by a forceps, were studied with the hypothesis that, as the scribbling and drawing movements, they obey the one-sixth power law, an empirical relationship which connects path curvature, torsion, and euclidean velocity. Our results show that obstetrical gestures have a constant affine velocity, which is different for each type of gesture and based on this idea this quantity is proposed as an appropriate classification feature in the hand human gesture recognition field.

#### I. INTRODUCTION

Hand posture and gesture recognition provide an alternative to obtain a more intuitive communication between human and traditional machines. The main applications in this field include the medical gesture recognition, performed in order to achieve an objective assessment of surgical skills [1], [2]. In recent years, some researchers have been focusing to find the similarities and differences among different surgical gestures. Most of the works are oriented to know what kind of variables (spatial and temporal) should be used to perform the classification. In any case, the variables involved should be related with both kinematic and dynamic aspects of trajectories generation. Some early studies have suggested that the CNS (Central Nervous System) associates representations for the movement based on geometrical and temporal attributes instead of motor execution or muscle activation [3], [4]. In particular, during drawing movements, humans tend to decrease the instantaneous tangential velocity of their hands at the same time the motion curvature increases and similarly, the velocity increases when the trajectory becomes straight [5], [6]. This relationship has been shown to be well described using a two-third power law, an empirical law that shows the correlation between local geometry and kinematics of human hand motion in planar drawing trajectories [7], [8].

However, if planar drawing movements follow this power law does not imply that it is an explicit relationship for every human movement planned by the CNS. Although, it has been generalized for some types of human movements and also for motion perception and prediction [9]. In [10], the twothird power law is applied to the smooth pursuit motion of

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eye, specifically controlled by distinct neural motor structures and a particular set of muscles. Whereas Vieilledent *et. al* have studied some curved locomotor trajectories with the hypothesis that, also during locomotion, movements obey this relationship [11]. Another important result in this field is related to the affine velocity of each trajectory. Pollick *et al.* show that if instead of computing the Euclidean speed, the affine velocity is calculated, then the unique function that involves the curvature and generates an affine invariant velocity is specifically the two-third power law [8]. It means that the hand writing trajectories implies motion at constant affine velocity. This fact shows that the power law and kinematic aspects of movement can be described by examination to the affine space instead of the euclidean one [12]. Actually, affine concepts have been applied to the analysis of images motion in [13].

Furthermore, it was demonstrated that the two-third power law is not enough to explain general three-dimensional drawing movements. Experimental results suggest that the movement at constant affine velocity is the main principle and the two-third power law could be a special case. In this way, a new power law (one-sixth power law) has been proposed to facilitate the description of spatial drawing movements [14], [15]. Based in these results, it was probed, that for the specific case of three-dimensional scribbling gestures, the one-sixth power law explains the data better than the two-third power law.

In this paper, an analysis of the affine velocity on obstetrical gestures is presented. Constant affine velocity is demonstrated for this kind of hand human gestures and the histogram behavior for each gesture, is presented. Additionally, some modifications of the power law's exponents across two different gestures are described. However, calculating the average over all subjects and gestures, the power law exponents are mostly in concordance with constant spatial affine velocity. This is an important result related to the importance of non-euclidean geometry in the medical gesture segmentation and recognition field.

#### II. ONE-THIRD AND ONE-SIXTH POWER LAWS

The inverse relationship between euclidean velocity  $v$  and curvature  $\kappa$  during planar drawing hand trajectories is defined by the two-third power law (1):

$$
v = \alpha \kappa^{-\frac{1}{3}},\tag{1}
$$

where v and  $\kappa$  are defined for a planar motion by:

$$
v = \sqrt{\dot{x}^2 + \dot{y}^2} \quad \kappa = \frac{|\dot{x}\ddot{y} - \ddot{x}\dot{y}|}{(\dot{x}^2 + \dot{y}^2)^{\frac{3}{2}}},\tag{2}
$$

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and where  $\alpha$  is a gain factor. In this case,  $\dot{x}, \dot{y}$  and  $\ddot{x}, \ddot{y}$  are the first and second derivatives of  $x, y$  relative to time. Previous works have demonstrated that in drawing movements, the gain factor  $\alpha$  is approximately constant for simple elliptical movements, but is piecewise constant for more complex trajectories [14], [16], [17].

Meanwhile, the affine velocity  $v_a$  for planar motion is defined by:

$$
v_a = |\dot{x}\ddot{y} - \dot{y}\ddot{x}|^{\frac{1}{3}}.
$$
 (3)

This equation describes the cube root of the signed area of the parallelogram generated by the first and second position derivative of motion with respect to time. With some algebraic manipulations of (2) and (3), it is possible to express the Euclidean Velocity as :

$$
v = v_a \kappa^{-1/3}.
$$
 (4)

If (4) is compared with (1); one can conclude that motion with constant affine velocity is equivalent to motion that obeys the two-third power law. If one performs the same transformation, one can obtain the equations that describe the spatial motion in drawing trajectories. Formally, for a spacial trajectory  $v, \kappa$  and the torsion  $\tau$  are defined by:

$$
v = \sqrt{\dot{x}^2 + \dot{y}^2 + \dot{z}^2}
$$
 (5)

$$
\kappa = \frac{\sqrt{(\ddot{z}\dot{y} - \ddot{y}\dot{z})^2 + (\ddot{x}\dot{z} - \ddot{z}\dot{x})^2 + (\ddot{y}\dot{x} - \ddot{x}\dot{y})^2}}{(\dot{x}^2 + \dot{y}^2 + \dot{z}^2)^{\frac{3}{2}}}
$$
(6)

$$
\tau = \frac{\left| \frac{dr}{dt}, \frac{d^2r}{dt^2}, \frac{d^3r}{dt^3} \right|}{\left| \frac{dr}{dt} \times \frac{d^2r}{dt^2} \right|},\tag{7}
$$

where  $||\bullet||$  and  $\times$  denote vector magnitude and cross product, respectively. Spatial affine transformations conserve the volume enclosed by the shape. Then, the spatial affine velocity at any point is defined by the volume of the parallelepiped generated by the first, second, and third derivative at that point [14]:

$$
v_a = \left| \frac{dr}{dt}, \frac{d^2r}{dt^2}, \frac{d^3r}{dt^3} \right|^{\frac{1}{6}},
$$
 (8)

where  $\vert \bullet \vert$  denotes the scalar triple product. Using some algebraic manipulations of  $(5)$ ,  $(6)$ ,  $(7)$ , and  $(8)$ , it is possible to prove that spatial motion at constant affine velocity entails the one-sixth power law as following:

$$
v = v_a(\kappa^2|\tau|)^{-\frac{1}{6}} = v_a \kappa^{-1/3}|\tau|^{-1/6}
$$
 (9)

## III. ONE-SIXTH POWER LAW APPLIED ON OBSTETRICAL GESTURES

## *A. Experimental Procedure*

An analysis of several data sets, acquired by an instrumented obstetrical forceps coupled with the BirthSIM simulator, is presented (Fig. 1) [18]. With this device, a medical practitioner can proceed to forceps blades placement.



Fig. 1. Obstetrical Forceps

The BirthSIM simulator consists of anthropomorphic models of the maternal pelvis and the fetal head. The instrumented forceps allow to measure displacements inside the pelvis. In the experiments, six obstetrical residents were asked to perform 30 forceps blade placements providing for each trainee 60 trajectories: 30 left blade trajectories and 30 right blade trajectories. The forceps placements are carried out in two different sessions of 15 forceps blade placements (Fig. 2). In each trajectory, the fetal head is positioned according to the ACOG (American College of Obstetrics and Gynecology) classification on an outlet LOA+5 presentation (Left Occiput Anterior location and station +5cm from the ischial spines plan).

## *B. Data Processing*

The position data were interpolated using a cubic splines to calculate the different derivatives with smoother trajectories. Based on the splines computation, variables such as velocity v, curvature  $\kappa$ , and torsion  $\tau$  were calculated using their analytical derivatives. In order to avoid uncertainties when the torsion is zero, a threshold was used in the different calculations.

#### *C. Linear Regression*

Several simulations were performed in order to examine if the relationship between the velocity, curvature, and absolute value of torsion could represent the obstetrical gestures. Fig. 3 presents some examples of the correlation found between



Fig. 2. Left and right blade trajectories



Fig. 3. Linear regression obtained for a sample data set (Left and Right Hands)

 $k^{\frac{1}{3}}|\tau|^{-1/6}$  and the norm of the Euclidean velocity  $|v|$  using a gradient descent algorithm [19].

These results show that there is a proportional relationship between both quantities (v and  $k^{\frac{1}{3}}|\tau|^{-1/6}$ ). That means that obstetrical gestures, like scribbling movements, are also governed by the one-sixth law. In order to analyze the variance found in each linear regression, a histogram was computed for each gesture. Due to the data variance has a similar behavior than a Gaussian one, then a Gaussian density function was fitted to the results. Fig. 4 shows the results calculated for a sample data set acquired from the experiments. Despite the fact that the left gesture is easier than the right one (less complex and less rotation), the dispersion is higher than the right one. This is due to the fact that the left hand is the less skillful hand for the people involved in the experiments. However the maximum dispersion to the linear regression is similar for both gestures.

#### *D. Power Law to Describe Obstetrical Gestures*

The survey carried out in this paper includes an analysis over the one-sixth power law. In this case, the exponents of (9) were not taken as fixed constant. In order to calculate the best exponents that fit the data acquired in this work, a logarithmic linearization of (9) was performed:

$$
\log(v) = \log(v_a \kappa^{\alpha} |\tau|^{\beta}) = \log(v_a) + \alpha \log(\kappa) + \beta \log(|\tau|). \tag{10}
$$

This expression can be rewritten as  $z = \gamma + \alpha x + \beta y$ , where  $z = \log(v)$ ,  $x = \log(\kappa)$  and  $y = \log(|\tau|)$ . Based on this expression, in order to calculate  $\alpha$ ,  $\gamma$  and  $\beta$ , a multivariate linear regression was performed using a gradient descent algorithm [19] (Fig 5 and Fig 6).



(b) Histogram Sample Data - Right Blade

Fig. 4. Histogram of a Sample Data (Left and Right Blade)

In Fig. 7, the average values for both exponents ( $\alpha$  and  $\beta$ ) are presented for each subject involved in this experiments. The results show that the exponent  $\alpha$  has an approximate average value of  $-\frac{1}{5}$  ranging approximately from  $-\frac{1}{7}$  to  $-\frac{1}{4}$ . The exponent  $\beta$ , on the other hand, presents an approximate average value of  $-\frac{1}{12}$  varying from values as  $-\frac{1}{16}$  to  $-\frac{1}{8}$ . The deviation for each gesture shows that the left trajectories have a higher variance than the right ones for each exponent calculated.

Based on the values of the constant  $\gamma$ , the affine velocity was calculated taking into account the relationship  $\gamma$  =  $log(v_a)$ . Fig. 8 presents the values of affine velocity for each gesture performed for each person involved in this experiments. As for the previous results the dispersion for the gesture performed by the left hand is higher compared



Fig. 5. Multivariate Linear Regression - Left Blade



Fig. 6. Multivariate Linear Regression - Right Blade



Fig. 7. Free Exponents Calculation

with the right one. Additionally, Fig. 8 shows that the affine velocity of the right gestures, in every case, is lower than the values calculated for the left gestures. The results obtained are clustered in such a way that is possible distinguish between both gestures for each subject.

#### IV. CONCLUSION

Hand gestures recognition is an interesting interaction paradigm in a variety of medical applications. In particular, greater efforts have been directed to find similarities and differences among obstetrical gestures. The main question has been focused to figure out what spatial and temporal variables should be used in this field. In this paper, an experimental relationship between velocity, curvature and



Fig. 8. Affine Velocity for each gesture

torsion is used to calculate a new suitable feature to classify gestures: *The affine Velocity*. The experiments involved in this survey include two obstetrical gestures acquired using an instrumented forceps. The affine velocity calculations allow us to classify between gestures for each subject involved in the experiments. The results obtained present a new alternative to analyze medical gestures by examining the affine space rather that the euclidean one. Our ongoing research in this field is focused on increasing the number of gestures and analyzing the correlation between novices and experts surgeons. Additionally, affine velocity could be used to segment complex medical movements such as surgical gestures in order to obtain a more accurate classification.

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