Identification of the Vestibulo-ocular Reflex Dynamics

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***Abstract*— The vestibulo-ocular reflex (VOR) plays an important role in our daily activities by enabling us to fixate on objects during head movements. Modeling and identification of the VOR improves our insight into the system behavior and helps in diagnosing various disorders. However, the switching nature of eye movements, including the VOR, makes the dynamic analysis challenging. In this work we are using integration of subspace and prediction error methods to analyze VOR dynamics. The performance of the method is evaluated using simulation studies and experimental data.**

I. INTRODUCTION

The Vestibulo-ocular reflex is an involuntary eye movement that serves to stabilize retinal images during head movements. The reflex consists of slow compensatory eye movements in the opposite direction to head rotation and fast re-orienting eye movements usually in the same direction as the head movement. The switching mechanism between the slow and fast phases relies on omnipause neurons (OPN) activities that release firing in burst neurons (BN) during fast phases only. The switching mechanism can extend the linear range of the VOR responses [1]. Fig. 1 shows an example of recorded conjugate eye position and velocity (average of the left and right eye position) during 1/6 Hz sinusoidal head rotations in the dark using electro-oculography (EOG). Sample slow and fast phase segments are marked with red and blue rectangles.

It is common in the literature to remove the fast phases from the VOR data and employ envelope approaches to analyze the reflex dynamics; i.e. replacing the removed data using interpolation, usually in a velocity record. However, this ignores the effects of initial conditions due to the switching at each slow phase segment and therefore biases analysis of the dynamics [1].

There are two algorithms in the literature to identify VOR dynamics in the presence of switching. The first one is the modified extended least squares (MELS) algorithm [2]. MELS is an iterative algorithm based on a NARMAX (Nonlinear AutoRegressive, MovingAverage eXogenous) parametric modeling that includes the initial conditions at each slow phase segment as an unknown parameter to be estimated. In this method, it is assumed that the output additive noise is Gaussian, white and zero-mean. In the presence of non-Gaussian noise, which is the case in EOG recordings [3], the accuracy of the algorithm can be affected. The second algorithm developed by [4] is called hybrid extended least squares (HybELS): it is also an iterative

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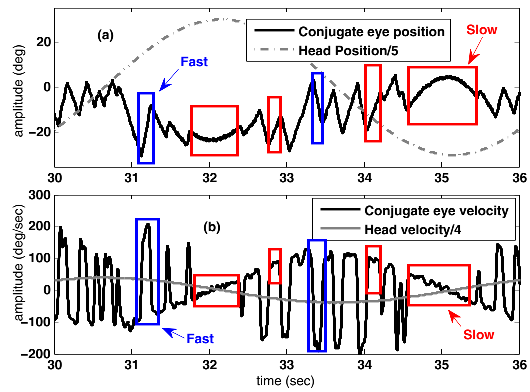

Fig. 1. VOR in response to sinusoidal head rotation recorded with EOG; (a) Conjugate eye position and scaled head position (deg); (b) Conjugate eye velocity and scaled head velocity (deg/sec). Sample slow and fast phase segments are marked with red and blue rectangles, respectively.

and parametric algorithm that identifies the parameters of slow/fast segments simultaneously. However, HybELS does not require estimates of the initial conditions, but rather relies on state continuity in the transitions between fast and slow phases. Therefore, when identifying each segment, the history of the signal from the previous segment is used instead of estimating initial conditions. The drawback with this method is that in the presence of noise, replacing the initial conditions with noisy data introduces biases in the model identification and the performance of the algorithm is also affected. The performance of both methods is similar in identifying VOR slow phases [4]; the HybELS is faster since there are fewer parameters to estimate. However, poor convergence in the presence of high noise levels is still a problem.

In this work, we are using an integration of prediction error minimization (PEM) [5] with subspace method identification [6] to find VOR dynamics in the dark. This method is implemented in the MATLAB identification toolbox (The MathWorks Inc., Natick, MA, USA) as function '*ssest*' to find a state space model. The PEM algorithm uses numerical optimization methods to minimize the cost function defined as a weighted norm of the prediction error, i.e. the difference between the measured output and the predicted output of the model. PEM is applicable to a wide spectrum of model parameterizations and results in models with excellent asymptotic properties due to its kinship with maximum likelihood. However, the PEM algorithm has its own drawbacks, such as requiring explicit parameterization of the model or laborious search over surfaces that may have local minima [5]. Therefore, good initialization of the parameter values is crucial. The *ssest* function employs a non-iterative subspace approach to initialize the parameter set and then refines the parameter values with PEM. Subspace methods are developed to estimate state space models for linear systems with no a priori knowledge about the system. Such methods are computationally efficient and can be extended to identify systems with different types of noise [7]. The *ssest* method also allows identification of merged data segments and estimates the initial conditions for each data segment either as an independent estimation parameter or as the value that produces the best least squares fit.

We evaluate here the performance of the *ssest* method in identifying VOR dynamics using simulation studies and compare this method to the previously developed MELS method [2]. We also apply this method to the identification of experimentally recorded VOR data in the dark.

The remainder of this paper is organized as follows. Section II provides a review on model formulation and the algorithm. Simulation and experimental results are presented in Section III, followed by a brief discussion in Section IV.

II. Methods

In this work we focus on identifying the VOR slow phase dynamics. We assume perfect classification tools are available to mark slow/fast segments beforehead. Fast phase dynamics can be identified with the same method.

A. Model Formulation

In our recent works [8], [9], we have introduced a physiologically relevant nonlinear hybrid model for VOR nystagmus in the dark. In this model, local nonlinear computations in brainstem circuits enable the model to replicate target-distance related behavior of the VOR. The input signal is head velocity (deg/sec), H_v, sensed by semicircular canals, and the output is conjugate eye position (deg), E. Here, for the purpose of evaluating the performance of the identification algorithms, we use a linear version of our hybrid model; i.e. the nonlinear surfaces are replaced with linear ones. With the linear model, the slow phase conjugate system is simplified to a linear low pass 1st order system with a gain, g, and a time constant, T, after the semicircular canals. The semicircular canals are modeled with 1st order high pass dynamics [10] as: $\frac{T_c s}{T_c s+1}$. Fig. 2 depicts the simplified model, assumed for representing VOR dynamics. Although the overall input-output system has second order dynamics, it is of interest to *separate* the canal time constant from that of the conjugate system, because the first is continuous, while the second is switched. Moreover, assuming 2nd order dynamics doubles the number of unknown initial conditions and over parameterizes the problem. Therefore, we will search for the canal time constant by first filtering the input, H_v, through a unity gain high pass filter with a variable time constant, T_c. We constrain the canal time constant search in the range of 2 to 20sec as reported in experimental studies [11] in increments of 1 sec. We then use the filtered H_v, now the canal signal H_c, as the input signal to the identification problem to estimate g and T of the VOR model. Finally, we compare the goodness of the fit to data for different selected T_c and accept the value that results in the best fit.

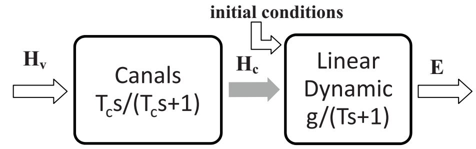

Fig. 2. Assumed structure of the slow phase VOR. H_v is head velocity (deg/sec), H_c is canals signal (spikes/sec) and E is eye position (deg).

B. State Space Identification

The *ssest* function estimates continuous or discrete state space models in the following general form:

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) + Ke(t) \\ y(t) = Cx(t) + Du(t) + e(t) \end{cases} \quad (1)$$

where A,B,C,D and K are state space matrices, $u(t)$ is the input, $y(t)$ is the output, $x(t)$ is the vector of states and $e(t)$ is the disturbance. If multiple experiment data sets are identified using *ssest*, initial conditions for each experiment are estimated individually, but common dynamics are forced.

Assuming first order dynamics for the conjugate VOR after the semicircular canals, i.e. $E = \frac{g}{Ts+1}H_c + \frac{T}{Ts+1}E_0 + noise$, where E_0 is the initial condition at each slow phase segment, a state state representation of our model in Fig. 2 is:

$$\begin{cases} \dot{x}(t) = \frac{-1}{T}x(t) + \frac{g}{T}H_c + Ke(t) \\ E = x(t) \end{cases} \quad (2)$$

With this formulation: $A = \frac{-1}{T}$, $B = \frac{g}{T}$, $C = 1$ and $D = 0$. It is possible to fix $C = 1$ and $D = 0$ for initialization of the parameters and limit the algorithm to only estimate A, B, K and the initial conditions.

Slow phase segments of the VOR dataset are merged as several experiments with different numbers of samples. It should be noted that the minimum number of samples for each data segment should be greater than the number of unknown parameters in the identification problem.

III. Results

We first validate our identification approach on simulated data to demonstrate its unbiased convergence to true parameters and then test it with experimental data.

A. Simulation Results

In this section, simulation results are presented to evaluate the performance of the state space identification algorithm. VOR data is obtained from simulation of a linear version of the VOR hybrid model [9] in Simulink. Knowing the nominal parameters of system dynamics as well as the switching instances between fast and slow modes allows precise evaluation of the identification algorithms . The nominal values for the slow phase model are $T = 5.00$ *sec* and $g = -4.98$. The canal time constant is set to $T_c = 6$ *sec*. The model is simulated at 200Hz for 60 sec. Input-output data is divided into two non-overlapping segments for the purpose of identification and validation. The first half of

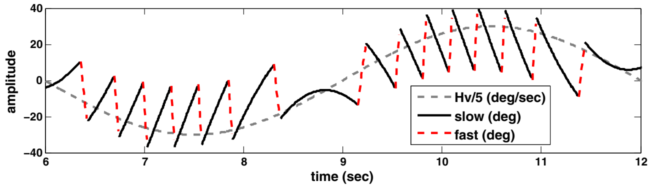

Fig. 3. Simulated input-output data used for identification.

data is used to identify the dynamics and the second half is used for cross validation of the identified dynamics to unseen data. '*Infinite step*' prediction is used to compute the model prediction for input signals given the switching instances. The quality of the model prediction ($\hat{y}$) fit to data (y) is evaluated by computing the variance accounted for as

$$\%VAF = \left(1 - \frac{var(y - \hat{y})}{var(y)}\right) \times 100 \tag{3}$$

where $var(.)$ is the variance of the signal. In the simulation studies we present %VAF between model prediction and *noiseless* output, even when there is additive output noise.

In clinical tests it is very common to use low frequency sinusoidal head rotation to test the VOR. It should be noted that although a low frequency sinusoidal input seems insufficient to be a persistently exciting input for dynamic identification, here, the switching mechanism increases the effective bandwidth of the input due to new *random* initial conditions at every switching instance. It is clear that higher bandwidth input signals, result in more accurate identification of the dynamics. Therefore, in order to be consistent with clinical test protocols, in this study we use a sinusoidal rotation at 1/6 Hz with 150 deg/sec peak velocity as the input signal, H_v. Fig. 3 shows the simulated VOR in response to sinusoidal rotation.

We compared the performance of the *ssest* algorithm and the MELS algorithm in identifying the dynamics of the VOR slow phase segments in several cases, including different white Gaussian noise levels and also realistic EOG noise. Realistic noise signals are neither white nor Gaussian, and are simulated according to the noise analysis study in [3]. Statistical properties of the identified parameters and %VAF are computed in 100 Monte-Carlo simulations with different Gaussian/realistic noise realizations. Here we present the results of this comparison in the following cases:

- *Case1*- Noise free input-output data
- *Case2*- White Gaussian output noise, SNR=40 db.
- *Case3*- White Gaussian output noise, SNR=20 db.
- *Case4*- White Gaussian output noise, SNR=10 db.
- *Case5*- Realistic output noise for EOG, SNR=31 db.

Fig. 4 shows the summary of this comparison. Searching for the canal time constant, $T_c = 6\ sec$, with both methods results in finding the correct value in all cases. Noise free output data also results in perfect estimation of the unknown parameters. However, as the noise levels increase in *Case2-4*, the MELS algorithm provides *biased* mean values for both T and g with small confidence interval, yet with high %VAF. This is because the prediction errors are biased and

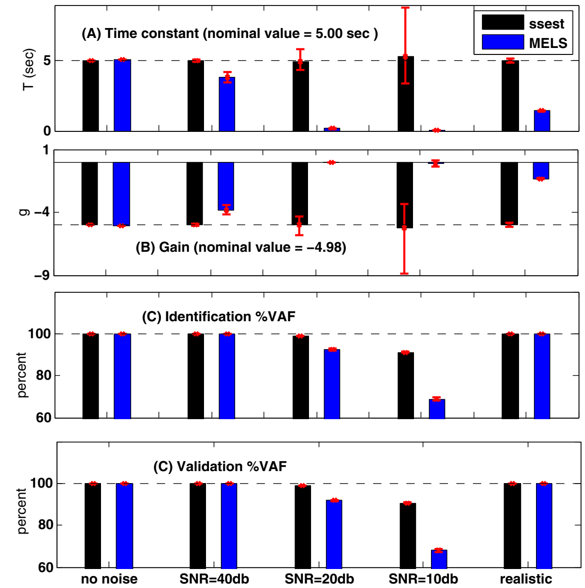

Fig. 4. Result of 100 Monte-Carlo VOR identification with *ssest* and MELS. (A) Identified time constant, T; (B) Identified gain, g; (C) %VAF of prediction on data used for identification; (D) %VAF of prediction validation data. Red bars show the 95% confidence interval of the values.

not white in many segments. On the other hand, as the noise level increases, the *ssest* algorithm identifies unbiased mean values of T and g with higher %VAF, with larger confidence intervals with higher noise levels, as expected. Non-Gaussian realistic additive noise, i.e. *Case5*, also results in unbiased identification of the dynamics and initial conditions of the VOR slow phase system with the *ssest* algorithm, while the parameters estimated by MELS are biased, despite a relatively high identification and validation %VAF.

Therefore, given low frequency sinusoidal input and realistic noise levels, the performance of the *ssest* algorithm in identifying VOR dynamics is considerably better than MELS, in terms of statistics of the residuals and robustness of estimates across noise properties.

B. Experimental Results

We evaluated the performance of the *ssest* method on experimental data. VOR data is recorded using EOG in the dark while the subjects are secured on a chair with the head restrained to a head rest. The angular head position was controlled by rotating the chair. Data is sampled at 500 Hz and is decimated to 250 Hz. Data calibration and drift removal are performed according to the procedure in [12]. Input-output data is then filtered to 55Hz using digital filtering. Rotation is performed at 1/6 Hz with peak velocity of 180 deg/sec for 65 sec. Data is first classified into slow and fast segments. The semicircular canal time constant is estimated using the approach described in Section II-A.

In order to obtain statistics on the identified dynamics we performed 100 Monte-Carlo identification-validation tests on the slow-phase segments. In each identification-validation trial, half of the total slow phase segments are selected randomly to be used to identify the dynamics and the other

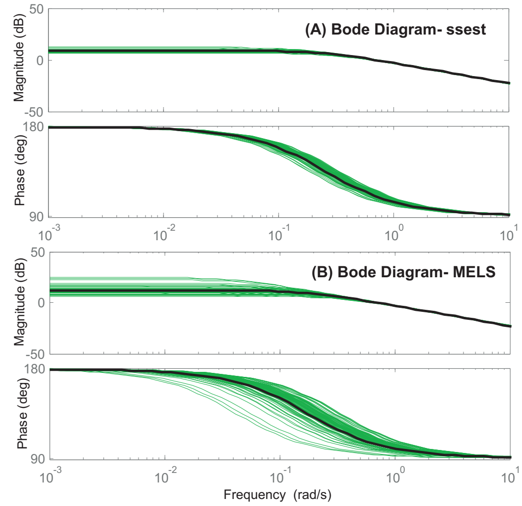

Fig. 5. Bode plot of 100 Monte-Carlo VOR identification. (A) *ssest* and (B) MELS. legend→ green: Bode of each identified system, black: average of the identified systems.

TABLE I

IDENTIFICATION RESULTS ON ONE NORMAL SUBJECT.

mean 95% interval	$T_c(sec)$	$T(sec)$	g	$\%VAF$ identification	$\%VAF$ validation
ssest	14 [14 14]	3.86 [2.95 5.32]	-2.78 [-3.93 -2.13]	99.66 [99.56 99.75]	99.61 [99.48 99.73]
MELS	14 [14 14]	5.66 [2.98 17.70]	-4.04 [-13.01 -2.12]	99.53 [99.33 99.64]	99.38 [98.73 99.62]

non-overlapping slow phase segments are used for validation. This is done to obtain 95% confidence intervals on the identified parameters. Since the true values of the VOR dynamics are unknown, we rely on %VAF and the statistics of the parameters as measures of the accuracy of our results. Fig. 5 depicts the bode plot of the 100 identified VOR slow phase dynamics using *ssest* and MELS for one healthy subject. The black line in the bode plots show the average of the identified systems. Table I also summarizes the results of this analysis. Fig. 6 shows an example of predictions with the average models from *ssest* and MELS to validation data. It is seen that despite high %VAF with the MELS algorithm, variation on the identified parameters is considerably larger than that of *ssest*. This suggests that the *ssest* method outperforms MELS in estimating the VOR dynamics, mainly beacuse of better zero-mean white residuals and repeatability.

IV. DISCUSSION

In this work, we used the state space identification approach (*ssest*) to identify the slow phase dynamics of the VOR. A linear model formulation is used to model the VOR dynamics. The challenge with identifying the VOR dynamics is its switching nature and the effects of initial conditions at each switching point that are usually ignored in *envelope* approaches. Here we evaluated the performance of the *ssest* method to estimate VOR dynamics using simulated data with different noise levels and distributions, i.e. realistic

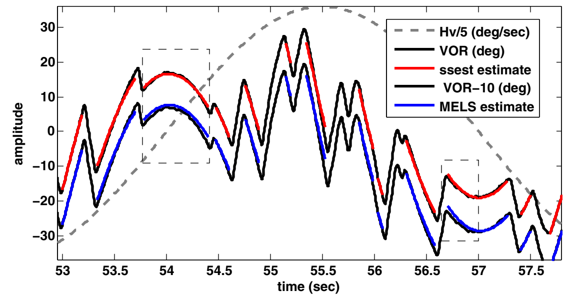

Fig. 6. Comparison of identified model prediction on validation data with *ssest* and MELS (shifted down by 10 deg). Dashed rectangles mark biased prediction with MELS.

non-Gaussian noise. The results are also compared to a former parametric method, MELS [2]. Since the former algorithm uses noisy data in its regressors matrix, i.e. it is based on 1-step ahead prediction, its performance is very sensitive to additive noise, resulting in biased identification. Comparison of *ssest* with MELS on experimental VOR data shows that the state space identification method is more accurate and robust in estimating a linear model for the VOR. There remains extension of the approach to the nonlinear characteristics of VOR data from controls and patients.

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