Modeling the Magnetic Disturbance of Pulsatile Blood Flow in a Static Magnetic Field

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***Abstract*— Non-invasive continuous monitoring of blood flow may be particularly valuable for early detection of different anomalies such as hypovolemia and internal bleeding. Recent studies have demonstrated the potential clinical benefits of photo-plethysmography in detecting hypovolemia before the onset of cardiovascular decompensation. The magnetic sensing method bears advantages of size, weight, and cost along with less stringent body placement rules. In this work, a detailed three-dimensional mathematical model for the acquisition of the ventricular response using the disturbance created by magnetized blood undergoing a stationary permanent magnet is presented. The proposed model accounts for the different magnetic properties of the blood such as the relaxation time and the magnetic saturation. The blood flow is simulated by means of Navier-Stocks equations with pulsatile inlet pressure. The blood is assumed to be in the deoxygenated state and has a diamagnetic properties. Moreover, a moving mesh technique is implemented in the Finite-Element model to represent the idle and the moving states of the blood which provides the capability to model the magnetized blood as a moving magnet. The simulated magnetic field at different sensor locations is found to be in good agreement with experimental observations from the literature. The presented model can provide basis for understanding the magnetic modulated blood signal as well as the practical constraints that might be encountered in the design of such devices.**

I. INTRODUCTION

The magnetic sensing method can be a transformative technology in enabling wearable pulse rate [1], and blood flow/pressure monitoring due to the following unique features: (i) Miniature, low power components (ii) Direct contact with skin is not needed and multi-site sensor placement (e.g. wrist or foot) is feasible. (iii) The technique is immune to body movement. Non-invasive continuous monitoring of the aforementioned critical parameters with current wireless wearable devices is a major challenge. Pressure cuff sensors are routinely used to measure arterial blood pressures. However, these sensors are bulky, intermittent, and do not allow continuous monitoring. A non-invasive cuff-less pulse transient time approach has been proposed for blood flow/pressure monitoring, yet this technique is not mature.

The main applications in focus are related to portable and continuous monitoring of blood flow, pressure, and oxygenation levels for home-health as well as ambulatory care settings. Nevertheless, recent studies [2], [3] can widely open the door for a larger range of applications by unveiling the relationship between the blood flow properties and different pathological conditions such as sickle cell disease and peripheral venous disease (PVA). However, a deep understanding of each condition along with the appropriate signal extraction and processing tools are essential. According to Webster [4], several measurements are clinically important: first desire is to know the concentration of O^2 in the cells (which is correlated the magnetic susceptibility of the blood), second is the blood flow and volume (which correlates with gases, nutrients, waste), and third is the blood pressure (which sometimes correlates with blood flow). According to Cordell [5] several conditions can be diagnosed by invasive flow methods:

- **Cardiac Output:** Assessing congestive heart failure & fluid volume status.
- **Atherosclerosis:** Assessing congestive heart failure & fluid volume status.
- **Occluded Arterial Disease:** assessing flow restoration immediately after revascularization.
- **Clot Monitoring:** monitoring for post vascularization thrombosis.
- **Extracorporeal:** assessing flow.

Initial studies by at least two separate research groups [6], [1] show promising results. However, limited work has been published on the basic principles and the studies were limited to idealized laboratory environments. In [7], [8] the authors implemented an empirical formula to represent the pulsatile nature of the blood flow. They constructed a two-dimensional model and calculated the magnetic field at the sensor. Although the simulation results were correlated to the measurements, the model did not account for the magnetization relaxation nature of the blood and lacked the realistic representation of the flow, geometry, and the magnetic interaction between the blood and the generated magnetic field. Finally, in [9], the author made an attempt to model the interaction between a ferromagnetic fluid and an external magnetic field following a two-dimensional representation. The author commented on the time decay of the blood magnetization and its potential effect on the output waveform; however, the simulation did not account for this phenomenon. Based on the limited work done regarding this topic bearing significant potential impact, the need to fully understand this phenomenon is urging.

In order to fully represent the behaviour of blood undergoing magnetic field, the following issues should be covered:

1) Magnetization properties of blood need to be investigated further to understand and the signal entitlement. Initial review of published literature suggests that the magnetic field perturbations are in the order of μT, which are practically detectable using miniature sensors. However, the information on changes in pertur-

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bation due to varying blood constituents and flow is not available.

2) Driver/receiver electronics that fit size and power constraints will be a challenge and need development. Our initial efforts will be limited to the first point in order to ensure that most scenarios related to blood pulse signal are uniquely identified with the magnetic sensing technique.

II. MATHEMATICAL FORMULATION

A. Ferrohydrodynamics

The fluid mechanics equations governing ferrohydrodynamics are conservation of linear and angular momentum equations given by [10]

$$\rho\left(\frac{\partial \mathbf{v}}{\partial t}+\mathbf{v}\cdot\nabla\mathbf{v}\right)=-\nabla p+2\zeta\nabla\times\omega+(\zeta+\eta)\nabla^2\mathbf{v}+\mathbf{F} \quad (1)$$

$$I\left(\frac{\partial \omega}{\partial t}+\mathbf{v}\cdot\nabla\omega\right)=2\zeta(\nabla\times\mathbf{v}-2\omega)+\gamma\nabla^2\omega+\mathbf{T} \quad (2)$$

where $\mathbf{v}[m/s]$ is the flow velocity, $\omega[1/s]$ is the spin velocity, $p[N/m^2]$ is the pressure, $\rho[kg/m^3]$ is the fluid density, $\eta[Ns/m^2]$ is the dynamic viscosity, $\zeta[Ns/m^2]$ is the vortex viscosity, $\gamma[Ns]$ is the shear coefficient of spin viscosity, and $\mathbf{F}[N]$ & $\mathbf{T}[Nm]$ represents body forces and torques per unit volume (magnetic forces and torques in this context) acting on the fluid.

B. Magnetization-Relaxation of Blood

In the presence of external magnetic field, the magnetic particles in the blood try to align their magnetic moments ($\mathbf{m}$) in the direction of the local magnetic field causing what is called magnetization of blood. However, this alignment is impeded by two processes [11]: (i) Brownian relaxation τ_B; at which the vector $\mathbf{m}$ rotates with the particle itself, and (ii) Néel mechanism; at which $\mathbf{m}$ rotates inside the particle with a reference time τ_N while the particle itself does not rotate. The Néel relaxation time constant depends on the magnetic anisotropy energy density, therefor its value changes with the applied magnetic field. If the external magnetic field is set to zero, the blood magnetization $\mathbf{M}$ shall decay to zero with an effective relaxation time of [12], [13]

$$\tau_{eff}=\frac{\tau_N\tau_B}{\tau_N+\tau_B} \quad (3)$$

Hence, the blood magnetization relaxation equation is given by [12]

$$\frac{\partial \mathbf{M}}{\partial t}+(\mathbf{v}\cdot\nabla)\mathbf{M}=\omega\times\mathbf{M}-\frac{1}{\tau_{eff}}(\mathbf{M}-\mathbf{M}_{eq}) \quad (4)$$

where $\mathbf{M}_{eq}$ is the equilibrium magnetization $[A/m]$ and is given by the Langevin equation

$$\mathbf{M}_{eq}=\mathbf{M}_{sat}\left(coth(\alpha)-\frac{1}{\alpha}\right)\frac{\mathbf{H}}{H} \quad (5)$$

where $\mathbf{M}_{sat}$ is the saturation magnetization of the blood and α depends on the applied magnetic field and temperature.

The magnetic properties of the blood are of a great influence on the generated magnetic signal. The main properties are:

- Relaxation time (τ_{eff}): the time required for magnetization/demagnetization.
- Saturation magnetization $\mathbf{M}_{sat}$: the maximum possible level of blood magnetization

τ_{eff} values are dependent on the hematocrit content, oxygenation level, and temperature. Recently, M. Cano and R. Castaneda-Priego [14] have proposed a model to calculate the magnetization of human blood by means of Brownian dynamics. A saturation magnetization of $\mathbf{M}_{sat}\simeq 0.6A/m$ is reported and compared to other measurements [15], [16], [17] which shows a good agreement with the simulated hemoglobin magnetic susceptibility.

C. Magnetic Field Equations

Maxwell's equations for static magnetic field are

$$\nabla\cdot\mathbf{B}=0 \quad (6)$$

$$\nabla\times\mathbf{H}=\mathbf{J} \quad (7)$$

where $\mathbf{B}[T]$ is the magnetic field density and $\mathbf{J}[A/m^2]$ is the current density. The relation between the magnetic density $\mathbf{B}$ and intensity $\mathbf{H}$ is

$$\mathbf{B}=\mu\mathbf{H} \quad (8)$$

where $\mu=\mu_0\mu_r$. Also, we have

$$\mathbf{M}=\chi\mathbf{H} \quad (9)$$

where $\mu_r=\chi+1$, μ is the permeability of blood, μ_0 is the permeability of vacuum, μ_r is the relative permeability of blood, and χ is the magnetic susceptibility of blood. Deoxygenated blood (venous) shows diamagnetic properties ($\chi<0$) while oxygenated blood (arterial) has paramagnetic properties ($\chi>0$).

The magnetized blood interacts with the external magnetic field to produce attractive force on each magnetized particle in the blood. The magnetic force presents itself as a body force and torque on the liquid. The magnetic force and torque on blood per unit volume is given by

$$\mathbf{F}=\mu_0(\mathbf{M}\cdot\nabla)\mathbf{H} \quad (10)$$

$$\mathbf{T}=\mu_0\mathbf{M}\times\mathbf{H} \quad (11)$$

where μ_0 is magnetic permeability of free space, M is the magnetization, and H is the magnetic field strength.

D. Assumptions

This paper is concerned with modeling the magnetic signal produced by magnetized blood undergoing a permanent magnet. It is clear from (10) and (11) that the internal magnetic force and torque are proportional to the magnetic field $\mathbf{H}$ and blood magnetization $\mathbf{M}$. Pai and Haik [18] calculated a magnetization level of about $100A/m$ at $10T$ external applied field. On the other hand, E. Tzirtzilakis [19] reported an insignificant change in the flow behavior under external magnetic field $B<2T$. For an external magnetic field generated

by a permanent magnet, the B level is typically under $2T$. Henceforth, the magnetic force and torque on the blood are neglected in this context leading to the elimination of (2). Moreover, we assume non-conducting medium.
The simplified system of equations is

$$\rho\left(\frac{\partial \mathbf{v}}{\partial t}+\mathbf{v}\cdot\nabla\mathbf{v}\right)=-\nabla p+(\eta)\nabla^2\mathbf{v} \tag{12}$$

$$\frac{\partial \mathbf{M}}{\partial t}=\frac{1}{\tau_{eff}}(\mathbf{M}_{eq}-\mathbf{M}) \tag{13}$$

$$\nabla\cdot\mathbf{B}=0 \tag{14}$$

$$\nabla\times\mathbf{H}=0 \tag{15}$$

III. RESULTS AND DISCUSSION

The model is developed with COMSOL (FEM package). The coupling between the magnetic and the flow systems is performed through the utilization of moving mesh technique. The moving mesh allows the blood to get magnetized during the idle state of the flow. Once the blood starts flowing, the magnetization is preserved. However, demagnetization starts once the blood leaves the magnet region. The magnetization/demagnetization of the blood is taken care of by (13). The magnetic susceptibility of blood has been measured experimentally [18], [20] to be -6.6×10^{-7} for the deoxygenated and 3.5×10^{-6} for the oxygenated blood, respectively. This paper is concerned with the deoxygenated blood. Moreover, the blood density $\rho=1050kg/m^3$ and the dynamic viscosity $\eta=3.45\times10^{-3}Ns/m^2$ are provided in [21]. The rest of the model parameters listed in Table I.
The blood is also assumed to be a Newtonian fluid with no-slip condition. The driving force for the blood flow in a blood vessel is the gradient of pressure across the vessel. Thus the pulsatile inlet boundary condition for the pressure is used at the inlet and zero pressure at the outlet. The following formula is used to reproduce the inlet pressure pulse

$$p=p_0\left[sin(2\pi ft)+\sqrt{(sin(2\pi ft))^2}\right] \tag{16}$$

where p_0 is a reference pressure and f is the frequency of the heart beat.
The permanent magnet is modeled as a disk with diameter $D=20mm$ and thickness $t=3mm$, as shown in Figure 1. The remanent magnetic flux density of the magnet is $1.2T$. The layer of skin between the blood vessel, magnet and sensor is modeled as a $1mm$ layer of non-magnetic tissue. In Figure 2 we show the axial profile of the velocity during one cycle at the center of the blood vessel. Figures 3 and 4 provide the disturbance in magnetic field density $1mm$ above the blood vessel at different distances from the magnet (at which the magnetic sensors are assumed to be located). The x-direction component of the magnetic field is neglected due to the insignificant change in its value. The filed densities B_y and B_z have a ventricular response that represent the heart cycle. Also, the magnetic field is maximum at $5mm$ from the magnet. However, the next maxima is at $20mm$ for B_y and $25mm$ for B_z. This behavior is also observed in [6] which is due to the interaction of both the stationary magnet and moving magnetized blood that is acting as a decaying magnet itself. We believe that the maximum at $5mm$ could not be detected in [6] due to the sensor saturation at close proximity of the magnet. Figures 5 and 6 provide the disturbance in magnetic field density for a reduced blood velocity of $v_{z,max}=0.25m/s$ at the center line of the blood vessel. The observed change in the sensor's measurements is quite significant. The maximum B_y field is now shifted to $10mm$ away from the magnet with a clear time shift between the first two measurements ($5mm$ and $10mm$) and the rest of the measurements. For B_x, the maximum field still appears at $5mm$ however, the second maxima is now at $10mm$. It is obvious that a significant reduction in the blood flow affects the measurement signal by shifting the maximum field towards the magnet. This might be a result of the decayed blood magnetization which has a longer time to demolish for slower velocities.

TABLE I: Model Parameters.

Parameter	Value	Description
μ_0	$4\pi\times10^{-7}Vs/Am$	Permeability of vacuum
χ	-6.6×10^{-7}	Susceptibility of blood
η	$3.45\times10^{-3}Ns/m^2$	Dynamic viscosity of blood
f	$1Hz$	Heart beat frequency
τ_{eff}	$0.5sec$	Effective relaxation time
D	$20mm$	Magnet diameter
d	$3mm$	Blood vessel diameter

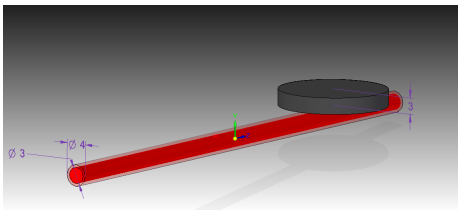

Fig. 1: Three-dimensional model geometry showing the blood vessel (red) and magnet (black).

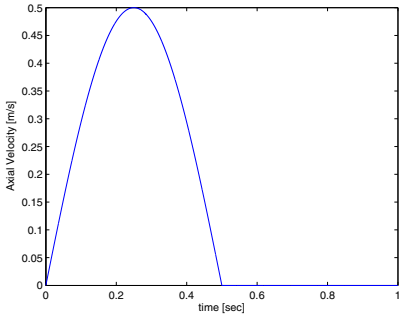

Fig. 2: Axial velocity v_z at the center of the blood vessel.

IV. CONCLUSION

We here addressed the problem of modeling the magnetic disturbance signal generated by blood undergoing a permanent magnet. The proposed model is based on Navier-Stocks

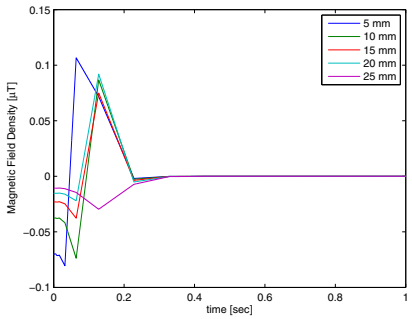

Fig. 3: B_y at different distances from the magnet and $v_{z,max} = 0.5m/s$.

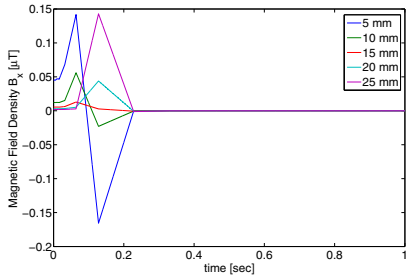

Fig. 4: B_z at different distances from the magnet and $v_{z,max} = 0.5m/s$.

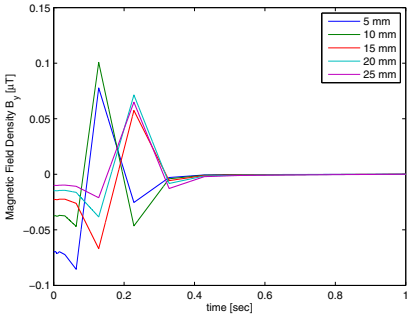

Fig. 5: B_y at different distances from the magnet and $v_{z,max} = 0.25m/s$.

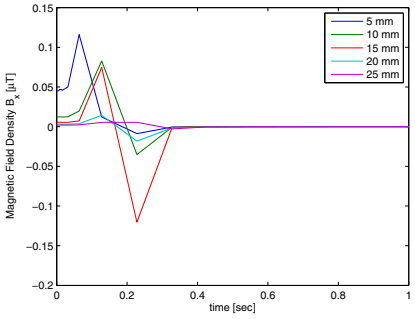

Fig. 6: B_z at different distances from the magnet and $v_{z,max} = 0.25m/s$.

equations coupled with magnetic relaxation and Maxwell's equations. The simulation results are found to be in good agreement with the experimental results acquired in [6], [1]. Such a model allows better understanding of this phenomenon and aids in the design and optimization of wearable devices for pulse rate measurements. This is, however, a preliminary study and future work shall consider more complicated geometries and shall also focus on modeling the magnetic signal for different abnormalities in the blood flow.

REFERENCES

[1] C. T. Phua and G. Lissorgues, "Non-invasive measurement of blood flow using magnetic disturbance method," 2009.

[2] C. H. Schmitz, H. L. Graber, and R. L. Barbour, "Peripheral vascular noninvasive measurements," *Encyclopedia of Medical Devices and Instrumentation*, 2006.

[3] M. Siebes and Y. Ventikos, "The role of biofluid mechanics in the assessment of clinical and pathological observations," *Annals of biomedical engineering*, vol. 38, no. 3, pp. 1216–1224, 2010.

[4] J. Webster, *Medical instrumentation: application and design*. Wiley. com, 2009.

[5] A. R. Cordell, A. S. Hudspeth, and F. R. Johnston, "Clinical blood flow measurement–evaluation of a current method.," *Annals of Surgery*, vol. 167, no. 5, p. 779, 1968.

[6] J. Joseph and V. Jayashankar, "Magnetic sensor for non-invasive detection of blood pulse and estimation of arterial compliance," 2010.

[7] C. T. Phua and G. Lissorgues, "Modeling of pulsatile blood flow in a weak magnetic field," *World Academy of Science, Engineering and Technology*, vol. 54, 2009.

[8] C. T. Phua and G. Lissorgues, "Mathematical modeling with experimental verifications of non-invasive blood flow acquired using the method of magnetic disturbance," 2010.

[9] F. L. Sinatra, *Understanding the Interaction Between Blood Flow and an Applied Magnetic Field*. PhD thesis, University of South Florida, 2010.

[10] R. E. Rosenzweig, *Ferrohydrodynamics*. DoverPublications. com, 1997.

[11] M. I. Shliomis, "Ferrohydrodynamics: Retrospective and issues," in *Ferrofluids*, pp. 85–111, Springer, 2003.

[12] M. Shliomis, "Magnetic fluids," *Soviet Physics Uspekhi*, vol. 17, no. 2, p. 153, 1974.

[13] Y. L. Raikher and M. I. Shliomis, "The effective field method in the orientational kinetics of magnetic fluids," *Adv. Chem. Phys*, vol. 87, p. 595, 1994.

[14] M. Cano, R. Castañeda-Priego, A. Barrera, J. Estrada, P. Knauth, and M. Sosa, "Magnetisation of red blood cells: a brownian dynamics simulation," *Revista mexicana de física*, vol. 58, no. 5, pp. 391–396, 2012.

[15] M. Zborowski, G. R. Ostera, L. R. Moore, S. Milliron, J. J. Chalmers, and A. N. Schechter, "Red blood cell magnetophoresis," *Biophysical journal*, vol. 84, no. 4, pp. 2638–2645, 2003.

[16] Y. Haik, V. Pai, and C.-J. Chen, "Development of magnetic device for cell separation," *Journal of Magnetism and Magnetic Materials*, vol. 194, no. 1, pp. 254–261, 1999.

[17] W. M. Spees, D. A. Yablonskiy, M. C. Oswood, and J. J. Ackerman, "Water proton mr properties of human blood at 1.5 tesla: Magnetic susceptibility, t1, t2, t* 2, and non-lorentzian signal behavior," *Magnetic resonance in medicine*, vol. 45, no. 4, pp. 533–542, 2001.

[18] V. M. Pai, Y. Haik, and C. J. Chen, "Bio-magnetic fluid dynamics," *ASME-PUBLICATIONS-FED*, vol. 237, pp. 761–774, 1996.

[19] E. Tzirtzilakis, "A mathematical model for blood flow in magnetic field," *Physics of Fluids*, vol. 17, p. 077103, 2005.

[20] T. Higashi, A. Yamagishi, T. Takeuchi, N. Kawaguchi, S. Sagawa, S. Onishi, and M. Date, "Orientation of erythrocytes in a strong static magnetic field," *Blood*, vol. 82, no. 4, pp. 1328–1334, 1993.

[21] K. M. Khanafer, P. Gadhoke, R. Berguer, and J. L. Bull, "Modeling pulsatile flow in aortic aneurysms: effect of non-newtonian properties of blood," *Biorheology*, vol. 43, no. 5, pp. 661–679, 2006.