An Analytical Model for Regular Respiratory Signal

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Abstract— **In disaster rescue, breathing motion detection is an important approach to searching survivors trapped under debris. Detection of breathing motion is realized by detecting the respiratory signal acquired by the sensing system. In this paper, modeling the regular respiratory signal is studied. Firstly, a preliminary model is built based on power of absolute value of cosine function. Then, this preliminary model is improved in terms of some practical considerations, such as the DC-component of the respiratory signal often is removed by signal processing, and a phase uncertainty occurs due to the data acquisition. Finally, an analytical harmonic-based random respiratory signal model is derived, which can be used as the signal model in the future research about breathing motion detection.**

I. INTRODUCTION

Breathing is a normal movement of human body, which is required to sustain life. Based on the natural understanding of breathing phases, the breathing motion can be modeled using three regular breathing states, i.e., inhale (IN), exhale (EX), and end-of-exhale (EOE), and one irregular breathing state (IRR) [1].

In disaster rescue, detection of breathing motion is an important approach for the rescue team to finding survivors under debris [2-6]. In external beam radiotherapy, real-time tracking method based on prediction of breathing motion allows beam delivery under free breathing conditions [1, 7-9]. In these applications, the breathing motion yields a respiratory signal in the data acquired by the sensing system. Then, detection or prediction of breathing motion is realized by detecting or analyzing this respiratory signal.

In this paper, we call the breathing motion without the IRR, regular breathing, and call the respiratory signal produced by the regular breathing, *regular respiratory signal*. Fig. 1 presents some experimental data acquired by the UWB impulse radar sensor employed in [6], where a typical regular respiratory signal including three regular breathing phases is contained.

In disaster rescue, the breathing detection system commonly detects a place in a short time, e.g., tens of seconds or a few minutes, to decide the existence of trapped survivors. Although in a long time period, the respiratory rate of a trapped person probably changes, it often keeps fixed approximately during the short detection period. Additionally, the unconscious breathing of a trapped person often is regular. Then, the acquired respiratory signal often is regular and nearly periodic. Therefore, a detector for the periodic regular respiratory signal is useful in disaster rescue.

According to detection theory, the choice of a detector depends upon many considerations. Of primary concern is the selection of a good mathematical model for describing the data statistically [10]. Commonly, the data model consists of the signal model and the noise model. In previous works related to breathing motion detection [2, 4-6, 11], less efforts have been made for modeling the respiratory signal. In this paper, we aim to build up an analytical signal model for the periodic regular respiratory signal.

This paper is organized as follows. In section II, the models adopted in previous works are reviewed, and a preliminary model is built up based on power of absolute value of cosine function. Then, this preliminary model is improved in the following two sections. In section III, the DC component is removed, and the signal power is introduced into the model as a parameter. In section IV, a random phase parameter is introduced, and the final model is obtained. In section V, the prior knowledge contained in the final model is analyzed. In section VI, an experimental result is presented. A conclusion is drawn in section VII.

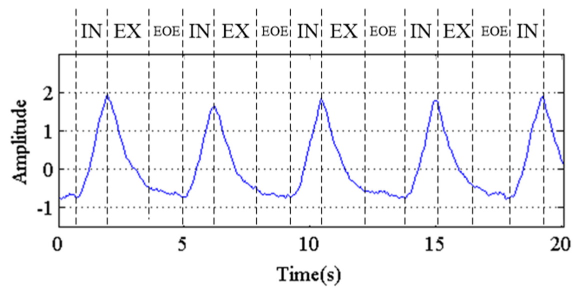

Fig. 1 An experimental regular respiratory signal

II. MODELING BASED ON TRIGONOMETRIC FUNCTION

A. A review of the models adopted in previous works

In this paper, we define the duty cycle of the regular breathing motion as follows,

$$D \triangleq \frac{L_{IN}+L_{EX}}{L_{IN}+L_{EX}+L_{EOE}}, \quad (1)$$

where D denotes the duty cycle, and L_{IN}, L_{EX}, and L_{EOE} are the time lengths of the IN phase, the EX phase, and the EOE phase, respectively. The regular breathing motions could have various duty cycles. For example, a person who just completes a 100m race should have a quite short EOE phase, leading to a

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large duty cycle, while a sleeping person often has a long EOE phase, leading to a small duty cycle.

Three models, listed in Table I, have been adopted in previous works, including the model based on cosine function (CM) [5, 8], the model based on absolute value of cosine function (ACM) [11], and the model based on even power of cosine function (EPCM) [6, 7, 9]. Fig 2, showing the results of using these models to fit the experimental data presented in Fig. 1, demonstrates that the EPCM obtains a better fit than the other two.

For the EPCM, the duty cycle [1] depends on its parameter m, and decreases with the increase of m, as shown in Fig. 3, where the graphs of the EPCM with $m = 1{\sim}5$ are presented.

A disadvantage of the EPCM is the possible significant deviation between the duty cycle, that the model can achieve, and the duty cycle of the actual respiratory signal. Fig. 3 presents a case that a significant deviation occurs, where the respiratory signal is plotted by the red dotted line.

TABLE I RESPIRATORY SIGNAL MODELS ADOPTED IN PREVIOUS WORKS

Model name	Mathematical expression*
CM	$M_C^{\langle f,d_C,A_C\rangle}(t) = d_C + A_C\cos(2\pi f t)$
ACM	$M_{AC}^{\langle f,d_{AC},A_{AC}\rangle}(t) = d_{AC} - A_{AC}\lvert\cos(\pi f t)\rvert$
EPCM	$M_{EPC}^{\langle f,d_{EPC},A_{EPC},m\rangle}(t) = d_{EPC} + A_{EPC}(\cos(\pi f t))^{2m}$

* In the mathematical expressions, t denotes the time variable, and the model parameters are given in the upper scripts of the symbols, which represent the models. Parameters d_C, d_{AC}, and d_{EPC} are DC terms, A_C, A_{AC}, and A_{EPC} reflect the amplitudes of breathing, f is the respiratory frequency, and parameter m determines the power order of the EPCM.

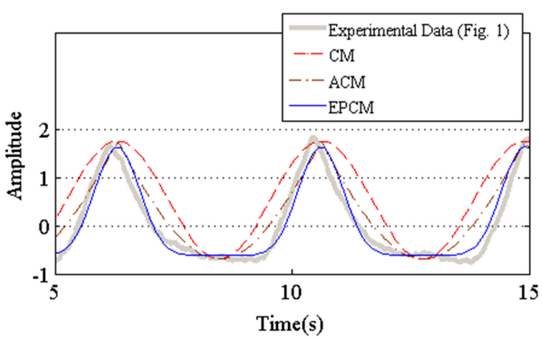

Fig. 2 The results of using models adopted in previous works to fit the experimental data presented in Fig. 1.

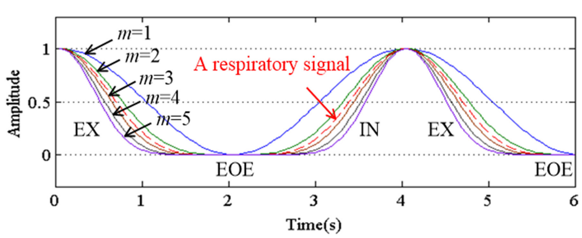

Fig. 3 The graphs of the EPCM for $m = 1{\sim}5$, and a respiratory signal painted by red dotted line.

B. Respiratory signal model based on power of absolute value of cosine function

We establish a respiratory signal model based on power of absolute value of cosine function (PACM), as follows,

$$M_{PAC}^{\langle f,d_{PAC},A_{PAC},n\rangle}(t) \triangleq d_{PAC} + A_{PAC}\lvert\cos(\pi f t)\rvert^n, \quad (2)$$

where d_{PAC} is a DC term, A_{PAC} reflects the amplitude of breathing, f is the respiratory frequency, and n is the power order.

The PACM contains the EPCM as its subset, since $M_{EPC}^{\langle f,d_{EPC},A_{EPC},m\rangle} = M_{PAC}^{\langle f,d_{PAC},A_{PAC},n\rangle}$, if $n = 2m$, $A_{PAC} = A_{EPC}$, and $d_{PAC} = d_{EPC}$. The graphs of the PACM for $n = 2{\sim}10$ are shown in Fig. 4.

According to Fig. 3 and 4, we can see that compared with the EPCM, the PACM improves the disadvantage of the duty cycle deviation to some extent, due to the increase of the achievable duty cycles.

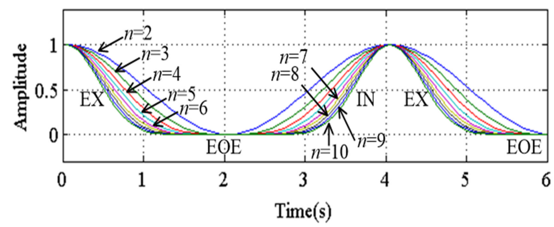

Fig. 4 The graphs of the PACM for $n = 2{\sim}10$.

III. HARMONIC-BASED MODEL DERIVED FROM THE PACM

The PACM represented by (2) contains a non-zero DC component. However, in practical detection systems, the DC component of respiratory signal commonly will be removed. For example, in [2], the DC component of the respiratory signal is cancelled by the clutter suppression algorithm proposed in [12]. Therefore, a model, for which the DC component is eliminated, will better fit the practical respiratory signal.

According to (17) (see appendix), we have

$$\lvert\cos\pi f t\rvert^n = \begin{cases} A_{even}^{\langle n\rangle}(0) + \sum_{q=1}^{\frac{n}{2}} A_{even}^{\langle n\rangle}(q)\cos(2q\pi f t) & n \in \mathbb{N}_e \\ \sum_{q=0}^{+\infty} A_{odd}^{\langle n\rangle}(q)\cos(2q\pi f t) & n \in \mathbb{N}_o \end{cases}, \quad (3)$$

where the definitions of the notations are given in appendix. Substituting (3) into (2), and removing the DC terms, we can get a new model with zero DC component,

$$M_{PAC-DC}^{\langle f,A_{PAC},n\rangle}(t) \triangleq \begin{cases} \left(\sum_{q=1}^{\frac{n}{2}} A_{PAC}A_{even}^{\langle n\rangle}(q)\cos(2\pi q f t)\right) & n \in \mathbb{N}_e \\ \left(\sum_{q=1}^{+\infty} A_{PAC}A_{odd}^{\langle n\rangle}(q)\cos(2\pi q f t)\right) & n \in \mathbb{N}_o \end{cases}. \quad (4)$$

The signal power of the respiratory signal represented by (4),

[1] A qualitative analysis for the duty cycle is enough for us to convey the ideas within this paper. So a clear mathematical definition for the duty cycle of the model, which can be derived from (1), is not included in this paper.

$$P = \begin{cases} \frac{1}{2}\sum_{q=1}^{\frac{n}{2}} \left(A_{PAC}A_{even}^{\langle n\rangle}(q)\right)^2 & n \in \mathbb{N}_e \\ \frac{1}{2}\sum_{q=1}^{+\infty} \left(A_{PAC}A_{odd}^{\langle n\rangle}(q)\right)^2 & n \in \mathbb{N}_o \end{cases}. \tag{5}$$

If we introduce the signal power P into the model as a parameter, then (4) can be rewritten as

$$M_{PAC-DC}^{\langle f,P,n\rangle}(t) = \begin{cases} \sqrt{P}\left(\sum_{q=1}^{\frac{n}{2}} \frac{A_{even}^{\langle n\rangle}(q)}{\sqrt{\frac{1}{2}\sum_{q=1}^{\frac{n}{2}}\left(A_{even}^{\langle n\rangle}(q)\right)^2}} \cos(2\pi qft)\right) & n \in \mathbb{N}_e \\ \sqrt{P}\left(\sum_{q=1}^{+\infty} \frac{A_{odd}^{\langle n\rangle}(q)}{\sqrt{\frac{1}{2}\sum_{q=1}^{+\infty}\left(A_{odd}^{\langle n\rangle}(q)\right)^2}} \cos(2\pi qft)\right) & n \in \mathbb{N}_o \end{cases}. \tag{6}$$

Define

$$H_{as}^{\langle n\rangle}(q) \triangleq \begin{cases} 0 & n \in \mathbb{N}_e \text{ and } q > \frac{n}{2} \\ A_{even}^{\langle n\rangle}(q) \Big/ \sqrt{\frac{1}{2}\sum_{v=1}^{\frac{n}{2}}\left(A_{even}^{\langle n\rangle}(v)\right)^2} & n \in \mathbb{N}_e \text{ and } q \le \frac{n}{2}, \\ A_{odd}^{\langle n\rangle}(q) \Big/ \sqrt{\frac{1}{2}\sum_{v=1}^{+\infty}\left(A_{odd}^{\langle n\rangle}(v)\right)^2} & n \in \mathbb{N}_o \end{cases} \tag{7}$$

resulting that (6) can be rewritten as

$$M_{PAC-DC}^{\langle f,P,H_{as}^{\langle n\rangle}\rangle}(t) = \sqrt{P}\left(\sum_{q=1}^{+\infty} H_{as}^{\langle n\rangle}(q)\cos(2\pi qft)\right) \quad n \in \mathbb{N}, \tag{8}$$

where the parameter vector of the model becomes $\langle f, P, H_{as}^{\langle n\rangle}\rangle$.

IV. Random Respiratory Signal Model

In this section, we call the beginning of the EX phase, breathing peak (BP). For the model M_{PAC-DC} represented by (8), the origin of time axis is located at a BP. Generally, an operator starts the data acquisition arbitrarily. We denotes the time interval, between the starting time of the data acquisition (ST) and the nearest BP before the ST, by t_Δ. If adopting the ST as the origin of time axis, which always is true in practical applications, then we get a new respiratory signal

$$M_{HRM}^{\langle f,P,H_{as}^{\langle n\rangle},t_\Delta\rangle}(t) \triangleq M_{PAC-DC}^{\langle f,P,H_{as}^{\langle n\rangle}\rangle}(t - t_\Delta), \tag{9}$$

where $t_\Delta \in [0, T_r)$, and $T_r = 1/f$, denoting the respiratory period. An example with $t_\Delta = 0.5T_r$ is presented in Fig. 5.

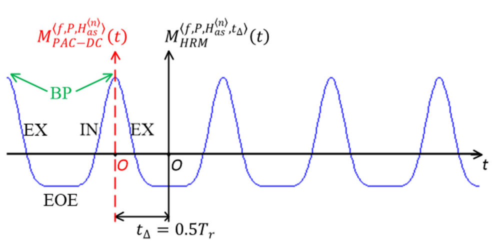

Fig. 5 The relation between the HRM and the DC-removed PACM.

Commonly, it is acceptable to model t_Δ as a random variable with uniform distribution in $[0, T_r)$. If we define $\varphi = 2\pi f t_\Delta$, then φ is uniformly distributed in $[0, 2\pi)$. Using φ instead of t_Δ, (9) can be rewritten as

$$M_{HRM}^{\langle f,P,H_{as}^{\langle n\rangle},\varphi\rangle}(t) = \sqrt{P}\left(\sum_{q=1}^{+\infty} H_{as}^{\langle n\rangle}(q)\cos(2\pi qft - q\varphi)\right). \tag{10}$$

Finally, by a series of derivations, we get a model represented by (10), which is the final model proposed in this paper. We call this final model, harmonic-based random model of the regular respiratory signal (HRM).

V. Prior Knowledge

The HRM represents a periodic signal by a linear combination of its harmonics. Why do not we model the respiratory signal based on its harmonics at the beginning of the modeling? In fact, if we do so, then a respiratory signal model based on harmonics (HM) can be written as follows,

$$M_{HM}^{\langle f,P,A'_q,\beta'_q,\varphi\rangle}(t) \triangleq \sqrt{P}\left(\sum_{q=1}^{+\infty} A'_q \cos(2\pi qft + \beta'_q - q\varphi)\right), \tag{11}$$

where f denotes the respiratory frequency, P denotes the signal power, A'_q denotes the amplitude of the qth order harmonic of the respiratory signal with unit power, β'_q denotes the phase of the qth order harmonic, and φ denotes the random phase parameter with uniform distribution in $[0, 2\pi)$.

Although the HM represented by (11) has a mathematical expression similar to the HRM represented by (10), the former contains less prior knowledge than the latter.

The HM provides no prior information about its harmonic amplitudes. But for the HRM, the harmonic amplitudes are determined by the signal power P and the function $H_{as}^{\langle n\rangle}(q)$. According to Fig. 6, which shows the graphs of $H_{as}^{\langle n\rangle}(q)$ with different n, the respiratory signal has a dominant fundamental harmonic, and with the decrease of the duty cycle, i.e., the increase of n, the amplitudes of the 2nd and 3rd order harmonics increase gradually. Furthermore, according to the HRM, it is clear that β'_q, the unknown phase parameter in the HM, equals zero.

The HRM contains additional prior knowledge. According to detection theory, more prior knowledge generally means a more specific detector with a better detection performance.

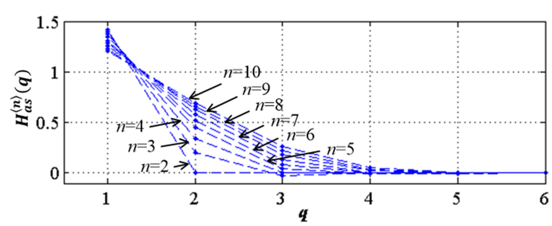

Fig. 6 The graphs of $H_{as}^{\langle n\rangle}(q)$ for $n = 2 \sim 10$

VI. Experimental Result

The result of using the HRM to fit the experimental data presented in Fig. 1 is shown in Fig. 7. Firstly, the value of the respiratory frequency f is obtained from the dominant frequency in the periodogram of the experimental data. Then, other parameters of the model, i.e., P, n and φ, are determined by the least square approach, which is realized based on a grid search method. The fitting result is shown in Fig. 7, where $f = 0.233\text{Hz}$, $P = 0.6097$, $\varphi = 0.922\pi$, and $n = 7$. The mean squared error is 0.0370.

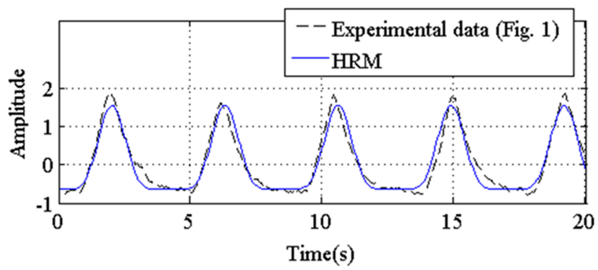

Fig. 7 The result of using the HRM to fit the experimental data presented in Fig. 1.

VII. Conclusion

In this paper, modeling *the regular respiratory signal* has been studied. In previous works, the CM, the ACM and the EPCM were adopted. The CM and the ACM have poor performances due to the significant waveform differences from the experimental respiratory signal (see Fig. 2). The EPCM shows a good fitting, but it may suffer a duty cycle deviation (see Fig. 3). The PACM is proposed in this paper as a preliminary model, which has the EPCM as its subset and shows an improvement in the duty cycle deviation (see Fig. 4). The PACM is developed in terms of some considerations for the practical respiratory signal, such as having a zero DC component and suffering a phase uncertainty due to the data acquisition, resulting in the HRM. The HRM represents the respiratory signal by a linear combination of its harmonics, and has additional prior knowledge about the harmonic amplitudes and phases, which can be used as the signal model in the future research about breathing motion detection.

Appendix

In the following computation, $\mathbb{N}$ denotes the set of natural numbers, $\mathbb{N}_e$ denotes the set of even natural numbers, and $\mathbb{N}_o$ denotes the set of odd natural numbers. According to Euler's formula, we have

$$(\cos x)^n = \left(\frac{e^{ix}}{2} + \frac{e^{-ix}}{2}\right)^n, \tag{12}$$

and using binomial theorem to expand the power $\left(\frac{e^{ix}}{2} + \frac{e^{-ix}}{2}\right)^n$, we can get

$$(\cos x)^n = \begin{cases} \frac{C_n^{\frac{n}{2}}}{2^n} + \sum_{k=1}^{\frac{n}{2}} \frac{C_n^{\frac{n}{2}-k}}{2^{n-1}} \cos(2kx) & n \in \mathbb{N}_e \\ \sum_{k=1}^{\frac{n+1}{2}} \frac{C_n^{\frac{n+1}{2}-k}}{2^{n-1}} \cos\big((2k-1)x\big) & n \in \mathbb{N}_o \end{cases}, \tag{13}$$

where C_n^k denotes the binomial coefficient. Let $\mathrm{s}(x)$ be a 2π-periodic signal, and for $x \in [-\pi, \pi)$,

$$\mathrm{s}(x) = \begin{cases} 1 & if\ x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right) \\ -1 & if\ x \in \left[-\pi, -\frac{\pi}{2}\right) \cup \left[\frac{\pi}{2}, \pi\right) \end{cases}, \tag{14}$$

then by Fourier series theory, we can get

$$\mathrm{s}(x) = \sum_{u=1}^{+\infty} \frac{4(-1)^{u-1}}{\pi(2u-1)} \cos\big((2u-1)x\big). \tag{15}$$

It is clear that

$$|\cos x|^n = \begin{cases} (\cos x)^n & n \in \mathbb{N}_e \\ \mathrm{s}(x)(\cos x)^n & n \in \mathbb{N}_o \end{cases}. \tag{16}$$

Substituting (13) and (15) into (16), we can get

$$|\cos x|^n = \begin{cases} A_{even}^{\langle n \rangle}(0) + \sum_{q=1}^{\frac{n}{2}} A_{even}^{\langle n \rangle}(q) \cos(2qx) & n \in \mathbb{N}_e \\ \sum_{q=0}^{+\infty} A_{odd}^{\langle n \rangle}(q) \cos(2qx) & n \in \mathbb{N}_o \end{cases}, \tag{17}$$

where $A_{even}^{\langle n \rangle}(0) = \frac{C_n^{\frac{n}{2}}}{2^n}$, $A_{even}^{\langle n \rangle}(q) = \frac{C_n^{\frac{n}{2}-q}}{2^{n-1}}$, $A_{odd}^{\langle n \rangle}(q) = \sum_{(k,u)\in \mathrm{Q}(q,n)} \frac{(-1)^{u-1} C_n^{\frac{n+1}{2}-k}}{2^{n-2}\pi(2u-1)}$, and the set $\mathrm{Q}(q,n) = \left\{(k,u) \in \mathbb{N}^2 \middle| 1 \le k \le \frac{n+1}{2}, |k+u-1| = q\ or\ |k-u| = q \right\}$.

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