Dipolar estimates of the cortical map

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***Abstract*— Various methods based on anatomical or mathematical models have been developed to estimate cortical potentials. Among them, the most popular are the surface Laplacians (SL) and the Electrical Source Imaging (ESI) approaches. In this paper, we develop an informed method named dipolar cortical mapping (DCM), aiming to find a balance between ESI methods based on anatomical models and methods without strong anatomical priors, such as surface Laplacians. Our method only uses easily available information on the electrode position and is based on a physiologically parametrized family of interpolating functions. Simulation results show that DCM competes with previously proposed surface Laplacians and with the model based Minimum Norm Estimates (MNE) computed with a Boundary Element Model (BEM).**

***Index Terms*— EEG, Cortical Source Mapping, Surface Laplacians, Electrical Source Imaging**

I. INTRODUCTION

Most of the brain activities recorded by scalp EEG are commonly considered to be those of radially-arrayed cortical pyramid cells [1] within parts of the cerebral cortex, usually involved in different cognitive and behavioural functions. For example, during a given cognitive task, several brain regions are activated and their identification, beyond its fundamental interest for brain mapping, is also interesting for brain computer interfacing (BCI) or clinical applications. Therefore, it is of high interest to estimate precise maps of cortical activity from raw scalp recordings.

This task is not obvious due to the noise and of the smearing effect of skull [2], [3]. Two main trends are to be distinguished when dealing with non-invasive cortical source estimation. A first class belongs to the Electrical Source Imaging (ESI) family of inverse problems. These methods use a so-called forward model, usually based on a discretized realistic model of the head, with patient specific geometry and with standardized conductivity parameters for the different tissues. The cortical sources are modelled as current dipoles existing under each mesh vertex of the discretized cortical surface and their projection on the EEG electrodes is computed using a forward model [4]. These ESI methods are based on patient-dependent propagation models leading to high computational costs, and are prone to errors due to inherent modelling imprecisions. A second class of methods estimate the cortical map directly from the EEG measurements, with weak or absent *priors* on the mixing model such as surface Laplacians (SL), based either on specific electrode configurations or on second order derivatives of (interpolated) scalp potentials. These estimators, acting as a high-pass spatial filter, eliminate much of the volume conduction distortion, improves spatial resolution and yields to a reference-independent estimate of dura (inner skull surface) potentials [3].

The aim of this paper is to improve and further analyse a method of dipolar cortical mapping previously introduced in [5]. The main idea is to impose a plausible smooth scalp projection constraint for every dipole, formalized as a basis function belonging to a parametrized family. We use this constraint to construct a propagation matrix whose inverse would yield cortical source estimation. We compare our methods with inverse ESI methods and with popular surface Laplacians based on spline interpolation techniques considering spherical [6] and realistic geometry [7].

II. CORTICAL ACTIVITY ESTIMATION

A. Electrical source imaging

If anatomical information is available, we can use ESI methods which construct and invert patient-dependent anatomical models. Assume that scalp recordings vector at a given time instant $\mathbf{v}$ can be expressed as a linear combination of dipole amplitudes $\mathbf{s}$ and propagation coefficients or gains $\mathbf{A}$ such that:

$$\mathbf{v} = \mathbf{A}\mathbf{s} \quad (1)$$

where $\mathbf{A}(M \times P, M \ll P)$ is known as lead-field or gain matrix. Then the general solution can be found by pseudo-inverting $\mathbf{A}$:

$$\hat{\mathbf{s}} = \mathbf{A}^{+}\mathbf{v} \quad (2)$$

where $\mathbf{A}^{+} = \mathbf{W}\mathbf{A}^{T}(\mathbf{A}\mathbf{W}\mathbf{A}^{T})^{-1}$ is a pseudo-inverse of $\mathbf{A}$. As it can be seen, there are an infinity of exact solutions parametrized by the weights matrix $\mathbf{W}$, and dozens of source estimation methods can be found in the literature. The simplest solution, yet adapted for superficial sources such as the cortical ones, is to simply consider the minimum norm solution obtained by taking $\mathbf{W}$ the identity matrix.

The propagation coefficients embedded in matrix $\mathbf{A}$ depend on the geometry of the head (distances and angles between the cortical surface mesh points, *i.e.*, sources, and electrodes placed on the scalp) and on the electrical properties of the head tissues (skull, skin, ...). Under the hypothesis that the conductivities are constant within a tissue type (*i.e.*, the tissue is homogeneous and isotropic), [8] states that wrongly estimated model conductivities do not have a significant impact on the performance of minimum norm estimates, except for a possible gain mis-estimation.

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B. Surface Laplacians

Physical considerations about current flow in the scalp, assuming that the head surface can locally be considered planar, lead to an estimate of the source below a position (x, y) on the head given by the surface Laplacian SL [9]:

$$Lap = \frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial y^2} \tag{3}$$

where Φ is the potential at position (x, y) on the surface. The first proposed Laplacian method [9] estimates directly the SL at selected sites using only the sampled values of Φ, *i.e.*, the electrode potentials $\mathbf{v}$. This local estimation is obtained by computing the difference between the potential at each electrode site and the average potential of its nearest four neighbours, provided that the distances between electrodes and the angles built by the electrodes configuration are equal [10]. Similar discrete Laplacian estimators, such as bipolar, quasi-bipolar and tri-polar [11] are based on various electrode configurations, .

More elaborated global methods depend on two main factors [10]: geometry and interpolating function. These methods consist in two steps: the first one interpolates the recorded scalp potential values $\mathbf{v}$ using some spline/radial basis functions in order to obtain a continuous Φ, while the second one apply a Laplacian operator similar to the one in (3), but adapted to the considered geometry. Classical interpolation solutions vary from spherical splines to thin-plate RBFs, while derivation assumes a given geometry of the head model: spherical [6], [12], ellipsoidal [13] or realistic [14], [7].

Unlike ESI solutions (2), surface Laplacians priors are based only on geometry and do not depend on any regularization or weight matrix.

III. Dipolar cortical mapping (DCM)

Our aim is to propose a method taking into account, like simple SL, only the basic geometrical information of the electrode positions, but using a more physiologically plausible interpolation scheme. To construct this interpolator, our basic assumption is that the potentials decrease with the square of the distance from the dipolar source, in the direction of the dipole.

A. One dipole case

Let s_k be the amplitude of an equivalent dipole k placed inside the brain below electrode k, at a given depth d_{kk} and pointing to it. Then, according to our proposed general rule, the potential v_{kk} generated by this dipole k on the electrode k writes:

$$v_{kk} = \frac{s_k}{K d_{kk}^2} \tag{4}$$

where K corresponds to some proportionality constant linked to the propagation properties of the head volume.

The potential generated by the same dipole on an electrode j can easily be written as:

$$v_{jk} = \frac{s_k \cos \alpha_{jk}}{K d_{jk}^2} \tag{5}$$

where α_{jk} is the angle between the vectors pointing to electrodes k and j, and d_{jk} is the distance between dipole k and electrode j. In the following, we will focus on the estimation of the cortical map morphology and ignore the amplitude information, thus we will simply set $K = 1$.

Different approximations of d_{jk} and $\cos \alpha_{jk}$ can be suggested, depending on the hypothesised geometry. A first approximation is the planar case, proposed in [5]: the neighbouring sensors with respect to v_{kk} are assumed to be distributed on a plane orthogonal to the direction of the dipole. This leads to approximate the distance d_{jk} between the dipole k and a neighbouring sensor j as $d_{jk}^2 = x_{jk}^2 + d_{kk}^2$, hence $\cos \alpha_{jk} = x_{jk}/d_{jk}$.Then equation (5) gives the planar $\mathrm{DCM_P}$ estimate:

$$v(x_{jk}) = s_k \frac{\frac{1}{d_{kk}^2}}{\left(\frac{x_{jk}^2}{d_{kk}^2} + 1\right)^{\frac{3}{2}}} \tag{6}$$

In other words, given the depth of the dipole d_{kk} and its amplitude s_k, the potential at a point in the plane depends only on the distance from this point to the electrode above the dipole.

A more elaborated approximation of the head surface is the spherical approximation. In this case, α_{jk} and d_{jk} can be expressed with respect to the depth of the dipole d_{kk} and the radius of the sphere r, which yields to the spherical $\mathrm{DCM_S}$ estimate:

$$v(x_{jk}) = s_k \frac{2r d_{kk} - x_{jk}^2}{2r(d_{kk}^2 + x_{jk}^2(1 - \frac{d_{kk}}{r}))^{\frac{3}{2}}} \tag{7}$$

The approaches described above depend on the depth d_{kk} below the electrode of the equivalent dipole, which needs to be given. As we have shown in our previous paper [5], this parameter influences the size of the activated region on the cortical surface above and, further, on the scalp: a dipole (and the corresponding activated cortical surface) will be visible on several neighbouring electrodes only if d_{kk} is large enough. It appears then that this parameter should depend on the spatial sampling of the scalp, thus on the distance between neighbouring electrodes. Numerical simulations will be given in the results section.

B. Multiple dipoles case

Up to now, we have only dealt with the potentials generated by a unique dipole of amplitude s_k below the sensor k. Assuming that we aim to estimate the cortical activity below each scalp electrode, we need to consider one dipole per electrode, thus M dipoles. Under the same hypothesis on the depth and the orientation, the potential at a given electrode j will write as the sum of the potentials generated by the $k = 1..M$ dipoles, which writes as a sum of weighted basis functions $h(x)$:

$$v_j = \sum_{k=1}^{M} v(x_{jk}) = \sum_{k=1}^{M} h(x_{jk}) s_k \tag{8}$$

Finally (see also [5]), when considering all electrodes, (8) can be written in matrix form as:

$$\mathbf{v} = \mathbf{H}\mathbf{s} \tag{9}$$

where $\mathbf{v}$ is the measured EEG, the column k of the matrix $\mathbf{H}$ contains the values of the basis function corresponding to the dipole situated below electrode k evaluated at distances x_{jk} (j being the index of the row) and the vector $\mathbf{s}$ contains the weights of these different interpolating functions, equal to the amplitudes of the dipoles that we want to estimate. Estimating the cortical potentials corresponds then to the weights vector $\mathbf{s}$ estimation and is obtained by simple matrix inversion:

$$\hat{\mathbf{s}} = \mathbf{H}^{-1}\mathbf{v} \tag{10}$$

C. Building bridges between SL and ESI

It should be highlighted that all previous equations (6) and (7) can be seen as an interpolation method based on families of parametrized basis functions, similar to the radial basis functions (RBFs) used in the surface Laplacian approaches. In particular, it is interesting to notice that the planar approximation (6) of the DCM corresponds to the second order derivative on a plane of a multi-quadric spline. In other words, our planar DCM solution is equivalent to a surface Laplacian obtained after interpolating the scalp potentials using multi-quadric RBFs, parametrized by the depth d_{kk}, instead of thin-plate splines (the detailed proof will be presented elsewhere).

This analogy to surface Laplacians can be visualized by comparing the transform matrix $\mathbf{H}^{-1}$ applied to the measured potentials $\mathbf{v}$ with the simple Hjorth discrete Laplacian montage from the measured EEG.Indeed, as in the case of the Hjorth's Laplacian, the elements of inverted matrix $\mathbf{H}^{-1}$ correspond to the weights given to the electrodes. In the basic Laplacian the weights are unitary on the diagonal, $-1/4$ on the neighbouring electrodes and 0 elsewhere. For the DCM the weights vary on the diagonal, because of the depth d_{kk} of the dipole, and on the off-diagonal with respect to the distance between the electrodes.

On the other hand, if the analogy between DCM and surface Laplacians appears clearly for the simpler approximations (6), one can notice that the general form (5) corresponds to an infinite homogeneous and isotropic propagation medium, for which the conductivity information was discarded, as we are only interested in the morphology of the cortical map.

IV. Simulation and Results

The aim of this section is to compare the performances of the proposed DCM methods with both surface Laplacians and ESI minimum norm inverse solutions.

A. Simulation set-up

A three layer Boundary Element Model (BEM) of the head was extracted from anatomical MRI using *Brainstorm* [15], yielding a mesh where each layer consists of 3242 points. The electrodes were simulated as a *BioSemi* sensor cap of either 64 or 128 electrodes. For source generation we assumed randomly placed dipole patches on the cortical layer which corresponds to the upper half of the brain mesh (1675 dipoles to avoid border effects), oriented radially to the cortical surface and having random amplitudes. The size of the patch vary randomly between 20 and 128 mesh points, corresponding roughly to 2.5 to 20cm^2. Potentials $\mathbf{v}$ on the electrodes, as well as the simulated cortical map were generated by the forward solution through Helsinki BEM library [16]. We considered both the case of one active patch per time instant (similar to the unique source in [7]) and the most complex case of multiple simultaneous activations. We also considered two noise levels perturbing the electrodes $\mathbf{v}$ by adding white Gaussian noise with signal to noise ratios (SNR) of 20dB and 10dB (similar to [7]).

To estimate the surface Laplacians we use following MATLAB toolboxes: CSD and SSL. The first provides current source density estimates using the spherical spline surface Laplacian algorithm suggested by [6] with the established computation parameters (50 iterations; $m = 4$; $\lambda = 10^{-5}$) proposed in [17]. The SSL toolbox provides two estimates: a spherical approximation based on New Orleans Spline Laplacians [3] and a realistic case [7].

ESI minimum norm estimates (MNE) were obtained by pseudo-inverting (2) a realistic BEM model giving the gain matrix between the cortical surface and the electrodes (either 64 or 128).

DCM values were computed using either (6) or (7). For the spherical case, we fitted the sphere using FieldTrip [18]. Several values were tested for the depth parameter d_kk. It appears that the best results are obtained when choosing d_{kk} as the mean distance between electrodes. For the simulated Biosemi caps of 64 and 128 electrodes used in this study, d_{kk} corresponds respectively to 27mm and 22mm. Only these results are presented here.

The considered performance measure was the correlation ρ between the computed (true) cortical activity $\mathbf{s}$ and estimated cortical activities $\hat{\mathbf{s}}$ by DCM, SL and ESI, all sampled in the cortical mesh points below the electrodes.

B. Results

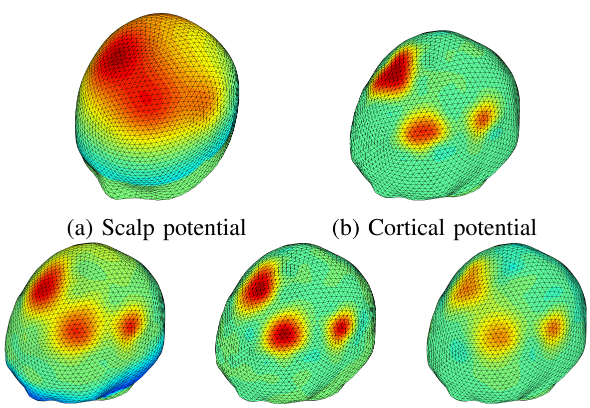
The results presented here were obtained after averaging 1000 simulations performed using the set-up described above. In other words, 1000 random BEM generated cortical maps were compared with the estimated maps obtained either by SL, DCM or MNE. Table I) presents the results for the unique active region and for multiple simultaneous activations for noise-free measurements and for 2 noise levels (20dB and 10dB). As it can be seen from table I, all methods perform badly for noisy signals and high-density measurements, with at most 69% correlation for CSD (which in fact overpasses the other methods only in this situation). Indeed, as the cut-off frequency of the surface Laplacian (high-pass) filters increases with the spatial density of the electrodes, the high density montages are more affected by noise. On the other hand, when the noise is absent, the estimations are better when the electrode density is high

	One			Multiple		
	Inf	20dB	10dB	Inf	20dB	10dB
DCMs	**84** / **88**	**84** / **87**	72 / 43	**84** / 85	**83** / 81	65 / 34
DCMp	80 / 80	80 / 79	**78** / 64	77 / 79	76 / 76	**71** / 57
CSD	70 / 67	70 / 66	70 / **67**	68 / 70	69 / 70	68 / **69**
SSLsph	81 / **88**	82 / 86	69 / 55	83 / **86**	**83** / **84**	64 / 43
SSLgeo	80 / 85	82 / 84	69 / 57	81 / 84	81 / 83	65 / 46
BEM	82 / 90	82 / 88	65 / 41	86 / 90	86 / 86	60 / 29

TABLE I: Correlation percentage ρ between the forward computed cortical map and the different estimations, with 64 / 128 scalp electrodes

regardless of the method. Finally, another general observation is that the number of active regions has a relatively low influence on the results, even if the slightly worse for the multiple activations case.

Comparing the methods, one can notice that DCM_S is better in no-noise or low-noise cases for all activations using 64 electrodes. For the higher density set-up, SSL_{sph} slightly performs better in the multiple activations case. Finally, the fully informed BEM, even if it performs better when the noise is weak, does not improve significantly the estimation of cortical activity. An example of different estimations obtained from 128 electrodes in the no-noise situation is given figure 1. The cortical maps were smoothed for visualization purposes.



(a) Scalp potential (b) Cortical potential

(c) DCM_S (d) MNE_{BEM} (e) SSL_{geo}

Fig. 1: Forward computed cortical (a) and scalp (b) maps and the obtained cortical activation estimations (c,d,e)

V. Conclusion

The goal of this paper is to propose algorithms based on simple geometrical assumptions with low computational cost. As a result we propose here a family of informed cortical map estimators (Dipolar Cortical Mapping, DCM) related both to surface Laplacians (SL) and to ESI minimum norm estimates (MNE). The DCM is based on a family of parametrized physiologically plausible radial basis functions that can be seen, depending on the considered approximation, either as an SL technique or a MNE solution. Besides, it uses easily available information even in the absence of imaging modalities, unlike the recently proposed SSL and the MNE techniques. Our proposed DCM shows good performance even for simple approximations of the head geometry, such as planar or spherical and remains reliable when multiple cortical areas are simultaneously active.

Future work will focus on a elaborating a more formal connection between the MNE, the DCM and the SL estimations of the cortical activity. Also the performance of presented methods should be tested in more realistic noise cases using multiple random noise dipoles. Another interesting perspective would be to relax the constraints imposed on the positions of the equivalent cortical dipoles used by the DCM (*i.e.*, below each EEG electrode) and thus allow sparser solutions.

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