Polynomial Chaos Decomposition applied to Stochastic Dosimetry: Study of the Influence of the Magnetic Field Orientation on the Pregnant Woman Exposure at 50 Hz

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Abstract— **Polynomial Chaos (PC) is a decomposition method used to build a meta-model, which approximates the unknown response of a model. In this paper the PC method is applied to the stochastic dosimetry to assess the variability of human exposure due to the change of the orientation of the B-field vector respect to the human body. In detail, the analysis of the pregnant woman exposure at 7 months of gestational age is carried out, to build-up a statistical meta-model of the induced electric field for each fetal tissue and in the fetal whole-body by means of the PC expansion as a function of the B-field orientation, considering a uniform exposure at 50 Hz.**

I. Introduction

Polynomial Chaos (PC) expansion is a probabilistic method consisting in the projection of the model output on a basis of orthogonal stochastic polynomials in the random inputs [1]. Because of the PC expansion results less computational time consuming than the Monte Carlo method [2], several studies in literature have already applied it for sensitivity analysis to evaluate the global variations in the output due to the uncertainties on the inputs [1-3]. The PC expansion has been also applied to the stochastic dosimetry to assess the variability of human exposure to electromagnetic fields (EMF) [4-6]. Indeed, the human exposure depends on several parameters, e.g. the posture of the subject, the position of the source and the morphology (age, dielectric properties of the tissues) that should be taken into account to build-up a "real" exposure scenario. The deterministic dosimetry would be too much time consuming to perform this analysis, therefore the stochastic dosimetry is used to build approximation models, called meta-models, that replace the expensive simulation. In this paper, as an example of application of the PC expansion to the stochastic dosimetry, the variability of the human exposure due to the change of **B**-field orientation is studied. In detail, the pregnant woman exposure to ELF-MF at 50 Hz has been taken into account. In literature several studies have already analyzed the pregnant woman exposure to ELF-MF by means of computational electromagnetics techniques [7-9]; however, most of these studies have considered only the front-to-back, lateral and top-to-bottom exposure scenarios, disregarding the other **B**-field orientations. The aim of this work is to generate a statistical model of the induced electric field **E** in the fetus at 7 months of gestational age (GA) exposed to a uniform **B**-field at 50 Hz by means of the Polynomial Chaos (PC) decomposition, in order to estimate the variation of **E** changing the **B**-field orientation.

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II. Procedure for Paper Submission

A. The Experimental Design

The simulations used to build-up the meta-model were conducted using the Magneto Quasi-Static low frequency solver of the simulation platform SEMCAD X v. 14.8 (by SPEAG, Zürich) [10]. An anatomical model of pregnant woman at 7 months GA, based on the model "Ella" of the Virtual Family [11] is used for the dosimetric analysis. A detailed description of the construction of the model is given in [12]. The conductivity values chosen for the woman and fetus tissues are described in [7]. The incident **B** vector of amplitude 1 μT at 50 Hz is supposed to rotate in the plane xz. The input parameter used for the rotation of **B** is the angle θ, which varies in the range [-180°; 180°]. The vector potential **A** and the magnetic flux density **B** are set to obtain a MF homogeneity equal to 100% for all the polarizations. The induced electric field (**E)** is considered as a vector average in a small contiguous tissue volume of 2 x 2 x 2 mm^3, as stated in [13]. The metrics of interest to build the meta-model are the 99th percentile and peak values of the RMS of **E** calculated in the fetus whole-body and in each fetal tissue.

B. The construction of the metamodel using the Polynomial Chaos

The Polynomial Chaos is a spectral method and consists in the approximation of the unknown response of a model in a suitable finite - dimensional basis $\psi_j(\mathbf{X})_{0\le j\le P-1}$ (made of orthogonal polynomials) [2] as follows:

$$\boldsymbol{Y} = M(\boldsymbol{X}) = \sum_0^{P-1} a_j \psi_j(\boldsymbol{X}) \quad (1)$$

where **Y** is the model response (in our case the induced **E** in the fetal whole-body and in each fetal tissue), **X** is the random input vector, which consists in the angle of rotation of **B** in the xz-plane. The random input vector has been generated through a Quasi- Monte Carlo method [2], which generates a sequential experimental design. The a_j are the coefficients to be estimated. In order to build-up the polynomial expansion, the input has been considered as uniform in the interval [-1; 1] and polynomials are Legendre polynomials. For each meta-model relative to the induced **E** in the fetal whole-body and in each fetal tissue the a_j coefficients have been estimated using the Least Angle Regression (LAR) algorithm [2], which provides a set of solutions rather than a unique solution as for ordinary least-square regression. The best solution among the ones generated is chosen through the estimation of the leave-one-out error at each step. An home-made Python script has been used for the Polynomial Chaos expansion by means of the openTURNS package [14]. The convergence of the polynomial expansion has been checked through a cross-validation based on the leave-one-out error estimation [2], enriching the experimental design up to 500 realizations. Furthermore, to check the goodness of the model, the trend of the percentage mean quadratic error between the output of an experimental design made of 40 points, selected with the Latin Hypercube Sampling (LHS) [15], and the relative output of the meta-models up to 500 realizations has been analyzed.

C. Statistic Fetal Exposure Assessment

A PC expansion is built separately for the 99th percentile and peak value of the induced **E** in the fetal whole-body and in each fetal tissue at 7 months GA. From these meta-models the statistical distributions of the induced **E** in the fetus, changing the incident **B**-field orientation respect to the pregnant woman body, are estimated considering a uniform exposure at 50 Hz.

III. RESULTS

In Fig. 1 the trend of the relative leave-one-out error is

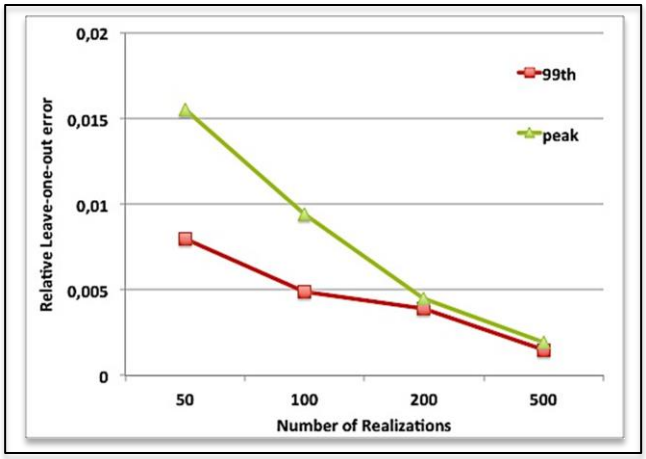

Figure 1. Relative leave-one-out error of the 99th percentile and of the peak of the induced **E** meta-models in the fetal whole-body increasing the size of the experimental design up to 500 realizations

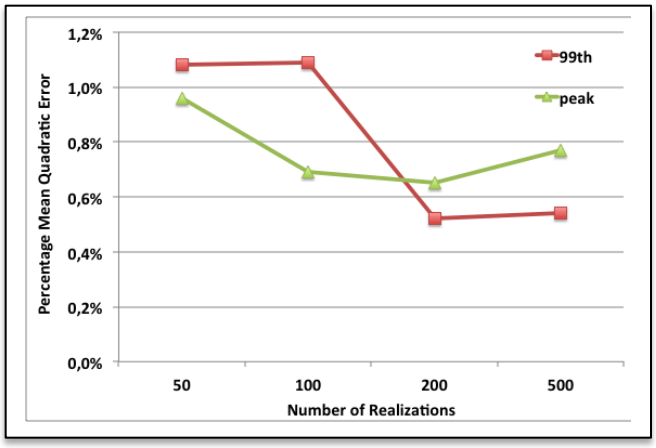

Figure 2. Percentage mean quadratic error between the outputs of the experimental design and the ones of the meta-model, estimated on 40 points generated by LHS, for the 99th percentile and the peak induced **E** in the fetal whole-body increasing the number of realizations up to 500.

reported to study the convergence of the PC expansion for both the 99th percentile and the peak value of the induced **E** in the fetal whole-body. As expected, the relative error decreases with the increment of the size of the experimental design.

In Fig. 2 the percentage mean quadratic error between the outputs of an experimental design made of 40 LHS points and the relative outputs of the meta-models up to 500 realizations is represented for the 99th percentile and the peak value, respectively. It can be observed that the percentage mean quadratic error is less than 1.5% for all the metrics.

The best meta-model has been chosen as a compromise between the lowest percentage mean quadratic errors of both the 99th and the peak of the induced **E**. In detail, 200 realizations has been used to build the meta-models with a percentage mean quadratic error of 0.52% and 0.65% for the 99th percentile and peak value, respectively.

In Fig. 3 the probability density functions (PDF) of the 99th percentile and the peak value are also represented. It can be observed that the 99th percentile presents a PDF in which the highest densities are within the range 17-20 μV/m, while the PDF of the peak value presents a maximum for 35.75 μV/m, and all the values are mostly in the range 34-38 μV/m.

IV. DISCUSSION

In this work the Polynomial Chaos expansion has been applied to the stochastic dosimetry to study the variability of human exposure to electromagnetic fields due to the change of the **B**-field orientation at 50 Hz, analysing, as an example, the fetal exposure at 7 months of gestational age. By means of the PC method the statistical distribution of the induced field in the fetus has been assessed. The method proved to be suitable to solve this type of problem with a low computational cost and with errors always within 2%. Preliminary results about the fetal whole-body exposure have been shown, in which a statistical meta-model has been built for each the 99th percentile and the peak value of the induced **E** with the PC decomposition and the probability density

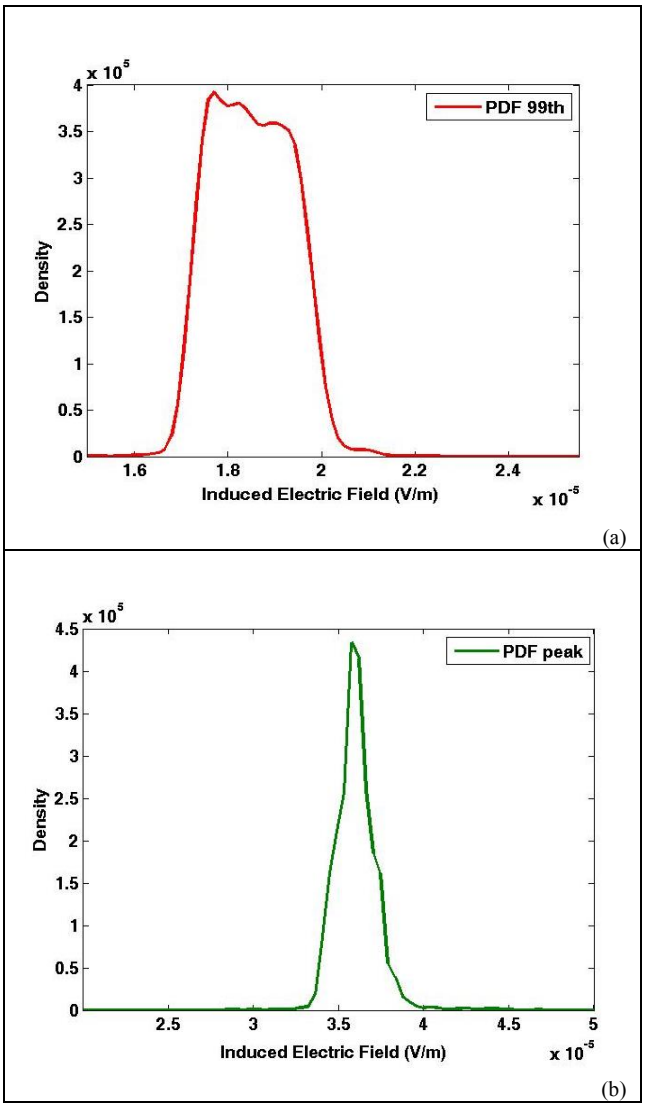

Figure 3. Probability density function (PDF) of the induced electric field E in the fetal whole-body. (a) PDF of **E** estimated from the 99th percentile meta-model built with 200 realizations, (b) PDF of **E** estimated from the peak meta-model built with 200 realizations.

functions (PDF) of both the 99th percentile and peak value have been analysed. From the PDF it has been observed that the 99th percentiles are more influenced by the variation of the B-field orientation, showing high densities in a broader range of values of the induced electric field (from 17 to 20 μV/m) than the PDF of the peak value, which remains in a narrower range around 35.75 μV/m, that presents the maximum density. Future work will be done, constructing induced electric field meta-models for each fetal tissue at 7 months GA, in order to generate a complete mapping of the fetal exposure as a function of the **B**-field orientation.

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