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Phase velocity and attenuation predictions of waves in cancellous bone using an iterative effective medium approximation

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***Abstract*— The quantitative determination of wave dispersion and attenuation in bone is an open research area as the factors responsible for ultrasound absorption and scattering in composite biological tissues have not been completely explained. In this study, we use the iterative effective medium approximation (IEMA) proposed in [1] so as to calculate phase velocity and attenuation in media with properties similar to those of cancellous bones. Calculations are performed for a frequency range of 0.4 – 0.8 MHz and for different inclusions' volume concentrations and sizes. Our numerical results are compared with previous experimental findings [2] so as to assess the effectiveness of IEMA. It was made clear that attenuation and phase velocity estimations could provide supplementary information for cancellous bone characterization.**

I. Introduction

Cancellous bone is a composite porous material with microstructure consisting of randomly distributed bone tissue surrounded by bone marrow. Quantitative ultrasound has been extensively used for the determination of cortical and cancellous bone material and structural properties. To this end, the estimation of the phase velocity and the attenuation coefficient has been used in previous studies [3-10] for ultrasound bone characterization.

However, when a plane wave impinges on a nonhomogeneous media such as bone, multiple scattering, material dispersion and absorption phenomena occur. Several multiple scattering theories have been proposed to investigate wave dispersion and attenuation induced by the randomly distributed particles in composite media [11-14]. Nevertheless, these theories are not able to provide reasonable dispersion and attenuation predictions for all types of inclusions and for a wide range of volume concentrations and wavenumbers.

To this end, an iterative effective medium approximation (IEMA) has been introduced in [1, 15], which predicts wave dispersion and attenuation in non-homogeneous materials that include particles with volume concentrations as high as 50%. This version of IEMA combines the self-consistent model of Kim et al. [12] with the quasicrystalline approximation of Waterman and Truell [8]. The iterative methodology uses the effective Lame' constants calculated from the static model of Christensen [13] in order to estimate the complex density and wavenumber at each frequency. In this way, the attenuation as well as the dispersion of an ultrasonic pulse propagating in composite media can be estimated, using the density as the main parameter that controls the iteration procedure.

In this work we estimate wave dispersion and attenuation in bone-mimicking porous media by making use of an iterative methodology. First, numerical calculations of the phase velocity and attenuation are performed in the frequency range from 0.4 – 0.8 MHz. Then, estimations are derived for the frequency of 0.5 MHz for different volume concentrations and particle diameters. The effectiveness of the presented methodology is investigated by comparing the numerical predictions with the experimental findings of a previous study in phantoms of cancellous bone [2]. The dependency of the phase velocity to frequency and particles' volume fraction is in excellent consistency with the experimental findings. Therefore, IEMA provides reasonable results and could be used for bone characterization.

II. Materials And Methods

A. The IEMA for particle suspensions

In this section a brief analysis of the iterative procedure is presented based on a previous study [1]. When a wave propagates in a composite medium it can be considered as a sum of: a) a mean wave travelling in the medium with the dynamic effective properties of the composite and, b) fluctuating waves obtained from the multiple scattering of the mean wave. This consideration forms a complicated self-consistent multiple scattering condition from which the dynamic effective properties of the composite could be determined. In order to make the matter simpler Kim et al. [16] proposed a simple self-consistent condition, which for a non homogeneous material assumes the form:

$$n_1 g_d^{(1)}(\hat{\boldsymbol{d}};\hat{\boldsymbol{k}},\hat{\boldsymbol{k}}) + n_2 g_d^{(2)}(\hat{\boldsymbol{d}};\hat{\boldsymbol{k}},\hat{\boldsymbol{k}}) = 0, \qquad (1)$$

where n_1, n_2 represent the volume fraction of the inclusions and the matrix, respectively, $\hat{\boldsymbol{k}}$ is the direction in which a $\hat{\boldsymbol{d}}$-polarized plane mean wave is propagated and $g^{(1)}, g^{(2)}$ are the forward scattering amplitudes taken from the solution of the scattering problems 1 and 2, respectively, as illustrated in Fig. 1.

The mean wave is both dispersive and attenuated with a complex wavenumber $k_d^{eff}(\omega)$ written as:

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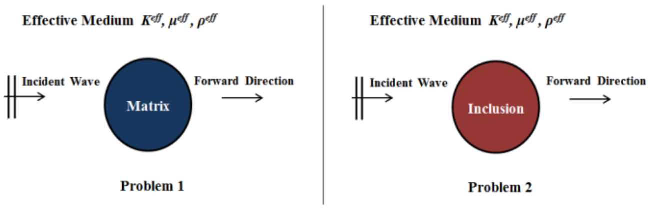

Fig. 1. The single scattering problems referred to the self-consistent condition.

$$k_d^{eff}(\omega) = \frac{\omega}{C_d^{eff}(\omega)} + i\alpha_d^{eff}(\omega), \tag{2}$$

where $C_d^{eff}(\omega)$ and $\alpha_d^{eff}(\omega)$ denote the effective and frequency dependent phase velocity and attenuation coefficient, respectively, of a longitudinal ($d \equiv$ P) or transverse ($d \equiv$ S) mean wave propagating with frequency ω.

Under the above explained considerations the steps of the IEM approximation for the determination of $C_d^{eff}(\omega)$ and $\alpha_d^{eff}(\omega)$ are the following: Replace the non-homogeneous medium with an elastic homogeneous and isotropic material with bulk and shear moduli K^{eff} and μ^{eff}, respectively, given by the static mixture model of Christensen [13]:

$$K^{eff} = K_2 + \frac{n_1(K_1-K_2)(K_2+\frac{4}{3}\mu_2)}{n_2(K_1-K_2)+(K_2+\frac{4}{3}\mu_2)}, \tag{3}$$

$$A\left(\frac{\mu^{eff}}{\mu_2}\right)^2 + 2B\left(\frac{\mu^{eff}}{\mu_2}\right) + C = 0 \ , \tag{4}$$

where A, B, C are functions of μ_1, μ_2 and n_1 given in [13] and the indices 1, 2 indicate material properties of the inclusion and matrix, respectively. Considering the effective density of the composite to be:

$$(\rho^{eff})_{step1} = n_1\rho_1 + n_2\rho_2, \tag{5}$$

evaluate through the material properties (4) and (5) and the relations:

$$C_P^2 = (\lambda + 2\mu)/\rho, \quad C_S^2 = \mu/\rho, \tag{6}$$

the real effective wave number $(k_d^{eff})_{step1}$ of the mean wave. This is the first step of the IEM approximation.

Utilizing the material properties of the first step, proceed to the second step where the scattering problems 1 and 2, illustrated in Fig.1, are solved and the forward scattering amplitudes $g^{(1)}$, $g^{(2)}$ are evaluated. Next, combining the scattering amplitudes as in relation (1), i.e.,

$$g_d(\hat{\boldsymbol{d}};\hat{\boldsymbol{k}},\hat{\boldsymbol{k}}) = n_1 g_d^{(1)}(\hat{\boldsymbol{d}};\hat{\boldsymbol{k}},\hat{\boldsymbol{k}}) + n_2 g_d^{(2)}(\hat{\boldsymbol{d}};\hat{\boldsymbol{k}},\hat{\boldsymbol{k}}) \tag{7}$$

and making use of the dispersion relation proposed in [11], find the new effective wavenumber of the mean wave as follows:

$$(k_d^{eff})_{step2} = (k_d^{eff})_{step1} + \frac{3n_1 g_d(\hat{\boldsymbol{d}};\hat{\boldsymbol{k}},\hat{\boldsymbol{k}})}{\alpha^3 (k_d^{eff})_{step1}}, \tag{8}$$

with a being the radius of a volume equivalent to the particle sphere. Evaluate the new complex density $(\rho^{eff})_{step2}$ via the $(k_d^{eff})_{step2}$ and the relations (4) and (6). Repeat the second step with the material properties (4) and the new density $(\rho^{eff})_{step2}$ until the self-consistent condition (1) to be satisfied. Then find from (8) and (3) the frequency dependent, effective phase velocity and attenuation coefficient of the mean wave.

B. *Material Properties*

Cancellous bone was assumed as a porous medium with properties derived from [2]. Water was considered as the material of the matrix of the composite and nylon as the content of the spherical particles. The elastic properties of both materials are shown in Table I.

C. *Multiple scattering calculations*

First, the particle diameter was set to 254 μm and the volume fraction of nylon to 7.9% as in [2]. The phase velocity and attenuation were estimated for frequencies from 0.4 – 0.8 MHz. Then, for a constant frequency 500 kHz and particle diameter 254 μm we gradually increased the volume concentration from 1.8 – 11.4%. The phase velocity and attenuation were predicted for increasing volume fraction. Finally, the phase velocity and attenuation were calculated by setting the frequency and volume concentration to 500 kHz and 7.9%, respectively and for different particle diameters (152 μm, 203μm, 254 μm and 305 μm).

III. Results

Figures 2-4 represent phase velocity estimations derived from IEMA as a function of frequency, inclusions' volume concentrations and diameters, respectively. In each figure the corresponding experimental results from [2] are also presented.

In Fig. 2 the phase velocity slightly decreases from 1506 m/s down to 1504 m/s with increasing frequency from 0.4 – 0.8 MHz, exhibiting a negative dispersion. On the other hand, the phase velocity increases from 1485 – 1519 m/s as the volume fraction increases from 1.8 – 11.4% (Fig. 3). The results in both sets of calculations are in total consistency with the experimental findings [2]. In particular, the relative errors are in the range 0.01– 0.12% in the first set of predictions and in the range 0.08 – 0.28% for the second case, respectively.

In Fig. 4, it can be seen that the phase velocity predicted from IEMA is almost constant with increasing inclusions' diameter. Specifically, the maximum phase velocity value is calculated as 1506.6 m/s for the diameter of 152 μm, while the minimum is 1505.3 m/s for the diameter of 305 μm. However, in the experimental study of [2] the phase velocity was found to increase with increasing particle diameter.

TABLE I. MATERIAL PROPERTIES OF NYLON AND WATER

	Nylon	**Water**
ρ (kg/m^3)	1100	1000
E (GPa)	4.96	300.50x10^{-9}
λ (GPa)	3.72	2.19
μ (GPa)	1.86	100x10^{-9}
ν	0.39	0.50

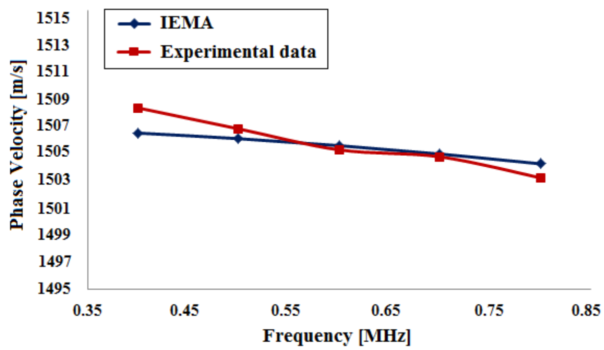

Fig. 2. Phase velocity dependence on the examined range of frequencies. Comparison to experimental results [2].

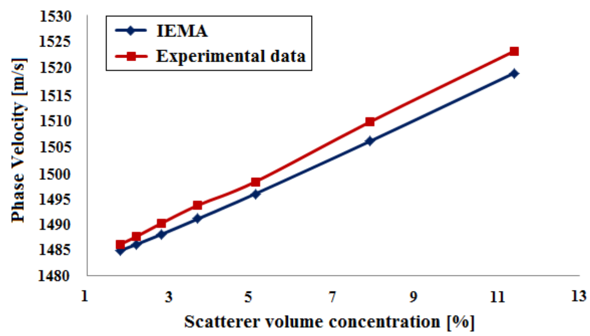

Fig. 3. Phase velocity dependence on the scatterer volume concentration. Comparison to experimental results [2].

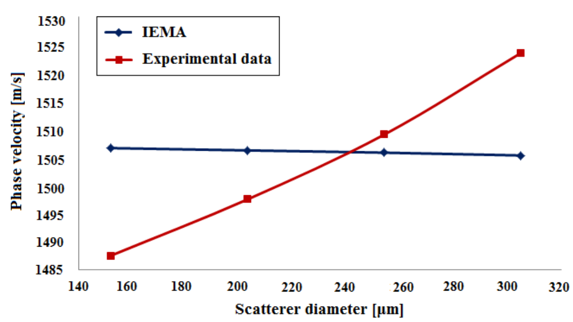

Fig. 4. Phase velocity dependence on the particle diameter. Comparison to experimental results [2].

Figures 5-7 present the numerical predictions of attenuation as a function of frequency, inclusions' volume concentrations and diameters, respectively. In Fig. 5 a gradual attenuation increase is observed as frequency increases. Specifically, the minimum attenuation value at 0.4 MHz is calculated as 0.06 m^{-1}, while the maximum is 0.82 m^{-1} at 0.8 MHz. In the next figures, similar attenuation behaviors are exhibited for increasing particles' volume concentrations and diameters. In particular, in Fig. 6 the attenuation coefficient show an increase in the range 0.01 – 0.21 m^{-1}, while in Fig. 7 it was found to increase from 0.03 m^{-1} to 0.23 m^{-1}.

IV. Discussion

In the present work, for the first time we make use of an iterative methodology in order to carry out wave dispersion

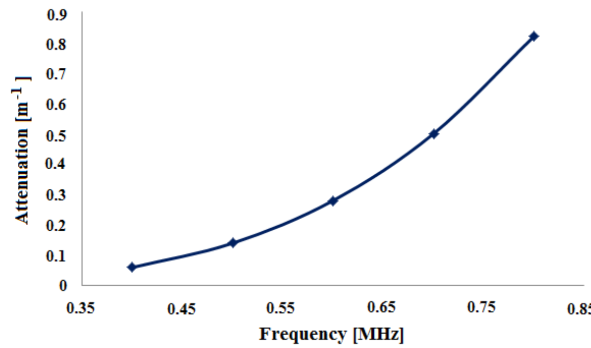

Fig. 5. Attenuation coefficient dependence on the examined range of frequencies.

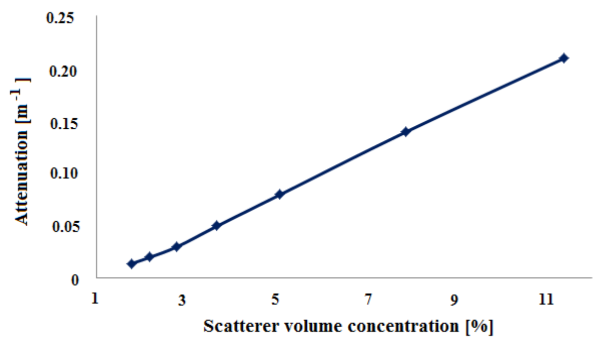

Fig. 6. Attenuation coefficient dependence on the scatterer volume concentration.

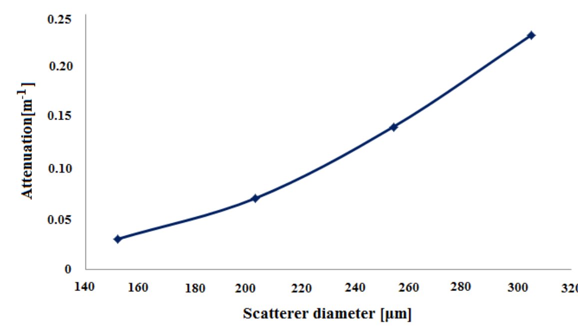

Fig. 7. Attenuation coefficient dependence on the examined particle diameters.

and attenuation estimations in bone-mimicking porous media. The phase velocity and attenuation variation was examined for different frequencies, particles' volume concentrations and sizes. By comparing the numerical results with experimental findings we also assessed the effectiveness of IEMA in bone characterization.

The phase velocity was found to decrease with increasing frequency, exhibiting a negative dispersion. This anomalous behavior of the frequency dependence of the phase velocity has been also reported in various studies in cancellous bone [17-23]. Despite the fact that several proposals have been made to explain this velocity trend, no conclusion has been drawn yet. In particular, in the study of Chakraborty *et al.* [22], a non-local extension of the Biot theory was presented, which can give rise to a negative dispersion under specific circumstances. Moreover, Haiat et al. [23] suggested that the coupling of multiple scattering and absorption may contribute to negative dispersion. In addition, the phase velocity was found to increase with increasing particles' volume concentrations. Although the variation of the phase velocity as a function of frequency and volume concentration was in excellent consistency with previous experimental and numerical findings [2, 19], this was not the case when the particle diameter increased. An almost constant phase velocity with increasing particle diameter has been also found in [20] investigating the influence of porosity and pore size on the ultrasonic properties of cancellous bone using a phantom material. This is attributed to the fact that phase velocity in cancellous bone is mainly dependent on the mechanical properties of osseous tissues rather than on bone structure [20]. However, further numerical research is needed in order to explain this discrepancy between the numerical and the experimental results.

On the other hand, the attenuation coefficient was found to increase gradually when the frequency increases. The same behavior was reported in [17] suggesting that when a negative dispersion is observed at specific bone regions, the attenuation coefficient increases almost linearly with frequency. Additionally, an attenuation increase was observed as the inclusions' volume concentration and diameter increased. This attenuation behavior has been also observed in a previous experimental study examining a cancellous bone phantom [21]. The numerical results indicate the significant impact of wave absorption, scattering and reflection phenomena due to the intense bone heterogeneity for larger diameters and higher particles' concentration.

However, the assumption that the scatterer geometry is spherical is not realistic and a better approach for the trabeculae structure would be a cylindrical shape. The incorporation of cylindrical inclusions into IEMA constitutes our future research so as to describe more realistic conditions.

Also, in spite of the fact that the water-nylon composite has a similar scattering behavior with cancellous bone, IEMA should be used in realistic bone-marrow composite media. Our ongoing research takes into consideration the material properties of bone calculated using scanning acoustic microscopy images in order to model healing bones. In this direction, the findings of this study can be extended so as to model ossification regions within callus (e.g. woven bone) that resemble porous media at some stages during healing.

V. Conclusions

In this work, we used an iterative methodology for the quantitative estimation of attenuation and wave dispersion in cancellous bone. The numerical results are in consistency with the experimental findings in cancellous bone mimicking-phantoms. Thus, it could be regarded as a starting point for the investigation of more realistic, inhomogeneous and anisotropic computational models of osteoporotic bones or the callus tissue in fracture healing. Our ongoing work will make use of IEMA in order to describe scattering in callus sub-regions from different healing stages based on scanning acoustic microscopy images.

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