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Subspace Electrode Selection methodology for EEG multiple source localization error reduction due to uncertain conductivity values

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Abstract— **This paper proposes a modification of the subspace correlation cost function and the Recursively Applied and Projected Multiple Signal Classification (RAP-MUSIC) method for electroencephalography (EEG) source analysis in epilepsy. This enables to reconstruct neural source locations and orientations that are less degraded due to the uncertain knowledge of the head conductivity values. An extended linear forward model is used in the subspace correlation cost function that incorporates the sensitivity of the EEG potentials to the uncertain conductivity value parameter. More specifically, the principal vector of the subspace correlation function is used to provide relevant information for solving the EEG inverse problems. A simulation study is carried out on a simplified spherical head model with uncertain skull to soft tissue conductivity ratio. Results show an improvement in the reconstruction accuracy of source parameters compared to traditional methodology, when using conductivity ratio values that are different from the actual conductivity ratio.**

I. Introduction

In epilepsy, an accurate information on the location of epileptic focus in the brain can be used to plan surgery for its removal [1]. The estimation of the neural source generators responsible for e.g. epileptic spikes, starting from electroencephalogram (EEG) data, is however subject to some sources of errors: noise in measurements, forward modeling errors and the ill-posedness of the inverse problem.

A first class of model-related errors are source modeling errors. A current–dipole source is suitable because it represents a population of active pyramidal cells at the microscopic level, but is only valid if the activity itself is limited to a focal region and if it stays focal over a period of time [2]. For patients suffering from epilepsy, focal brain activity is mostly the case. In order to reduce these source modeling errors, it is possible to use more complex source models. Distributed source models can represent an alternative where the inverse problem is highly underdetermined and regularization methods are required, e.g. [3]. Another source modeling approach consists of limiting the parameters of the multidipolar sources to be less than the number of electrodes e.g. the Recursively Applied and Projected–MUltiple SIgnal Classification (RAP–MUSIC) algorithm [4].

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A second class is the possible inaccurate geometrical modeling of the head [5]. When using patient-specific head models based on T1-segmented magnetic resonance images, the geometrical modeling error is limited. Another type of forward modeling errors can be electrode misplacements [6]. A fourth type of modeling-related errors are head models where the anisotropic behavior of the conductivity is not incorporated. Using diffusion tensor magnetic resonance imaging it is possible to estimate the nerve bundle direction. [7,8] have shown that anisotropically conducting compartments have a significant impact on EEG and MEG activity. Finally, large errors are introduced due to the use of inaccurate absolute conductivity values of the several tissues in the volume conductor head model, e.g. [9,10]. The uncertain conductivity values, more specifically the ratio of the skull conductivity to the conductivity values of the soft tissues, have a large influence on the EEG dipole localization accuracy and are the most dominant source of error [11].

A method was recently proposed for reducing the impact of uncertain conductivity values onto the solution of the EEG single dipole localization problem when using a spherical head model [12] and realistic head model [13]. The least squares cost function was modified and the iterative selection of a subset of electrodes in the EEG cap was introduced. The method enables to recover neural source locations and orientations that are robust to the uncertain skull to soft tissue conductivity ratio. Possibilities for recovering a limited number of neural sources in a conductivity robust way were given. This paper aims at studying the use of a modified subspace-based method, that is suitable for reconstructing a limited number of sources in the brain through modification of the RAP-MUSIC method.

II. Subspace Electrode Selection Methodology

A. Subspace correlation cost function

In this paper, we assume that electrical activity in the brain is modeled by equivalent current dipoles. This assumption ensures a unique solution to the EEG inverse problem [14], when reconstructing the source parameters from the EEG potentials recorded at various electrodes placed at the surface of the head. The estimation of multiple dipole parameters is known as a difficult nonlinear minimization problem because the cost functional contains many local minima. The traditional RAP-MUSIC cost function for the reconstruction of the *l*-th dipole position, is given by the following relation:

$$r_l^* = \arg\max_r (subcorr(\Pi^{\perp}_{A_{l-1}} L(r), \Pi^{\perp}_{A_{l-1}} \Phi_S)_1) \qquad (1)$$

with $L(r)$ being the lead field matrix of the forward model solution, Φ_S the signal subspace of the spatio-temporal measurement matrix. Here, $\Pi^{\perp}_{A_{l-1}} = (I - A_{l-1}A^{\dagger}_{l-1})$ is the projection matrix constructed by A_{l-1}, a matrix containing in each column the topographies of the already found *l-1* sources; $A^{\dagger}$ is the Moore Penrose pseudo inverse matrix. The subspace correlation term $subcorr(\cdot)_1$ computes the cosine of the first principal angle between the subspaces spanned by the columns of $L(r)$ and Φ_S. The computed value here is 0 if the two subspaces are orthogonal and a number between 0 and 1 if the two subspaces have at least one dimensional subspace in common. The Nelder-Mead simplex methodology is used here for the maximization in (1).

B. Sensitivity analysis

An approach for reducing the propagation of the uncertainty on the inverse solutions is thus needed. A sensitivity analysis can provide insights onto the impact of the conductivity values on the EEG potentials at the different sensors located on the scalp. From the semi-analytical solution of the EEG forward problem in [15], we can write the EEG potential along the N electrodes at a single instant as $V_m = L(r).d$ with $L \in R^{N\times 3}$ the lead field matrix and $d \in R^3$ the dipole orientation. When having actual dipole location r_{act}, actual dipole orientation d_{act}, actual conductivity ratio $\tilde{X}$ and we assume that the measurements have no noise and that the other modeling errors are negligible, then we have

$$V_{meas} \equiv V_m(r_{act}, d_{act}, \tilde{X}) \tag{2}$$

However, the assumed conductivity ratio is uncertain and thus different from the actual conductivity ratio $\hat{X} \neq \tilde{X}$. When solving the inverse problem, the measured EEG potentials are best approximated by

$$V_{meas} \approx V_m(r^*, d^*, \hat{X}) \tag{3}$$

with the recovered dipole location r^* and dipole orientation d^*. The term on the right in (3) can be written as a first order Taylor expansion [13] around $\tilde{X}$ with r_{act}, d_{act}:

$$V_m(r_{act}, d_{act}, \tilde{X}) + (\hat{X} - \tilde{X}) \left. \frac{\partial V_m(r_{act}, d_{act}, X)}{\partial X} \right|_{X=\tilde{X}} \tag{4}$$

where the second term depends on the propagation of the uncertainty to the forward problem. The Taylor coefficient $\alpha = \hat{X} - \tilde{X}$ which is unknown, can be approximated by fitting the data set $Y = V_{meas} - V_m(r, d, \tilde{X})$ with $S = \left. \frac{\partial V_m(r_{act}, d_{act}, X)}{\partial X} \right|_{X=\tilde{X}}$, see also [12,13].

The following transformation is performed so to obtain a forward linear model to be used in the cost function:

$$L(r) \rightarrow L(r) + \alpha W(r) \tag{5}$$

with $W(r) \in R^{N\times 3}$ being the derivative of the lead field matrix to the conductivity ratio i.e. $W(r) = \left. \frac{\partial L(r)}{\partial X} \right|_{X=\tilde{X}}$. By selecting sensors less influenced by the sensitivity, it is possible to reconstruct the dipole parameters in a more accurate way. The selection is intended to reduce the propagation of the uncertainty on the inverse solutions. Two approaches can be used here to compute the sensitivity of the potentials to the conductivity: a direct differentiation of the forward model solution and the differentiation using the principal vector U of the subspace correlation function i.e. $\partial U/\partial X$. Fig. 1 show a 2D mapping of the sensitivity using both approaches for dipoles situated in the center of the brain. One can identify the sensitive sensors (labeled between 1 and 27) from these figures.

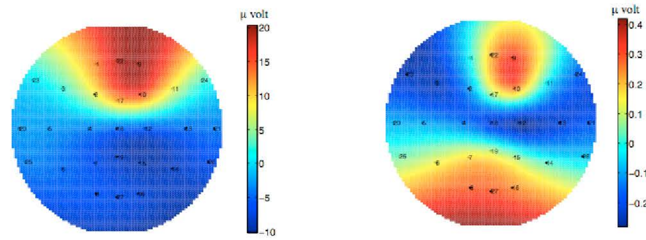

Figure 1. Sensitivity of potential values (left) and principal vector U (right) towards the conductivity ratio in 2D.

C. Subspace Electrode Selection and the RAP-MUSIC methodology

From the information provided by the sensitivity analysis, we alter the traditionally known RAP-MUSIC subspace-based cost function and incorporate the sensitivity of the potentials to the uncertain conductivity ratio X. The dipole positions can then be reconstructed by maximizing the following cost function:

$$r_l^* = \arg\max_r (subcorr(\Pi^{\perp}_{A_{l-1}}(L(r) + \alpha W(r)), \Pi^{\perp}_{A_{l-1}}\Phi_S)_1) \tag{6}$$

based on cost function (1) and transformation (5). The iterative scheme of the methodology is shown in Fig. 2.

The proposed methodology contains several steps that are iteratively carried out until the p-th dipole is recovered. In this flowchart, l $(l=1,\dots,p)$ is the dipole number, while k is the iteration number in the maximization of the subspace correlation function.

Step 1: Evaluation of the start value $r_l^{(0)}$ in the forward model, yielding $V_m(r_l^{(0)})$. An assumed conductivity ratio $\hat{X}$ is used here. Initialize $k=0$.

Step 2: Computation of the principal vector U using the potential value $V_m(r_l^{(k)})$.

Step 3: Calculation of the sensitivity of the principal vector values to the conductivity ratio for a certain assumed conductivity ratio value $\hat{X}$:

$$Y(r_l^{(k)}) = \left. \frac{\partial U(r_l^{(k)})}{\partial X} \right|_{X=\hat{X}} \tag{7}$$

in the k-th iteration for fixed $r_l^{(k)}$.

Step 4: Selection of the N_s least sensitive electrodes, based on the values of each electrode in (7). For the selection, we set a certain threshold for sensitivity values, in order to obtain a subset of potentials that are used in the inverse procedure. This step defines the selection operator in the k-th iteration for selection of the least conductivity dependent sensors.

Step 5: Calculation of the Reduced Conductivity Dependence (RCD) subspace-based correlation cost function:

$$subcorr(\Pi^{\perp}_{A_{l-1}}(L(r)+\alpha W(r)),sel(\Pi^{\perp}_{A_{l-1}}\Phi_S))_1 \quad (8)$$

Step 6: Based on the value (8), the next iterate $r_l^{(k+1)}$ is calculated. If the termination criteria of the maximization procedure are met, then stop the recovery of the *l*-th dipole. Otherwise, update $k=k+1$ and go to step 2. If the number of dipoles is reached, then stop the algorithm.

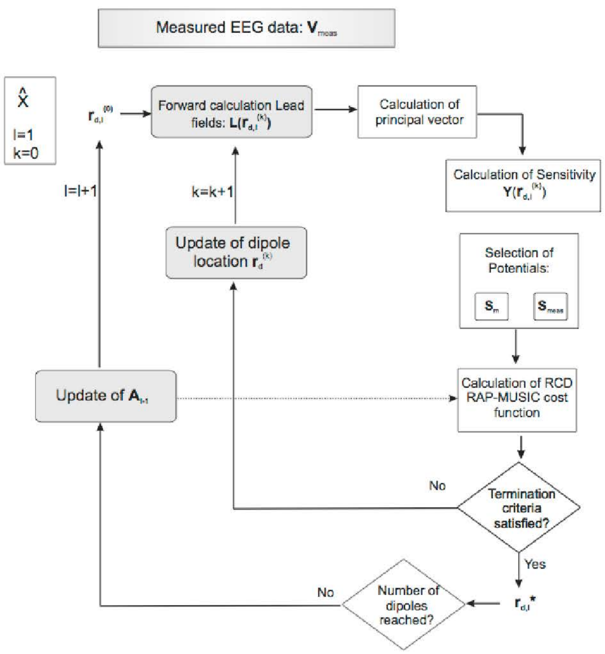

Figure 2. Iterative scheme of the subspace electrode selection methodology applied in RAP-MUSIC algorithm.

III. Results and Discussion

A. Simulation setup

The performance of the subspace electrode selection method in case of reconstruction of a limited number of multiple dipoles, is investigated by performing numerical experiments. Synthetic EEG data is generated with dipoles showing rhythmic activity (cosinusoidal waveform between 8 and 10 Hz) and dipoles representing an epileptic spike of 0.2s with onset at 0.4s is used. Using the forward model with certain actual conductivity ratio value $\tilde{X}$, a spatio-temporal matrix is generated for an EEG electrode setup (standard 10-10 setup with 27 electrodes):

$$V_{meas}=\left[L(\tilde{r}_1),L(\tilde{r}_2),...,L(\tilde{r}_p)\right]\left[\tilde{d}_1,\tilde{d}_2,...,\tilde{d}_p\right]^T \quad (9)$$

with $\tilde{d}_i=\tilde{u}_i\tilde{s}_i$. $\tilde{u}_i$ is a unit-norm dipole orientation vector and $\tilde{s}_i$ a n_t-dimensional vector expressing the waveform of the dipole strength of n_t time samples for each dipole.

The signal subspace Φ_S is calculated from the autocorrelation matrix $R_V=E(V_{meas}V^T_{meas})$ where *E(.)* is the mathematical expectation.

B. Recovery error reduction

We solve the EEG inverse problem for multiple assumed conductivity ratios starting from an EEG data set generated with actual $\tilde{X}$ value. Fig.3 shows the results for $p=4$ dipoles when recovering the dipoles using the traditional RAP-MUSIC method and when using the methodology elaborated in steps 1-6 of section IIC. The accuracy is measured through the dipole position error (DPE) which is the difference between the actual dipole location r_{act} that is associated to the measured EEG data and the recovered neural dipoles r^* calculated using that measured EEG dataset: $DPE=\left\|r^*-r_{act}\right\|_2$. The DPE increases when the assumed conductivity value is more different from the actual conductivity ratio. The results show that the DPE is overall reduced when recovering the 4 dipoles using the RCD based methodology.

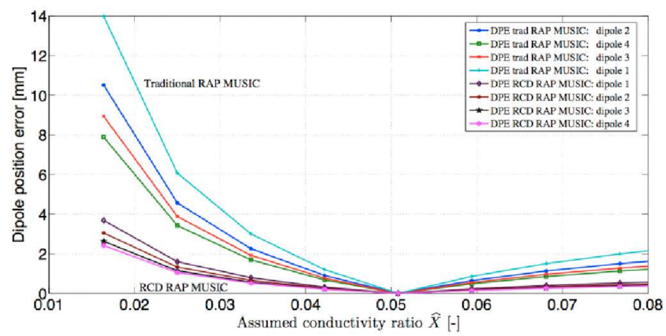

Figure 3. Average dipole position errors when recovering p=4 dipoles starting from EEG data with $\tilde{X}$=1/20. The number of selected sensors is N_s=13.

C. Noise robustness

EEG spatio-temporal data was generated with added zero mean white Gaussian noise. The noise level is defined as $n=\Sigma/V_{RMS}$. DPE are shown in Fig. 4 for different noise levels. The EEG inverse problem is solved in case no singular value decomposition or principal component analysis (PCA) is applied, while in the second case, a spatio-temporal dataset is used. We observe that the cost function (8) is noise robust. The fitting constant α is much better approximated when using as data *U* and *Y*. The principal vector and corresponding sensitivity (7) are less affected by noise. An efficiency ranging from 60% to 75% is observed in the reconstruction accuracy. Fig. 5 shows the total dipole position errors in case a different number of selected subsets N_S is employed in step 4 of section IIC. The total dipole position error is the summation of all DPE of the different dipoles.

When reducing the number of selected sensors, the accuracy of reconstruction is increased. Note that the number of selected sensors may not be reduced so that the inverse problem becomes ill-posed. Moreover, in case of noise levels higher than 0.2, the simulation results show that the reduction needs to be limited until N_s=*N*/2. The iterative procedure follows a path that is less affected by the uncertain parameter values.

The total amount of sensors N can be used throughout the algorithm but in each iteration, when calculating the cost function, N_s sensors are used. Moreover, the N_s sensors vary when solving the inverse problem. These selected sensors depend on the dipole location and orientation in the k-th iteration of the minimization process.

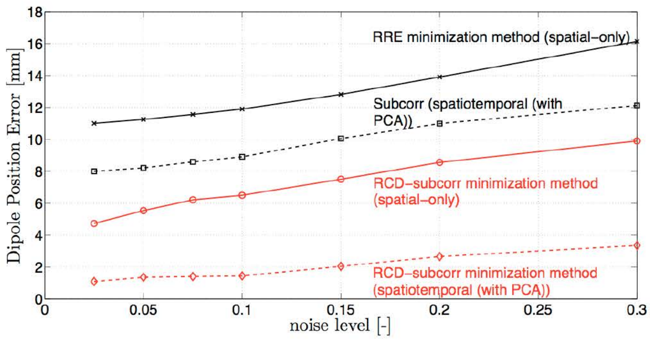

Figure 4. Dipole position error for p=1 dipole using spatial-only EEG data and spatio-temporal EEG data of the same dipole with the time signal being a spike in head model with $\tilde{\boldsymbol{X}}$=1/25, while the assumed conductivity ratio is $\hat{\boldsymbol{X}}$=1/9.

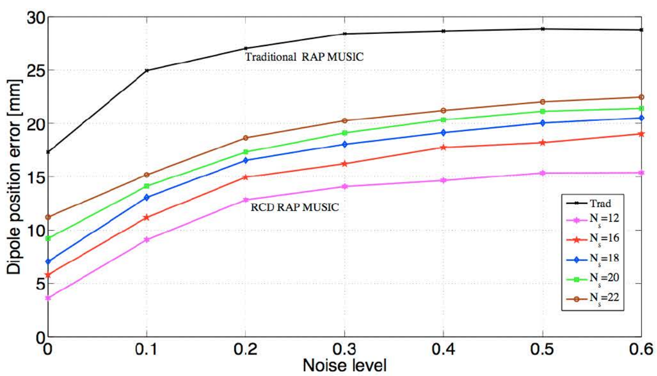

Figure 5. Total diople position error for $p=3$ dipoles in case of different selected subsets Ns for different noise levels in the EEG data. $\tilde{\boldsymbol{X}}$=1/13 while $\hat{\boldsymbol{X}}$=1/50.

IV. Conclusion

We modified the subspace correlation cost function and incorporated selection of the electrodes within the methodology, based on the sensitivity to the uncertain conductivity which is computed using the principal vector. A numerical study was performed in a three-shell spherical head model and shows a reduction of the dipole position errors compared to the traditional approach. The methodology is able to localize a limited number of dipoles in an accurate way. Moreover, the methodology is robust to noise in the measured EEG data. The presented methodology can also be applied onto other inverse problems where uncertain parameters are present in the forward modeling.

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