35th Annual International Conference of the IEEE EMBS
Osaka, Japan, 3 - 7 July, 2013

An EKF-based approach for estimating leg stiffness during walking

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***Abstract*— The spring-like behavior is an inherent condition for human walking and running. Since leg stiffness k_{leg} is a parameter that cannot be directly measured, many techniques has been proposed in order to estimate it, most of them using force data. This paper intends to address this problem using an Extended Kalman Filter (EKF) based on the Spring-Loaded Inverted Pendulum (SLIP) model. The formulation of the filter only uses as measurement information the Center of Mass (CoM) position and velocity, no *a priori* information about the stiffness value is known. From simulation results, it is shown that the EKF-based approach can generate a reliable stiffness estimation for walking.**

INTRODUCTION

Understand and emulate human gait has been one of the main goals in different scientific areas such as biomechanics, neuroscience or robotics.

Although several models have been proposed for representing legged locomotion, most of them agree in considering the elastic behavior of legs as a crucial characteristic that represents the biomechanics of legged systems. This assertion is based on the fact that humans and other vertebrate animals exhibit bouncing gaits like hopping, running and walking [1].

The Spring-Loaded Inverted Pendulum (SLIP) model, first proposed by [2] and then studied by others [3],[4], is a walking and running template that is capable of reproducing some important characteristics of human locomotion, such as the Center of Mass (CoM) trajectory and the Ground Reaction Forces (GRF). In this model, both legs act like massless springs attached to a mass located at the body CoM. The system dynamics are governed by the forces that the spring legs exerted on the body mass during the stance phase, that is, the moment where the feet make contact with the ground.

Like this model, and many others using spring-like legs, the choice of the value of leg stiffness directly impacts on the the resultant gait solution. In [5], different periodic walking solutions were obtained by means of varying the SLIP's model parameters. These solutions reveal different behaviors in terms of robustness against disturbances and energy efficiency.

Moreover, when the SLIP model is used along with experimental data, considerable differences between the model output and the measured GRFs and CoM are observed [6],[7]. This discrepancy could be caused by some differences in the parameters definition. For the case of leg stiffness, most calculation uses the vertical force-leg's compression ratio. This definition could lead to different results if different definition of the leg's length are used [8].

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This paper presents a different approach of calculating leg stiffness using an extended kalman filter (EKF). The formulation is based on the equations of motion of the SLIP model. The CoM trajectory is considered as the measurement information. Simulation results for different walking solutions using different stiffness values are presented in order to validate the filter performance.

I. METHODS

A. Bipedal Spring-Mass Model

According to the SLIP model (see Figure 1), a walking pattern can be generated by two massless legs represented by two springs with fixed rest length L_0 and stiffness k_{leg}. The body is represented by a point mass m located in the body center of mass $\mathbf{r} = \begin{bmatrix} x_{CoM} & y_{CoM} \end{bmatrix}^T$. During stance, the forces $\mathbf{F_{1,2}}$ are generated from the springs directed from the foot points $\mathbf{r_{FP}} = \begin{bmatrix} x_{FP} & y_{FP} \end{bmatrix}^\mathbf{T}$ to the body mass m. The swing leg doesn't affect the dynamics of the whole system since it's massless and generates no force.

A single step is defined by a *single support* of one leg, followed by a *double support* phase, and finally a single support of the leg which was initially in a swing phase. The first single support is initiated at *mid-stance*, *i.e.*, when the stance leg is vertically oriented with respect to the ground, condition that is obtained again at the end of the final single support. This is the highest point reached by the CoM, also called the *apex height* [3].

The beginning of the double support phase is given by the landing event, which is the instant when the swing leg hits the ground describing an angle of attack α_0 with the surface.

The equation of motion of the system is defined as [5]:

$$m\ddot{\mathbf{r}} = \mathbf{F_1} + \mathbf{F_2} - m\mathbf{g} \tag{1}$$

where $\mathbf{g} = \begin{bmatrix} 0 & g \end{bmatrix}^T$ corresponds to the gravity vector. The force of any leg during stance is

$$\mathbf{F_{1,2}} = k_{leg}\left(\frac{L_0}{\|\mathbf{r}-\mathbf{r_{FP}}\|} - 1\right)(\mathbf{r}-\mathbf{r_{FP}}) \tag{2}$$

The transitions from stance to swing is detected when any leg force has decreased to zero, while the *swing-to-stance* transition is reached when the current swing leg has touched the floor and the difference $y_{CoM} - y_{TD} = 0$, where the touch-down height is defined as $y_{TD} = L_0 sin(\alpha_0)$. The parameters of this model are the body mass m, the leg's rest length L_0, the angle of attack α_0, and the leg stiffness k_{leg}. In

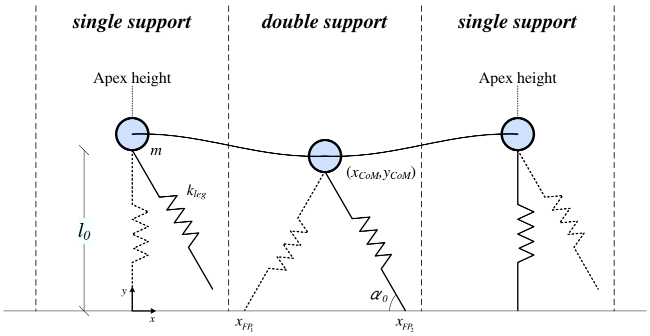

Fig. 1. SLIP model. A single step is defined by a sequence of single a double support phases and it begins when the CoM position is at the apex height. The step ends when the same condition is reached.

a symmetric solution both legs are considered with equal length and stiffness.

B. State and Parameter Estimation

The Kalman filter (KF) is an optimal solution for the estimation of states in a linear stochastic system, where its dynamics can be represented by models which describe its process and measurements [9]. The Extended Kalman Filter (EKF) is an extension of the original KF for the nonlinear systems. Besides estimating the system's states, these filtering techniques could be also applied for parameter estimation, where the unknown parameter is considered as an additional state to be estimated.

As mentioned above, the EKF is used in this work for the estimation of the SLIP model's states, as well as the identification of k_{leg}. The second-order system presented in (1) and (2) can be written using a state-space notation:

$$x_i = \mathbf{F}^{\theta}_{i-1}(\lambda)x_{i-1} + w_{i-1} \tag{3}$$

$$y_i = \mathbf{H}_i x_i + v_i \tag{4}$$

Equation (3) represents the process function, derived from equations (1) and (2). The state vector is defined as as

$$x = \begin{bmatrix} x_{CoM} & y_{CoM} & \dot{x}_{CoM} & \dot{y}_{CoM} & k_{leg} \end{bmatrix}^T \tag{5}$$

Measurements of the CoM trajectory and its velocities are considered as available, thereby

$$y = \begin{bmatrix} x_{CoM} & y_{CoM} & \dot{x}_{CoM} & \dot{y}_{CoM} \end{bmatrix}^T \tag{6}$$

The process function matrix $\mathbf{F}_{i-1}$ depends on the parameter vector $\lambda = \begin{bmatrix} m & l_0 & \alpha_0 & g & k_{leg} \end{bmatrix}^T$. Process and measurement noises, w and v, are considered zero-mean white noise with covariance matrices $\mathbf{Q}$ and $\mathbf{R}$, respectively.

It is important to mention that the system dynamics change according to the current phase (double or single support), which means that the process function definition is conditioned to the detection of the events that delimit the beginning or the end of an specific phase. The superscript θ in the $\mathbf{F}$ matrix denotes this hybrid behavior, where θ can takes values of 1 or 2, depending if the current phase of the system is double or single support, respectively.

The conditions that lead the system to a transition depend on the contact of the leg with the ground. Since the foot point positions $\mathbf{r_{FP}}$ are also known, this information is used for establishing the transitions between phases.

II. Simulation Results

In order to verify the feasibility of this proposal some simulations were performed. The SLIP dynamics were solved to generate data sets that later will be used as measurement information for the EKF. Three walking solutions with three different values of k_{leg} were generated. The system equations were solved using the Runge-Kutta integration method with a step size of $0.1\,ms$.

The stiffness values, as well as the other system parameters, were chosen in order to have periodic stable solutions, based on the results obtained by [5]. For all three proposed scenarios the body mass was $80\,kg$ and the leg's rest length was adjusted to $L_0 = 1\,m$. The chosen k_{leg} values correspond to $k_1 = 11.8\,kN/m$, $k_2 = 15.7\,kN/m$, $k_3 = 23.5\,kN/m$.

Concerning the EKF implementation, the initial estimate of k_{leg} was intentionally set at a high value, $k_{leg} = 2.8\,kN/m$, in all the three scenarios. The rest of the states' initial values was taken from the corresponding data sets.

Figure 2(a) shows the vertical position of CoM as a result from the EKF implementation. In Figure 2(b) one-period solution for the three simulations are presented. As expected, as k_{leg} goes higher, the vertical oscillations of the CoM are smaller. This is a normal situation in human walking, namely, the subject switches to a higher velocity wich it reflects into stiffer legs.

The results of the stiffness estimation for the three walking solutions can be seen in Figure 2(c). For the three cases, the k_{leg} estimation steady-state solutions converge to a value very close to the corresponding true stiffness. The estimation error evolution is shown in Figure 2(d). The RMS errors for each estimation are $e_1 = 0.47\,kN/m$, $e_2 = 0.06\,kN/m$ and $e_3 = 1.157\,kN/m$.

From this preliminary results, it can be concluded that the EKF-based approach can generate a reliable stiffness estimation using the SLIP walking model, with no more *a priori* information than the CoM trajectory and its velocities. Moreover, the hybrid nature of the SLIP model is well represented in the EKF formulation, which can be observed in the vertical CoM position for all the three proposed scenarios. The conditions of transition based on the feet points switched the system properly to the next phase. In terms of implementation this means that the process function changes to the corresponding model in a precise way.

III. Conclusions

This paper presented an alternative stiffness estimation technique using an extended kalman filter based on the SLIP walking model. The simulation results showed good agreement between the true k_{leg} values and its estimation at three different scenarios of low, medium and high stiffness. An interesting characteristic of this proposal is the simultaneous state estimation and parameter identification of the

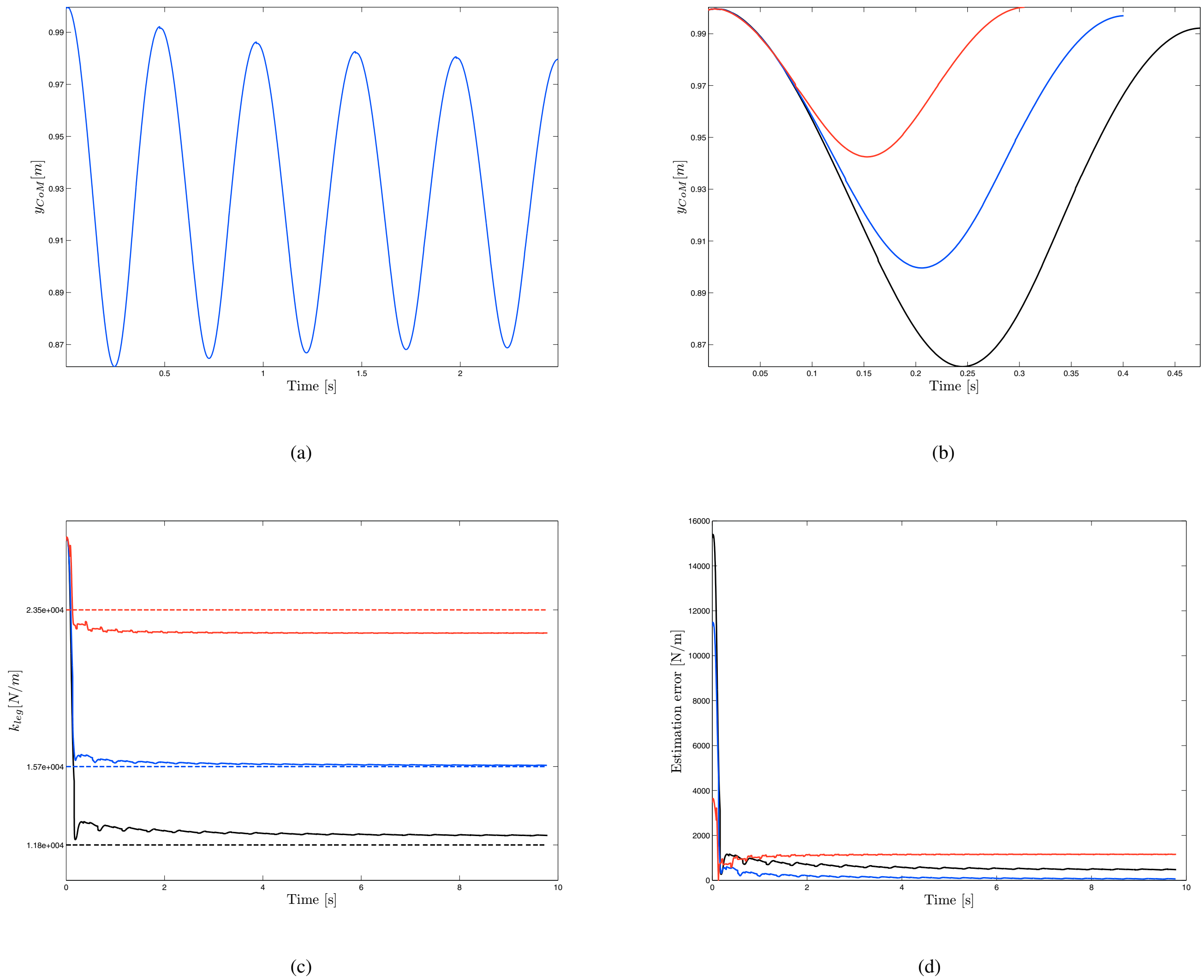

Fig. 2. Simulations Results - (a) Vertical CoM position obtained from one of the three walking solutions using the EKF. (b) A single step extracted for each solution. It can be observed the differences between vertical excurtions of the CoM for the three values of k_{leg}. (c) The k_{leg} estimation for the three walking solutions. (d) The corresponding estimation error. Black curve corresponds k_1, while the blue and the red ones belongs to k_2 and k_3, respectively.

system model, using only CoM kinematics information. This contrasts with other techniques that rely on the GRFs to obtain an approximate value of k_{leg}.

Even though the SLIP model is an ideliazed representation of legged locomotion, this approach may also be implemented using experimental data collected from humans. Future work is oriented to add more realisitic features to this model in order to implement the EKF-based approach in a real leg stiffness identification problem for rehabilitation applications.

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