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Coordinate Rotation Based Low Complexity Architecture for 3D Single Channel Independent Component Analysis

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Abstract— **In this paper, we have proposed a low complexity architecture of the Single Channel Independent Component Analysis Algorithm (SCICA) targeted for remote cardiac health monitoring by introducing the concept of coordinate rotation. The ECG is widely used for the diagnosis of cardiovascular diseases, and in remote healthcare applications, it is necessary to obtain clean ECG data by separating noise from the obtained signal. Independent Component Analysis is popularly used to solve the signal separation problem, however it requires as many signal sensors as the number of independent signals involved, and also requires the number of signals involved to be known a priori. This is not always possible in remote healthcare environments and thus, this motivates us to explore the use of the SCICA algorithm which requires only one input sensor for signal separation. SCICA is computationally intensive and thus there is a need for a low-complexity implementation of the algorithm for the development of healthcare devices. The proposed architecture is validated in terms of functionality and computational complexity and it has been observed that its performance compares favorably with the conventional algorithm.**

I. Introduction

Cardiovascular diseases (CVD) are one of the prime causes of mortality throughout the world [1]. The electrocardiogram (ECG) is widely used for the diagnosis and prognosis of patients who suffer from such diseases. With the advent of mobile phones and advancements in signal processing, VLSI, communication and information technologies along with the rapid growth in the area of pervasive computing and internet of things, remote cardiac health monitoring has emerged as a recent thrust area of engineering research in healthcare domain. However, a major challenge in designing such systems is to obtain clean ECG data from the noisy signal. This problem resembles the Blind Source Separation (BSS) model which is solved by applying Independent Component Analysis (ICA) [2][3]. ICA has been found to be useful for noise and artifact removal from ECG recordings [4] [5]. However, it requires the number of sensors to be equal to the number of mixed signals involved. Also, the number of independent sources has to be known a priori in order to separate the signals. But in remote healthcare systems, the number of mixed signals including noise could vary depending on environmental factors. This encourages us to explore the use of undetermined ICA models in which the number of sensors required is lesser than the number of signals. The Single Channel Independent Component Analysis (SCICA) model is a special case of undetermined ICA, in which the independent sources can be extracted from a single channel of mixed signals [6]. However, this algorithm is computationally intensive and a direct implementation would result in a complex hardware design which would contribute to high power consumption. This is not feasible for remote healthcare applications as it would result in the limited battery lifetime of devices. Thus, there is a need for an algorithmic-architectural holistic approach to optimize SCICA, so that without losing algorithmic accuracy, a low complexity architecture may be proposed. To the best of our knowledge, we propose a low-complexity architecture of the SCICA algorithm for the first time, by introducing the concept of coordinate rotation. We envisage that this would help in the development of healthcare devices that require low-area and low-power, and thus would lead to better battery lifetime and more user-friendly designs of such devices. The SCICA also involves an ICA step which contributes to its computationally intensive nature. Our previous research [7][8], has shown that the Coordinate Rotation Digital Computer (CORDIC) can be used for the low-complexity architecture of FastICA enabling its reusability in different FastICA stages. In this paper, we extend this coordinate rotation concept, and propose a low-complexity architecture of 3D SCICA, considering 3 different signals are mixed and only one sensor is available for data acquisition. Subsequently we validate the proposed design (Section IV A) and analyze its hardware complexity (Section IV B). Further optimizations to the implementation of the CORDIC based SCICA algorithm will be discussed in our future work.

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II. Theoretical Background

A. Single Channel Independent Component Analysis

Mixed signals $\boldsymbol{x}$ can be modeled as $\boldsymbol{x} = \sum_i s_i \boldsymbol{a}_i = \boldsymbol{A}\boldsymbol{s}$, where $\boldsymbol{s}$ is a column vector with independent sources $s_1, s_2, \dots, s_n$ as its elements and $\boldsymbol{A}$ is a mixing matrix with its columns, $\boldsymbol{a}_i$ (also called weights) chosen such that they form a basis in the signal space [6]. Similarly the unmixing matrix $\boldsymbol{W}$ can be modeled as $\boldsymbol{W} = \boldsymbol{A}^{-1}$ and $\boldsymbol{s} = \boldsymbol{W}\boldsymbol{x}$. Thus, each of the separated signals in the observation domain $\boldsymbol{x}_s^{(i)}$, can be obtained by $\boldsymbol{x}_s^{(i)} = \boldsymbol{A}_{(:,i)}\boldsymbol{W}_{(i,:)}\boldsymbol{x}$ so as to ensure the perfect reconstruction decomposition $\boldsymbol{x} = \sum \boldsymbol{x}_s^{(i)}$ [6]. The steps involved in SCICA are summarized below [6]:

- *Pre-processing:* Temporally whiten the signal and break the signal up into N contiguous blocks.
- *ICA*: Apply an ICA (such as FastICA) algorithm to learn the mixing matrix $\boldsymbol{A}$.

- *Post-processing:* Calculate the magnitude transfer function of each $a_i(t)$ and then cluster into groups γ_p using a clustering algorithm (such as k-means algorithm)
- Calculate the source signals in the observation domain using $x_p(t) = f_p(t) * x(t)$ where [6]:

$$f_p(t) = \frac{1}{N}\sum_{i\in\gamma_p} a_i(-t) * w_i(t) \qquad (1)$$

where, $a_i(t)$ and $w_i(t)$ are finite impulse response filters associated with $\boldsymbol{A}_{(:,i)}$ and $\boldsymbol{W}_{(i,:)}$ respectively.

B. Coordinate Rotation Digital Computer (CORDIC)

The CORDIC is an efficient low-complexity implementation technique for vector rotation and arctangent computation [9] – [11]. In the rotation mode, the angle of rotation θ and the initial vector (x_0, y_0) is given as input and the final vector is (x_f, y_f) computed as follows

$$\begin{bmatrix} x_f \\ y_f \end{bmatrix} = \begin{bmatrix} cos\theta & sin\theta \\ -sin\theta & cos\theta \end{bmatrix}\begin{bmatrix} x_0 \\ y_0 \end{bmatrix} \qquad (2)$$

For brevity, this set of equations can also be written as $x_f = Rot_x(x_0, y_0, \theta)$ and $y_f = Rot_y(x_0, y_0, \theta)$ where $Rot_{x/y}$ denotes the x/y component of the CORDIC unit output. Similarly in the vectoring mode, the angle between the initial vector and the principal coordinate axis is computed.

III. Proposed Low-Complexity Architecture

In this paper, as a part of our preliminary research, we concern ourselves with proposing a low-complexity architecture of the SCICA algorithm in the cases where 3 sources are mixed and only one sensor is present to acquire the mixed signal. The envisaged application, as discussed in Section I, is to obtain clean ECG in the remote cardiac health monitoring environment amid the artifact and noise [4][5]. However, since our intention here is to introduce the concept of low complexity SCICA architecture, we would like to validate its performance considering 3 generic signals. As shown in Fig. 1 and discussed in Section II, SCICA uses an ICA algorithm as a core unit. The FastICA algorithm (FICA) is used here due to its faster convergence speed and accuracy [12][13]. The FICA consists of two steps, Preprocessing and the main Iterative step [13]. The preprocessing step can be divided into two steps, centering and whitening [13]. While the centering step can be implemented with use of an add and accumulation unit, the whitening step requires the computation of Eigen values and the Eigen vectors. Further, the Iterative step of the FICA algorithm involves the use of extremely costly arithmetic operations including multiplications and divisions.

In our previous research it was shown that the 2D CORDIC based FICA, which involves the iteration, normalization and component estimation stages, can be used to implement N-D FICA, where N > 2 [7][8]. As the CORDIC can also be used for the implementation of Eigen Value Decomposition in hardware [14] – [20], such an architecture would reuse the CORDIC block for the whitening stage and the iterative stages. We will use this concept here, to propose the SCICA architecture which is discussed below. The iteration stage of 2D FICA implementation involves the following equation [7]:

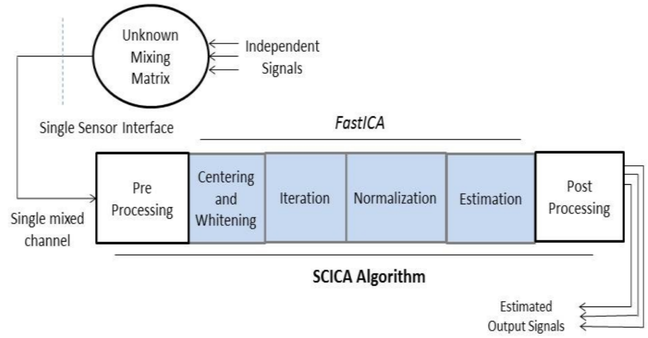

Figure 1. Block diagram represensentation of the SCICA Algorithm

$$\begin{bmatrix} w_{1,1}^{(p+1)} \\ w_{1,2}^{(p+1)} \end{bmatrix} = \begin{bmatrix} E\left[z_{1,j}\left\{z_{1,j}\underline{w_{1,1}^{(p)}} + z_{2,j}\underline{w_{1,2}^{(p)}}\right\}^3\right] \\ E\left[z_{2,j}\left\{z_{1,j}\underline{w_{1,1}^{(p)}} + z_{2,j}\underline{w_{1,2}^{(p)}}\right\}^3\right] \end{bmatrix} - 3\begin{bmatrix} \underline{w_{1,1}^{(p)}} \\ \underline{w_{1,2}^{(p)}} \end{bmatrix} \qquad (3)$$

where p denotes the number of iteration stage, $z_{i,j}$ represents the i^{th} whitened data containing number of samples j where $i = \{1,2\}$, $j = (1, m)$ and where m denotes the frame-length. $w_{1,q}^{(p+1)}$ is the 1st column of the unmixing matrix after p^{th} iteration where $q = \{1,2\}$ and $\underline{w_{1,q}^{(p)}}$ is the normalized value of the basis vector $w_{1,q}^{(p)}$ used in the p^{th} iteration. As shown in [7], this 2D FICA computation can be implemented using the CORDIC unit (Fig. 2 (a)), in the following manner:

$$\begin{bmatrix} w_{1,1}^{(p+1)} \\ w_{1,2}^{(p+1)} \end{bmatrix} = \begin{bmatrix} E\left[z_{1,j}\{Rot_x(z_{1,j}, z_{2,j}, \theta_p)\}^3\right] \\ E\left[z_{2,j}\{Rot_x(z_{1,j}, z_{2,j}, \theta_p)\}^3\right] \end{bmatrix} - 3\begin{bmatrix} \underline{w_{1,1}^{(p)}} \\ \underline{w_{1,2}^{(p)}} \end{bmatrix} \qquad (4)$$

where θ_p is the arctangent of the two components of the vector $\underline{w_{1,q}^{(p)}}$. Similarly, the second stage, i.e. normalization of the vector (Fig. 1) obtained after the iteration computation can be computed using the following equations:

$$\begin{bmatrix} \underline{w_{1,1}^{(p+1)}} \\ \underline{w_{1,2}^{(p+1)}} \end{bmatrix} = \begin{bmatrix} Rot_y(0,1,Vec_\theta(w_{1,1}^{(p+1)}, w_{1,2}^{(p+1)}) \\ Rot_y(0,1,Vec_\theta(w_{1,1}^{(p+1)}, w_{1,2}^{(p+1)}) \end{bmatrix} \qquad (5)$$

where $Vec_\theta(w_{1,1}^{(p+1)}, w_{1,2}^{(p+1)})$ is a concise notation for the output of the CORDIC in the Vectoring mode (Fig. 2(b)). Finally, the FICA computation can be completed with the estimation of the independent components (Fig. 1), using the following:

$$s'_{1,j} = Rot_x(z_{1,j}, z_{2,j}, Vec_\theta(w_{1,1}{}^c, w_{1,2}{}^c)) \qquad (6)$$

where $w_{i,j}{}^c$ denotes the j^{th} component of the i^{th} converged normalized unmixing vector (Fig. 2(c)). Similarly as given in [8], the equations of the iteration step of the CORDIC based 3D FICA implementation are as follows:

$$\begin{bmatrix} w_{1,1}^{(p+1)} \\ w_{1,2}^{(p+1)} \\ w_{1,2}^{(p+1)} \end{bmatrix} = \begin{bmatrix} E[z_{1,j}\{G_{3D}\}^3] \\ E[z_{2,j}\{G_{3D}\}^3] \\ E[z_{3,j}\{G_{3D}\}^3] \end{bmatrix} - 3\begin{bmatrix} w_{1,1}^{(p)} \\ w_{1,2}^{(p)} \\ w_{1,2}^{(p)} \end{bmatrix} \qquad (7)$$

where $G_{3D} = \left\{Rot_x^{l2}(z_{3,j}, Rot_x^{l1}(z_{1,j}, z_{2,j}, \theta_{1,p}), \theta_{2,p})\right\}$, $\theta_{1,p} = Vec_\theta(\underline{w_{1,1}^{(p)}}, w_{1,2}^{(p)})$ and $\theta_{1,p} = Vec_\theta(w_{1,1}^{(p)}, Vec_x\left(\underline{w_{1,1}^{(p)}}, w_{1,2}^{(p)}\right))$[8].

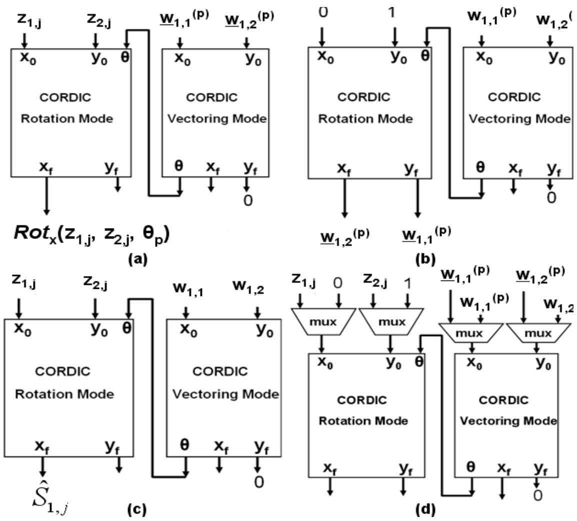

Figure 2. CORDIC based 2D implementation (a) Iteration Stage (b) Normalization Stage (c) Estimation Stage (d) Multiplexed Architecture

The normalization step can also be written as:

$$\begin{bmatrix} \frac{w_{1,1}^{(p+1)}}{w_{1,2}^{(p+1)}} \\ \overline{w_{1,3}^{(p+1)}} \end{bmatrix} = \begin{bmatrix} Rot_y(0, Rot_x(0,1,\theta_{2.(p+1)}), \theta_{1.(p+1)}) \\ Rot_x(0, Rot_x(0,1,\theta_{2.(p+1)}), \theta_{1.(p+1)}) \\ Rot_y(0,1,\theta_{2.(p+1)}) \end{bmatrix} \quad (8)$$

and the estimation equation is:

$$s'_{1,j} = Rot_x(z_{3,j}, Rot_x\left(z_{1,j}, z_{1,j}, Vec_\theta({w_{1,1}}^c, {w_{1,2}}^c)\right), Vec_\theta({w_{1,3}}^c, Vec_x({w_{1,1}}^c, {w_{1,2}}^c)) \quad (9)$$

Since none of the aforementioned stages (4) – (6) are concurrent, a multiplexed architecture can be adopted for 2D FICA (Fig. 2(d)), which can be extended easily to 3D FICA [8]. Thus, the CORDIC based FICA can be used for the implementation of SCICA, resulting in a CORDIC based SCICA architecture. As the CORDIC can also be used for the implementation of Eigen Value Decomposition in hardware [14] – [20], the CORDIC block can be reused for the whitening stage and the iterative stages, thus reducing the area and complexity of hardware, as it would preclude the use of additional processing blocks that would otherwise be required in the conventional implementation of the SCICA algorithm.

IV. Experimental Results And Discussion

A. Experimental Setup and Results

We generated MATLAB models for the conventional as well as CORDIC based SCICA to carry out experiments. In the first experiment, we used 2 mixed signals in a single channel of data and observed that the CORDIC-based SCICA algorithm was able to separate the independent components with accuracy (Fig. 3). The Root Mean Square Error values (RMSE) obtained from comparing the two separated signals obtained from the conventional and proposed architectures were 0.0117 and 0.0119. We then used 3 signals in the mixed single channel of data and tested our proposed architecture in three difference case studies by using 3 different weight $a_1 = (0.43, 0.09, 0.51)$, $a_2 = (0.49, 0.91, 0.53)$ and

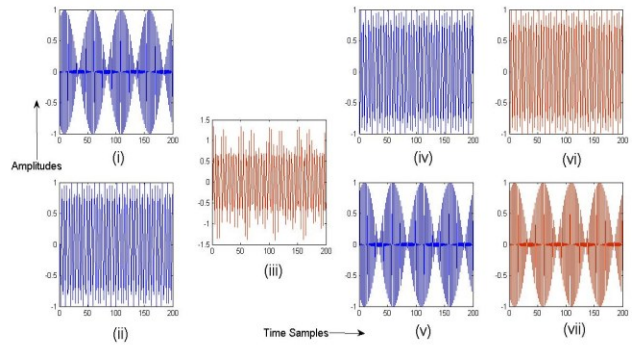

Figure 3. (i)(ii) Independent source signals (iii) Single channel mixture (iv)(v) Estimated signals obtained by conventional SCICA (vi)(vii) Estimated signals obtained by CORDIC Based SCICA

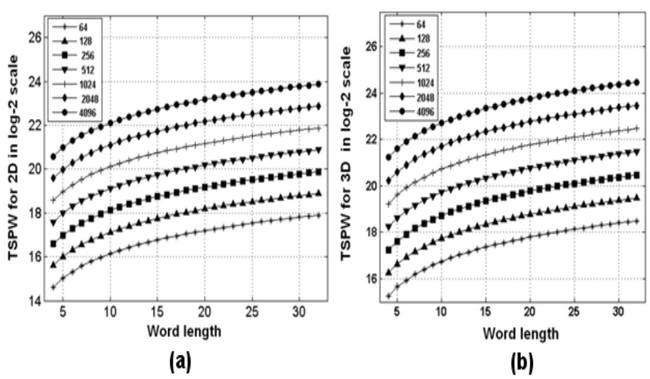

Figure 4. Transistor Saving per Word Length (TSPW) in CORDIC based 2D and 3D SCICA architecture

$a_3 = (0.41, 0.57, 0.49)$ (Fig. 5). It should be noted from the values of $a_1(2), a_2(2)$ and $a_3(2)$ that these case studies signify the high, low and moderate noise environments respectively, in which the signal of interest is attenuated as required while the other weight coefficients that correspond to the noise and artifact are kept approximately the same. The separated signals obtained from each case using the CORDIC based SCICA were then compared with the initial source signals using Root Mean Square Error (RMSE) and Standard Deviation (Std. D.) as performance metrics. While the time series representation of the separated signals is shown in Fig. 5, it should be noted that the frequency spectra of these signals can easily be reproduced for the better understanding of the reader. It is evident from Fig. 5 and Table 1 that the CORDIC based SCICA maintained the functionality of the original algorithm and is able to extract the signals with the original algorithmic efficiency.

TABLE I. Error Analysis of 3D CORDIC based SCICA

Source	Case 1		Case 2		Case 3	
	Std. D.	*RMSE*	*Std. D.*	*RMSE*	*Std. D.*	*RMSE*
160 Hz	0.0147	0.6489	0.5649	1.1813	0.9207	1.4035
100 Hz	1.2112	1.0762	0.0911	0.7056	1.2726	0.3825
278 Hz	0.0074	1.2957	1.0611	0.8628	1.3049	0.1172

B. Hardware Complexity Analysis

The CORDIC reuse is accompanied with a penalty of the requirement of multiplexers. This penalty is as follows:

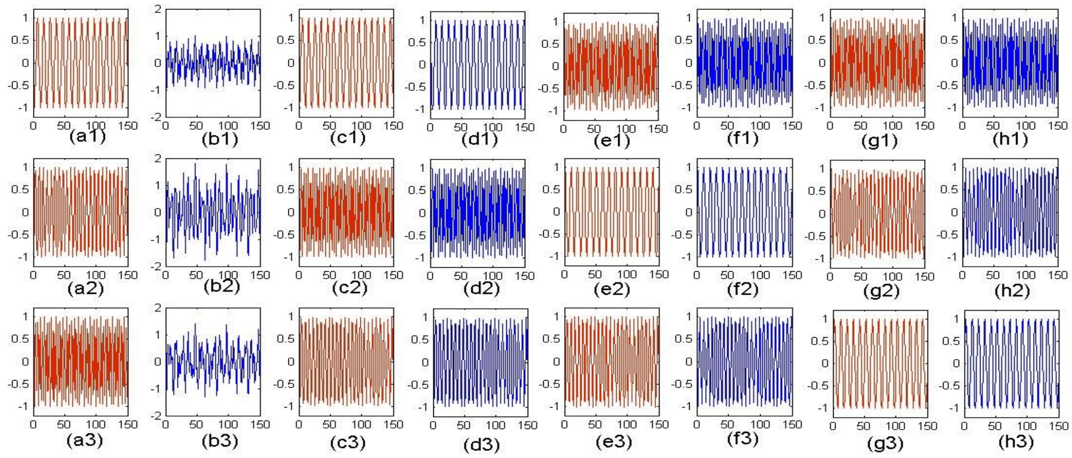

Figure 5. (a1)-(a3): Independent source signals used for testing. (b1)-(b3): Mixed signals for each case. (c1)-(c3) Signals obtained using the CORDIC based SCICA on first case (d1)-(d3): Signals obtained using conventional SCICA on first case (e1)-(e3): Signals obtained using the CORDIC based SCICA on second mixture. (f1)-(f3): Signals obtained using conventional SCICA on second mixture. (g1)-(g3): Signals obtained using the CORDIC based SCICA on third mixture. (h1)-(h3): Signals obtained using conventional SCICA on third mixture.

$P_{2D} = (5)TC_{mux}^{2:1}$ and $P_{3D} = (10 + 2(k + 1))TC_{mux}^{2:1}$where k is the number of stages in the CORDIC unit and $TC_{mux}^{2:1}$ is the transistor count of a 2:1 b bit multiplexer [8]. The Transistor Savings per Wordlength (TSPW) for 2D and 3D can be seen in Fig. 4.

TABLE II. COMPUTATIONAL SAVINGS FOR THE CORDIC BASED (MUL – MULTIPLICATION, ADD – ADDITION, SQRT – SQUARE ROOT, DIV – DIVISION) 2D AND 3D FICA

Dim	**Iteration**		**Normalization**				**Estimation**	
	Mul	***Add***	***Mul***	***Add***	***Sq Rt***	***Div***	***Mul***	***Add***
2D	*2m*	*m*	*2*	*1*	*1*	*2*	*2m*	*m*
3D	*3m*	*2m*	*3*	*2*	*1*	*3*	*3m*	*2m*

V. CONCLUSION

In this paper, we proposed a co-ordinate rotation based low-complexity architecture of the SCICA algorithm for signal separation, tested its algorithmic efficiency and its low-complexity and compared it with the conventional architecture of the Single Channel ICA. This architecture results in a significant reduction in hardware complexity which will further lead to a decreased consumption of power and area. Since ECG is band-limited in which various frequencies are mixed, SCICA will have to be applied in an intelligent manner and this forms part of our future research.

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