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Multi-dipole EEG Source Localization using Particle Swarm Optimization

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Abstract— **The multi-dipole EEG source localization problem is (usually) highly nonlinear with a non-convex cost function. Moreover, the gray matter tissue is located in several disjunct regions in the head which leads to a non-continuous solution space. For solving this problem an efficient algorithm which can handle multi-source activities is needed. In this paper, a modified particle swarm optimization (MPSO) method is proposed to solve the multi-dipole EEG source localization. The method is tested on synthetic EEG signals generated from two strong active sources and a noisy background source. The results show that using the new method is a reliable choice when we deal with a strong multi-active source scenario, in which a single dipole source localization may fail.**

I. INTRODUCTION

Epilepsy is one of the most common neurological diseases, and is present in up to 1% of the world's population. Many patients with epilepsy never receive the treatment which make them seizure free; consequently, treatment of epilepsy by medications is a major challenge, according to the World Health Organization [1]. Although intracranial surgery involves inherent risks, these risks are smaller than the risks of uncontrolled seizures. An electroencephalogram (EEG) is the most important method that is used in the clinical daily routine to find the source of activities inside the brain. The procedure of the EEG source localization deals with two problems: 1) a forward problem to find the scalp potentials for the given current source(s) inside the brain, 2) an inverse problem to estimate the source(s) that fit with the given potential distribution at the scalp electrodes.

The EEG source localization problem is (usually) highly nonlinear and requires efficient algorithms for its solution. The most widely used optimization methods for solving the EEG inverse problem can be classified into two groups: gradient methods, which use function and derivative information (e.g., Levenberg-Marquardt [2]), and search methods (non-gradient techniques) which use only function values (e.g., Nelder-Mead downhill simplex [3]). In both cases the methods minimize the cost function by iteratively adjusting the parameters of the dipole sources. The gradient-based optimization is fast to converge and effective when there is only one dipole in the source model and the data is noiseless. But when we use the multi-dipole model and have noisy data the local optimization approaches are not always effective since they are often trapped in local minima [4]–[8].

The dimension of the EEG source localization problem can be reduced by factoring out the linear parameters but still a fundamental problem remains: the least squares cost function is highly non-convex with respect to the locations of the dipoles. The gray matter tissue is located in several disjunct regions in the head which leads to a non-continuous solution space and makes the problem more difficult to solve using standard optimization methods [9]. Moreover, by applying the physiological constraints, such as orthogonality (sources are orthogonal to the gray matter surface) and the sparsity, the problem has a non-differentiable cost function [10]–[12]. In addition, the final solution often depends on the initial approximation and the number of local minima of the cost function [13] since reasonable initial guesses are difficult to make

Metaheuristic algorithms for global optimization have been used in the solution of the EEG inverse problem [14]–[16], and most of them reported high accuracy on the estimation of multiple dipoles with simulation and realistic studies. Nevertheless, a strict statistical study on the variability of these results under realistic conditions has not yet been performed, and the establishment of realistic confidence intervals as a function of the parameter space of the metaheuristic algorithms remains an open task.

Particle swarm optimization is a swarm intelligence algorithm for numerical optimization problems [17], [18]. In previous studies done by the authors, [19], [20], a modified particle swarm optimization (MPSO) method was proposed for solving the EEG source localization. In [19], by several examples, it is shown that, where a deterministic method, i.e., DIvide RECTangle (DIRECT) failed the MPSO could find the optimal solution significantly faster than other improved version of the PSO, as well as GA. In addition that, the MPSO is less prone to be trapped in local minima. In [20] and [21], it was shown that the MPSO is able to solve the EEG source localization in a real clinical setup.

In this paper, the MPSO method is extended to multi-dipole EEG source localization. Our main goal is to propose a novel technique for EEG source localization in the daily routine clinical application.

II. METHOD

A. Forward problem

The characteristic frequencies of the signals in the kHz range and below make the capacitive and inductive effects of the tissue negligible. Therefore, the quasi-static approximation of Maxwell's equations for the potential Φ can be used. If we denote the domain of interest as Ω (with boundary $\partial\Omega$) and let the tissue conductivity be σ, we have Poisson's equation

$$\nabla \cdot (\sigma \nabla \Phi) = \nabla \cdot \mathbf{j}^s \text{ in } \Omega, \tag{1}$$

subject to the conditions

$$\hat{\mathbf{n}} \cdot (\sigma \nabla \Phi) = 0 \text{ on } \partial\Omega, \tag{2}$$

$$\Phi(\mathbf{x}_{\text{ref}}) = 0. \tag{3}$$

The source current $\mathbf{j}^s(\mathbf{x}) = \delta(\mathbf{x} - \mathbf{x}_0)\mathbf{M}$ is modeled by a mathematical dipole at position $\mathbf{x}_0 \in \Omega$ with the moment

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$\mathbf{M} \in \mathbf{R}^3$.

The reciprocity principle was introduced by Helmholtz [22] and then adapted to the EEG problem by Rush and Driscoll [23] when they proved the applicability of reciprocity to anisotropic conductors. The concept allows switching the role of the electrodes and dipole sources. One of EEG electrodes is used as a common reference for the measurements (ground). We assume that p sources are present in the system. These sources are defined by its position, $\{\mathbf{x}_1, \cdots, \mathbf{x}_p\}$ and dipole moment (orientation and amplitude) $\{\mathbf{M}_1, \cdots, \mathbf{M}_p\}$. As we will be using reciprocity, we also define distributions of electric fields in the domain, $\mathbf{E}_j(\mathbf{x})$ for $j = 1, 2, \cdots, N_{\text{elec}} - 1$ where N_{elec} is number of EEG electrodes. These are the electric field distributions which are present in the system when we inject a unit current source to electrodes i, and withdraw a unit current at the reference electrode.

The theorem of reciprocity states that

$$u_{\text{elec}_j} = \alpha \sum_{n=1}^{p} \mathbf{M}_n \cdot \mathbf{E}_j(\mathbf{x}_n). \tag{4}$$

In this expression, the constant α depends on several factors, such as whether we use a voltage or a current source to calculate $\mathbf{E}_j$ as well as how the channels in the measurement system are set up. For simulated data in our system, $\alpha = 1$ (A^{-1}) when a unit current source is used to calculate $\mathbf{E}_j$.

For the reciprocity method, (4) holds for all electrodes and we obtain

$$\mathbf{u}_{\text{elec}} = \sum_{n=1}^{p} \mathbf{L}(\mathbf{x}_n)\mathbf{M}_n, \tag{5}$$

which

$$\mathbf{L}(\mathbf{x}_n) = \begin{bmatrix} E_{1,x}(\mathbf{x}_n) & E_{1,y}(\mathbf{x}_n) & E_{1,z}(\mathbf{x}_n) \\ E_{2,x}(\mathbf{x}_n) & E_{2,y}(\mathbf{x}_n) & E_{2,z}(\mathbf{x}_n) \\ \vdots & \vdots & \vdots \\ E_{N_{\text{elec}-1},x}(\mathbf{x}_n) & E_{N_{\text{elec}-1},y}(\mathbf{x}_n) & E_{N_{\text{elec}-1},z}(\mathbf{x}_n) \end{bmatrix} \tag{6}$$

B. Inverse Problem

In a parametric inverse method, the number of dipoles is assumed to be fixed and their locations and moments are chosen such that the potentials at the electrodes, $\mathbf{u}_{\text{elec}}$, that are computed according to (5), approximate the measured potentials $\mathbf{u}_{\text{meas}}$ well according to some criteria. Here we follow the common practice and choose the parameters such that we have the best fit in least squares sense. For one dipole we get the following minimization problem

$$J = \min_{\substack{\mathbf{x} \in \Omega_{\text{brain}} \\ \mathbf{M} \in \mathbf{R}^{\text{d}}}} \| \mathbf{u}_{\text{meas}} - \mathbf{L}(\mathbf{x})\mathbf{M} \|, \tag{7}$$

where Ω_{brain} is the brain domain and d the dimension. Since this is a least squares problem and $\mathbf{u}_{\text{elec}}$ depends linearly on the dipole moment it is convenient to separate the parameters in (7) and solve for the dipole moment $\mathbf{M}$ first. Define, for fixed $\mathbf{x} \in \Omega_{\text{brain}}$,

$$J(\mathbf{x}) = \min_{\mathbf{M} \in \mathbf{R}^{\text{d}}} \| \mathbf{u}_{\text{meas}} - \mathbf{L}(\mathbf{x})\mathbf{M} \| . \tag{8}$$

According to the normal equations for linear least squares problems, optimality is obtained for

$$\mathbf{M}_{\text{opt}}(\mathbf{x}) = (\mathbf{L}^T(\mathbf{x})\mathbf{L}(\mathbf{x}))^{-1}\mathbf{L}^T(\mathbf{x})\mathbf{u}_{\text{meas}}. \tag{9}$$

Substituting (9) into (8) yields after some manipulation

$$J(\mathbf{x}) = \left(\mathbf{u}_{\text{meas}}^T[\mathbf{I} - \mathbf{L}(\mathbf{x})(\mathbf{L}^T(\mathbf{x})\mathbf{L}(\mathbf{x}))^{-1}\mathbf{L}^T(\mathbf{x})]\mathbf{u}_{\text{meas}}\right)^{1/2}. \tag{10}$$

Here the optimization problem is the function of the source position only, thus the complexity of the inverse problem is reduced.

III. PARTICLE SWARM OPTIMIZATION

A. Standard Particle Swarm Optimization

The Particle Swarm Optimization concept was first introduced by Kennedy and Eberhart [17], [18] in 1995 based on the social system behavior such as movement of the school of birds or the flock of fishes for finding food. Each individual in the swarm is called a particle. In the original version particles move according to the following formula:

$$\begin{cases} \mathbf{V}_i^{t+1} & = \mathbf{V}_i^t + c_1 Rand()(\mathbf{P}_i - \mathbf{X}_i^t) + c_2 Rand()(\mathbf{P}_g - \mathbf{X}_i^t), \\ \mathbf{X}_i^{t+1} & = \mathbf{X}_i^t + \mathbf{V}_i^{t+1}. \end{cases} \tag{11}$$

where $\mathbf{X}$ and $\mathbf{V}$ represent the particle position and velocity, respectively. $\mathbf{P}_i$ and $\mathbf{P}_g$ are the personal best (*pbest*) and global best (*gbest*), respectively. i represents the particle index and t is the time step. $Rand()$ denotes a normally distributed one-dimensional random number with mean zero and standard deviation one. Parameters c_1 and c_2 are the *cognitive* and *social learning rates*.

The PSO algorithm introduced by Kennedy and Eberhart has been proven to be powerful but needs to select various parameters, such as the maximum velocity coefficient, the swarm size, the neighborhood size as well as the cognitive and social learning rates. Moreover, the parameter selection in a specific problem is not straightforward. The PSO algorithm has a risk to trap in a local minima and lose its exploration–exploitation ability [24]. In the following section we describe a modified PSO (MPSO) [19], which can help to cure the aforementioned drawbacks.

B. Modified Particle Swarm Optimization

In modified PSO (MPSO) [19] we use the concept of *authority* mixed with the mutation and EP and apply it to the particle's behavior. In this method M particles are selected among the swarm population by the q-tournament selection method and then mutated by the EP method [25]. By evaluating the fitness value of all the particles, the global best position is determined. For each particle, the nearest elite particle, $\mathbf{P}_e$, is determined by the Euclidean distance. The velocity and the position of the particles are updated according to the global best position, the nearest elite position, and the personal best position.

Moreover, to maintain the exploration ability and increase the exploitation ability we introduced the concept of *authority* in the MPSO. The concept of *authority* means that in some steps the particles which are closer to the global best can influence the swarm's performance and decision more than other particles. This is because when the *gbest* particle is moving close to the minima, it cannot move faster than its velocity weight which is a small value during the last iterations. When PSO comes close to a minima (local or global) it can only find the global one when it has sufficiently many particles around *gbest*. Thus, PSO needs a lot of iterations to gather enough particles around *gbest*.

We extract the $R = 5$ closest particles to the *gbest* and let them fly freely based on their memory and knowledge. This allows the PSO to have more information around *gbest* before lots of particles come close to it and stuck with each other. Now, the velocity update is divided into two parts as

$$\mathbf{V}_i^{t+1} = w\mathbf{V}_i^t + c_1 Rand()(\mathbf{P}_i - \mathbf{X}_i^t) + c_2 Rand()(\mathbf{P}_g - \mathbf{X}_i^t) + c_3 Rand()(\mathbf{P}_e - \mathbf{X}_i^t), \quad (12)$$

where $i = 1, 2, ..., N-R$, c_3 and $\mathbf{P}_e$ denote the constant and position of the nearest elite, respectively. Then, the second part is

$$\mathbf{V}_r^{t+1} = w\mathbf{V}_r^t + c_1 Rand()(\mathbf{P}_r - \mathbf{X}_r^t) \quad (13)$$

where $\mathbf{V}_r$, $\mathbf{X}_r$, and $\mathbf{P}_r$ are the velocity, position and personal best of the r-th particle, respectively, for $r = N-R+1, ..., N$ and w is the inertia weight advised by [26]. The R nearest particles to *gbest* are re-selected in each iteration to ensure that the particles which moved away from the *gbest* lose their authority and update their velocity according to (12). The following parameters are selected for the MPSO coefficients: w = linear from 0.9 to 0.4, $c_1 = 0.8$, $c_2 = 0.4$ and initial swarm size = 30, in addition the MPSO uses adaptive swarm size [27] during its searching progress.

IV. Multi-dipole Source Localization

Theoretically, it would be possible to calculate the objective function for all combinations of p sources in N_{gray} possible locations in the gray matter, i.e.,

$$\binom{N_{\text{gray}}}{p} = \frac{N_{\text{gray}}!}{(N_{\text{gray}} - p)!p!}, \quad (14)$$

evaluations. In practice, this is generally not feasible as the number of gray matter points in the configuration space is too large and cannot be explored exhaustively. The PSO is flexible and straightforward to extend to multi-dipole source localizations. For p source locations, $6p$ unknown parameters should be estimated in 3D, i.e., $3p$ dipole position parameters in Cartesian space (x, y, z) and $3p$ dipole moments (M_x, M_y, M_z). Thus the $i-th$ particle of the swarm can be represented by the vector $\mathbf{X}_i \in \mathbb{R}^{np}$ and $\mathbf{V}_i \in \mathbb{R}^{np}$, where $n = 1, 2, 3$ is the problem dimension. For $n = 3$ we get,

$$\begin{cases} \mathbf{X}_i = ((x_1, y_1, z_1), \cdots, (x_p, y_p, z_p), (M_{x_1}, M_{y_1}, M_{z_1}), \cdots, (M_{x_p}, M_{y_p}, M_{z_p}))_i, \\ \mathbf{V}_i = ((V_{x_1}, V_{y_1}, V_{z_1}), \cdots, (V_{x_p}, V_{y_p}, V_{z_p}), (V_{M_{x_1}}, V_{M_{y_1}}, V_{M_{z_1}}), \cdots, (V_{M_{x_p}}, V_{M_{y_p}}, V_{M_{z_p}}))_i. \end{cases} \quad (15)$$

With this configuration for the particles we can now again use (12) and (13) to minimize the cost function. The minimization problem (7) then becomes,

$$J = \min_{\substack{\mathbf{x} \in \Omega_{\text{brain}} \\ \mathbf{M} \in \mathbb{R}^d}} \| \mathbf{u}_{\text{meas}} - \sum_{n=1}^{p} \mathbf{L}(\mathbf{x}_n)\mathbf{M}_n \|^2, \quad (16)$$

To reduce the unknown parameters we can use the method explained in Section II-B. For two dipoles with positions $\mathbf{x}_1 \in \Omega_{\text{brain}}$, $\mathbf{x}_2 \in \Omega_{\text{brain}}$ and orientation $\mathbf{M}_1 \in \mathbb{R}^3$ the optimal components $\mathbf{M}_{2_{\text{opt}}}$ are found in least squares sense as the solution of the linear equations $\mathbf{u}_{\text{meas}} - \mathbf{L}(\mathbf{x}_1)\mathbf{M}_1 = \mathbf{L}(\mathbf{x}_2)\mathbf{M}_2$, i.e.,

$$\mathbf{M}_{2_{\text{opt}}}(\mathbf{x}) = (\mathbf{L}^T(\mathbf{x}_2)\mathbf{L}(\mathbf{x}_2))^{-1}\mathbf{L}^T(\mathbf{x}_2)\tilde{\mathbf{u}}_{\text{meas}}. \quad (17)$$

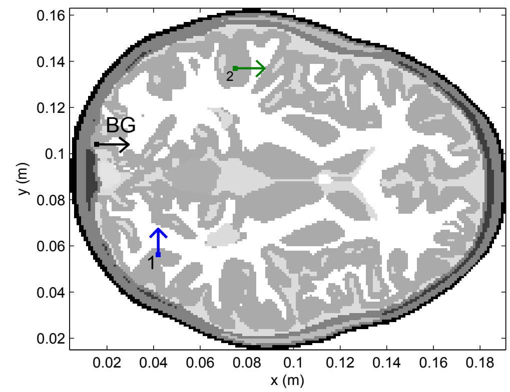

Fig. 1: Position and orientation of the two active spikes, with the same amplitude equal to 10 μAm and background sources, 1 μAm, in 2D setup

where $\tilde{\mathbf{u}}_{\text{meas}} = \mathbf{u}_{\text{meas}} - \mathbf{L}(\mathbf{x}_1)\mathbf{M}_1$.

V. MPSO results

To test the ability of the MPSO to localize multiple sources, we generate sets of simulated potentials for 30 channel electrodes for two active spike sources placed in both brain hemispheres in a 2D case, see Fig. 2. The following conductivities were then assigned to the FE compartments based on their segmentation labels and the isotropic reference model [28]: skin = 0.43 S/m, skull = 0.0042 S/m (skull to skin conductivity ratio of approximately 1:100), CSF = 1.538 S/m, gray matter = 0.33 S/m, and white matter = 0.142 S/m.

The background dipole is fixed at the occipital lobe for all cases. The two spike sources have the same amplitude equal to 10 μAm, one with radial direction and the other with tangential direction. Fig. 2 shows the cost function when only a single dipole is used to estimate the potential for this test case. As we can see in Fig. 2 the global minimum is located 10.3 mm from the source in the left hemisphere and 88.4 mm from the source in the right hemisphere. The relative error is equal to 0.52 and clearly a single dipole is not enough in this case.

We ran the multi-dipole MPSO for 100 cases in which the position of active sources were selected randomly inside the gray matter. The MPSO had 30 initial particles and the optimization was stopped if the relative error $\leq$ 0.08

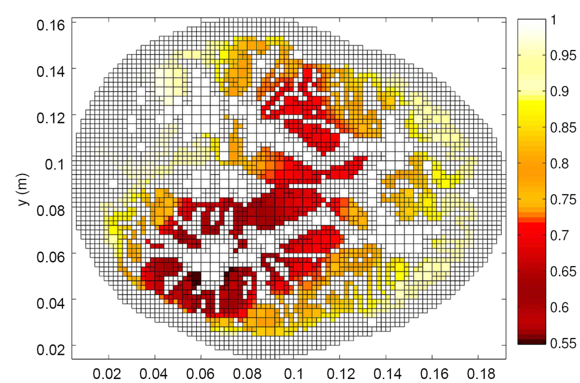

Fig. 2: The cost function when a single dipole source is used.

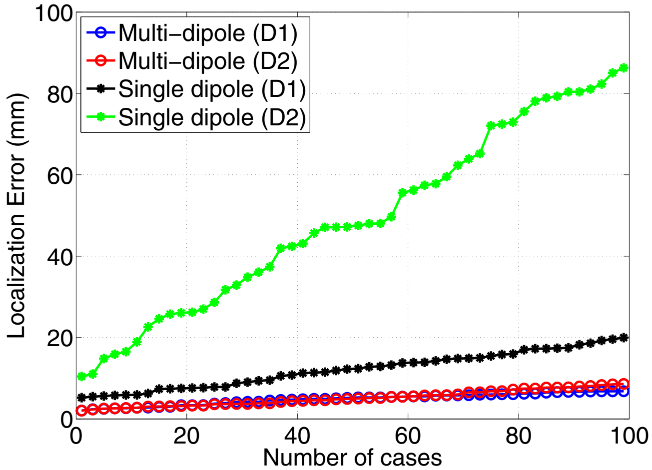

Fig. 3: Localization error for 100 cases (the results are sorted in ascending order to make the plot easier to interpret).

(this value was obtained when the exact dipole positions and orientations were selected as input for the optimization problem). Fig. 3 shows the localization error for single dipole localization compared to the multi-dipole MPSO localization for 100 cases.

Table I presents the mean localization and orientation error for both multi-dipole MPSO and single dipole source localization done by an exhaustive search. The results in Table I show a significant source localization improvement compared to the one dipole localization approach. In our case the head model had 2 879 gray matter points. Thus all possible unique combinations of the two sources are 4 142 881. The multi-dipole MPSO found the optimal solution with only 800 evaluations in average, which is 0.019% of the total number of possible choices.

TABLE I: Mean of the localization and orientation errors for 100 cases.

	Multi-Dipole		Single Dipole	
	Source 1	Source 2	Source 1	Source 2
Mean LE $\pm$ std (mm)	4.6±1.5	5.8±1.9	12.1±4.3	48.2±23.8
Mean OE $\pm$ std (deg)	2.0±1.1	3.4±1.4	8.3±2.3	10.2±5.7

VI. Conclusion

In this paper, the ability of a new optimization method was tested for multi-dipole EEG source localization. The new method is a modified version of particle swarm optimization. The positions and orientations of dipoles are optimized to obtain the best least squares fit with the measured EEG signals. The results showed that using multi-dipole MPSO source localization is a reliable choice when we deal with a strong multi-active source scenario, since a single dipole source localization may fail in that case. Future work includes to apply the proposed method on real EEG signals generated from two separate sources, such as auditory evoked potentials.

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