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Cortical Dipole Imaging using Truncated Total Least Squares Considering Transfer Matrix Error*

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Abstract— **Cortical dipole imaging has been proposed as a method to visualize electroencephalogram in high spatial resolution. We investigated the inverse technique of cortical dipole imaging using a truncated total least squares (TTLS). The TTLS is a regularization technique to reduce the influence from both the measurement noise and the transfer matrix error caused by the head model distortion. The estimation of the regularization parameter was also investigated based on L-curve. The computer simulation suggested that the estimation accuracy was improved by the TTLS compared with Tikhonov regularization. The proposed method was applied to human experimental data of visual evoked potentials. We confirmed the TTLS provided the high spatial resolution of cortical dipole imaging.**

I. Introduction

Noninvasive EEG (electroencephalogram) recordings with low restriction on the measurement environment are effective to analyze brain function in daily life. However, the spatial resolution of the EEG data is limited because of a small number of scalp electrodes and low conductivity of a skull. Therefore, it was difficult to specify brain electrical activity directly from the potential distribution measured on the scalp surface. Cortical dipole imaging that estimates the equivalent dipole source distribution on a virtual layer within a brain from the scalp potential has been proposed to solve this problem [1], [2]. According to this method, brain electrical activity is represented by the equivalent dipole distribution without being restricted in the number and the direction of the dipole sources.

The cortical dipole distribution is estimated from the scalp potentials by solving an inverse problem of the transfer matrix from the dipole layer to the scalp surface based on a head model. The solution of the inverse problem is influenced not only by the measurement noise but also by the error in the transfer matrix. The measurement noise originates in the measurement environment such as the variance of electrode impedance, the noise, and the artifact caused by eye blinks or body movements. On the other hand, the transfer matrix error originates in the distortion of the head model design such as gaps of an electrode displacement, individual difference of the head shape, and the non-uniform electrical conductivity. Therefore, it is important to reduce both the measurement noise and the transfer matrix error for the EEG inverse solution of the cortical dipole imaging.

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Several spatial inverse filters have been proposed to reduce the influence on the measurement noise. Tikhonov regularization was applied to truncate the noisy components [3]. The parametric projection filter incorporated with the statistical information on the noise was proposed [4], [5]. However, the transfer matrix error was not taken into consideration by these methods. In the present study, we examined the solution in consideration of the transfer matrix error of the EEG inverse problem aiming to improve the accuracy of cortical dipole imaging. The truncated total least squares (TTLS) method is proposed to reduce the influence of the transfer matrix error [6]. In this method, after scaling the covariance of the transfer matrix error equals to that of the measurement noise, the solution is estimated by minimizing the influence from both error and noise. The TTLS has been applied to bioluminescence topography inverse problem [7] and an ECG inverse problem [8].

In the present study, the TTLS was applied to the cortical dipole imaging. In computer simulations, we compared the restorative ability of TTLS with the conventional Tikhonov regularization. Moreover, the method to estimate the regularization parameter which adjusts the accuracy and noise reduction was also examined. Based on the simulation results, the TTLS-based cortical dipole imaging was applied to human EEG recordings of visual evoked potential (VEP).

II. Method

A. Cortical Dipole Imaging

We used a head model to estimate the cortical dipole distribution from scalp potential. The head volume conductor is approximated by an inhomogeneous three-concentric sphere model that represents the scalp, the skull, and the brain [2]. The radii of the scalp, the skull, and the cortex were set to 1.0, 0.94, and 0.87, respectively. The electrical conductivity of the cortex and the scalp was set to 1.0, and that of the skull was set to 0.0125. A dipole layer was virtually established inside of the cortex with arbitrary radius. 1280 radial dipoles were located on this layer at equal intervals. The signal sources in a brain can be equivalently represented by the dipole layer distribution without any restriction on the number and direction of dipole sources.

The observation of the scalp potential g is modeled using the transfer matrix A from the dipole layer to the scalp surface as follows:

$$g = A f + n \tag{1}$$

where f is the dipole distribution and n is the measurement noise. The transfer matrix A is determined from the geometry of the head model, the electrical conductivity involved, and the electrode and equivalent dipole source arrangements. In addition to measurement noise, there is an error in the electrode displacements, the geometry, and the electrical conductivity of the head model against the realistic head. We call it a transfer matrix error, E. The observation process considering the transfer matrix error E is expressed by

$$g = (A + E) f + n \tag{2}$$

The inverse problem should be solved in order to estimate the dipole distribution f from the measured scalp potential g. As the method to construct the inverse filter, we employed the TTLS [6] method that reduces both the measurement noise and the transfer matrix error.

B. Inverse Techniques

The transfer matrix is ill-conditioned and the EEG inverse problem can be characterized as an ill-posed problem. The dipole distribution was solved using a least squares method as follows:

$$\min \|Af - g\|_2 \tag{3}$$

where $\| \|_2$ denoted the Euclidian norm of vector space. In order to obtain a stable solution, a regularization method such as Tikhonov regularization is typically used to suppress the influence of the measurement noise. According to Tikhonov regularization, the inverse solution f_0 is calculated using a regularization parameter γ as follows:

$$f_0 = (A^{\mathrm{T}} A + \gamma I)^{-1} A^{\mathrm{T}} g \tag{4}$$

where A^{T} is a transposed matrix of A.

In the least squares method, when an error exists in a transfer matrix, the estimation includes a bias. The total least squares (TLS) method is proposed to reduce the influence by the transfer matrix error. The TLS method is generalized version of the original lest squares method, and it is motivated by linear models $A f = g$ in which both A and f have errors. Instead of using standard least squares formulation, we state the problem with TLS formulation as follows:

$$\min_{\tilde{A},\tilde{g}} \|(A, g) - (\tilde{A}, \tilde{g})\|_F \quad \text{subject to} \quad \tilde{g} = \tilde{A} f$$

where $\| \|_F$ denotes the Frobenius norm, $\tilde{g}$ and $\tilde{A}$ are the error versions of g and A, respectively. Corresponding to truncated singular value decomposition (SVD) in least squares method, the regularization in TLS includes the TTLS method. By the SVD, the augmented matrix (A, g) is decomposed using the mth-order orthogonal matrix U and the (n+1)th-order orthogonal matrix V as follows:

$$(A, g) = U \Sigma V^{\mathrm{T}} \tag{5}$$

where

$$U = (u_1, \dots , u_m),\ V = (v_1, \dots , v_{n+1}) \tag{6}$$

$$U U^{\mathrm{T}} = I_m,\ V V^{\mathrm{T}} = I_{n+1} \tag{7}$$

$$\Sigma = (z_{ij}) \in R^{m\times(n+1)}, z_{ij} = \begin{cases} \sigma_i \ for\ i\ = j \\ 0\ for\ i\ \neq j \end{cases} \tag{8}$$

$$\sigma_1 \geq \dots \geq \sigma_r > 0,\ \sigma_{r+1} = \dots = \sigma_{n+1} = 0 \tag{9}$$

Here, σ_i are the singular values, u_i are the left singular vectors, and v_j are the right singular vectors. r is the number of non-zero singular values. By using a right singular vector and regularization (truncation) parameter k, the solution f_0 is derived by

$$f_0 = -V_{12}V_{22}^{+} = -\frac{V_{12}V_{22}^{T}}{\|V_{22}\|_2^2} \tag{10}$$

where

$$V = \begin{pmatrix} V_{11} & V_{12} \\ V_{21} & V_{22} \end{pmatrix},\ V_{11} \in R^{n\times k},\ V_{22} \epsilon R^{1\times(n+1-k)} \tag{11}$$

In TTLS, the measurement noise and the transfer matrix error is summarized by performing SVD of an augmented matrix. Then, the bias caused by the transfer matrix error is reduced by restricting the singular values which amplifies the measurement noise and a transfer matrix error with a truncation parameter.

C. Estimation of Regularization Parameter

In order to construct the inverse filters, it is necessary to determine the regularization parameter γ in Tikhonov regularization and to determine the truncation parameter k in TTLS. In simulation, the optimal parameter is determined by minimizing the relative error (RE) between the actual dipole distribution f and estimated dipole distribution f_0.

$$RE = \| f_0 - f \|_2 \,/\, \| f \|_2 \tag{12}$$

However, the actual dipole distribution f is unknown. In such cases, the L-curve method has been proposed to estimate the regularization parameter [9]. The L-curve is a log-log plot of the estimated signal norm $\|f_0\|_2$ and residual norm $\|A f_0 - g\|_2$. According to the L-curve method, the parameter was determined by minimizing both $\|f_0\|_2$ and $\|A\ f_0\ -\ g\|_2$, simultaneously. The corner of L-shaped curve corresponds to the optimal value.

As the parameter estimation method, the curvature method [10] that searches for maximum curvature of L-curve minimal product method [11] that searches for the point of minimum area of $\| A f_0 - g \|_2$ and $\| f_0 \|_2$ have been proposed. Moreover, to improve the accuracy, a triangle algorithm [12], a corner algorithm [13], and an adaptive pruning algorithm [14] have been investigated. These methods still have disadvantages that the estimated parameter has bias. Especially, there were cases when the error of the curvature method became extremely large. Then, we proposed new algorithm that considering both the curvature and the parameter interval. When we pay attention to L-curve of TTLS, the interval became narrow near the corner against the parameter change, while the interval became wide far away from the corner. Then, the maximum curvature point was estimated as the regularization parameter when the interval of adjoining parameters is smaller than a threshold level. The threshold was considered as the average over all intervals of adjoining parameters. We expected that small bias and variation could be accomplished by this method.

III. Results

A. Simulations

The computer simulation was carried out to evaluate the performance of the inverse filters and the parameter estimations. 128 electrodes have been arranged on the scalp surface of the head model. 1280 radial dipoles were set at the spherical dipole layer with the radius of 0.85 inside of the brain. Two radial dipole sources with the eccentricity of 0.6 were installed in the left rear head and the forward right head. Gaussian white noise (GWN) was used as the measurement noise. The noise level ($NL = \|n\|_2 / \|g\|_2$) was set to 0.1. The elements of the transfer matrix depend on the distance between electrodes and equivalent dipoles. The transfer matrix error was expressed with GWN weighted with an exponential function against the distance. The error level of the transfer matrix (EL) is expressed by $EL = \|E\|_F / \|A\|_F$.

The actual dipole distribution and the observed scalp potential distribution when EL = 0.1 are shown in Figs. 1 (a) and (b), respectively. Although two peaks that caused by the two dipole sources are observed in the dipole distribution, they cannot be found in the scalp potential. The dipole distribution estimated from the scalp potential using Tikhonov regularization and TTLS are shown in Figs. 1 (c) and (d), respectively. In order to eliminate the influence from the parameter estimation, the optimum value that minimizes the *RE* between the actual and estimated dipole distributions was used for regularization. When the regularization parameter was the optimal value, the dipole distribution of the TTLS was more localized than that of Tikhonov regularization.

The RE between the actual and estimated dipole distributions when setting the EL of the transfer matrix as 0.01, 0.05, and 0.1 is shown in Fig. 2. The result shows the averaged RE and its standard deviation over 8 simulations with different initial values for noise. The relative error of the TTLS was significantly smaller than that of the Tikhonov regularization. When the EL of a transfer matrix increased, the RE of Tikhonov regularization increased. On the other hand, the RE of the TTLS was stable against the EL.

Next, we examined the parameter estimation methods that were suitable for the TTLS. Figure 3 shows the RE between the actual dipole distribution and the estimated dipole distribution with various parameter estimation methods. The variation in parameter error was large in the curvature algorithm. That is, there was a case when an error was large extremely. The REs of the corner method was large because of the parameter bias. By the proposed method which used the curvature method considering with the parameter interval, the amplitude and variation of the parameter error became small. The proposing method was able to obtain the parameter nearest to the optimal value.

B. Application to VEP

Based on the simulation results, the proposed method was applied to VEP data. The experiment was conducted by obtaining the consent from healthy subject according to the University of Illinois Ethical Review Board regulation. 94 electrodes were arranged according to the expanded international 10-20 method and the electrode positions were measured by the 3D digitizer to construct the transfer matrix from the dipole layer to the scalp potential. The difference between the head model and real head geometry was assumed to be the transfer matrix error. Half visual field pattern

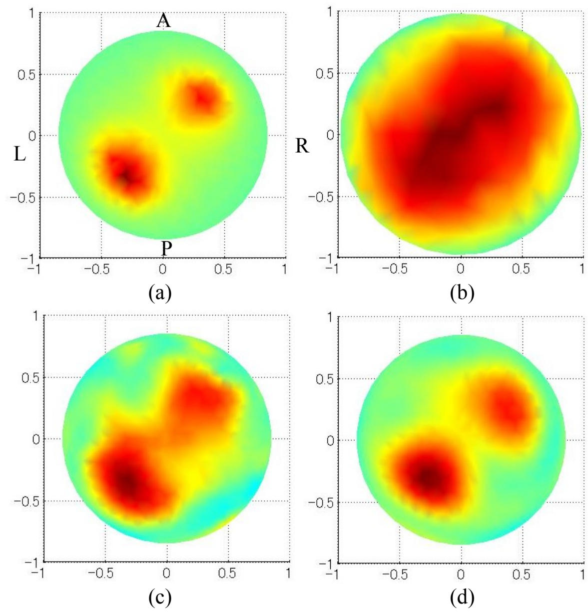

Figure 1. Simulation results. (a) actual dipole distribution, (b) scalp potential, and dipole distributions estimated by (c) Tikhonov regularization and (d) TTLS.

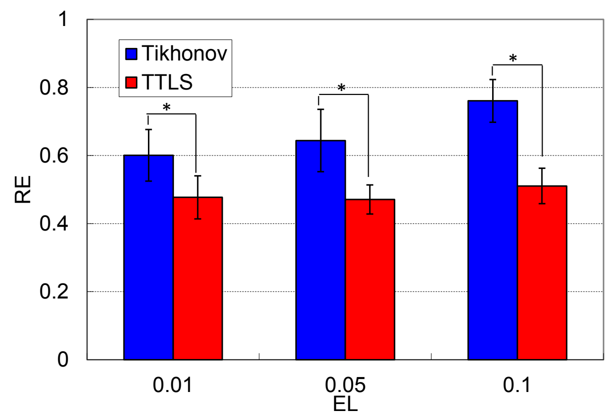

Figure 2. Relative error (RE) of dipole distributions estimated by Tikhonov regularization and TTLS when changing the error level (EL) of the transfer matrix. (N=8, *<0.01)

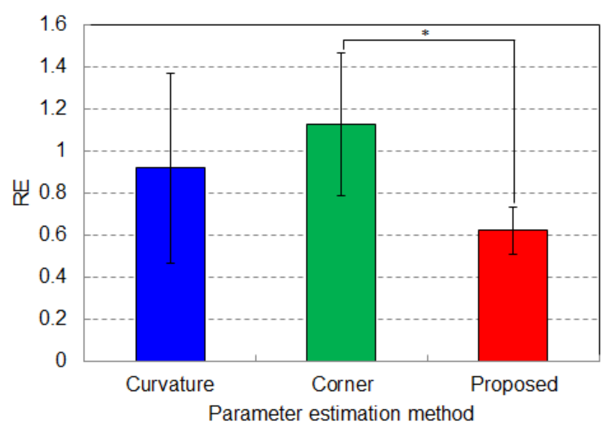

Figure 3. Relative error of dipole distributions estimated by TTLS when the regularization parameter was calculated by the curvature method, corner algorithm, and the proposed method. (N=8)

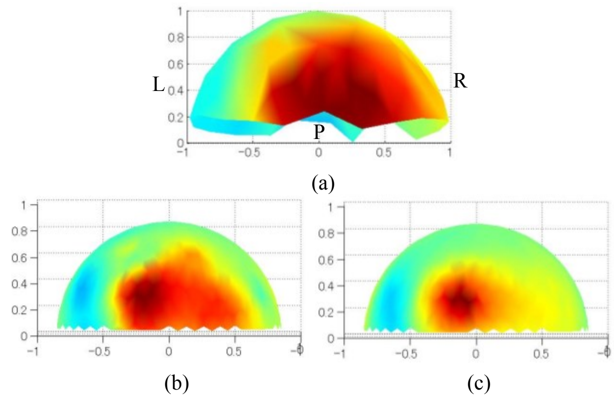

Figure 4. Estimated results of VEP. (a) Scalp potential distribution, and estimated dipole distribution with (b) Tikhonov regularization and (c) TTLS.

reversal check boards with the interval of 0.5 s served as visual stimuli and 400 reversals were recorded to obtain averaged VEP signals. The EEG data was obtained with the sampling frequency of 1kHz. The dipole distribution was estimated from the scalp potential at a positive peak observed about 100ms after a stimulus (P100). The dipole distribution of VEP was estimated using Tikhonov regularization and the TTLS. The modified curvature method was used for estimation of the regularization parameter. The scalp potential distribution of VEP is shown in Fig. 4(a). The dipole distributions estimated by Tikhonov regularization and TTLS are shown in Figs. 4(b) and (c). The plots show the result of having displayed from the back of the head. VEPs are produced from the calcarine sulcus of the primary visual cortex located at the occipital region. Positive potential was wide-spread over the whole back of the head in scalp potential distribution. In the dipole distributions, the signal was localized at primary visual cortex. Especially, the result of the TTLS was more localized than that of Tikhonov regularization.

IV. Discussion

In EEG inverse problem of cortical dipole imaging, we considered the error involved in the transfer matrix from the dipole sources to the scalp potential. As shown in Fig. 2, when the transfer matrix error increased, the RE increased in Tikhonov regularization. On the other hand, a significant change was not found in the RE of the TTLS in spite of the EL of the transfer matrix. It is considered that the TTLS is effective to eliminate the transfer matrix error in cortical dipole imaging. When there is an error in the transfer matrix, the solution of Tikhonov regularization has bias. TTLS can reduce the bias by taking the transfer matrix error into consideration. In order to realize unbiased estimation strictly, the condition is limited when each element of the measurement noise n and the transfer matrix error E is mutually non-correlated, the average is zero, and the covariance is equal. As a result, the dipole distribution was estimated with small error compared with Tikhonov regularization.

The most effective parameter estimation was our proposed method based on the curvature and the corner method. Because the change of the evaluation value against a regularization parameter is large in the curvature method, the regularization parameter can be determined exactly. However, since the differentiation of L-curve is calculated in the curvature method, the interval of the regularization parameter needs to be small. That is, the wider the parameter interval, the lower the accuracy of parameter estimation. Furthermore, while the interval of the parameter is narrow near the corner of L-curve, the interval of the parameter is wide at the place away from the corner. Then, the threshold was established for the parameter interval to limit the search area of a parameter near the corner of L-curve. As a result, the estimated accuracy of the parameter was improved by our method. This method was effective also for the parameter estimation of Tikhonov regularization.

We are planning to design the inverse filter which reduces the transfer matrix error using the statistical information on noise. Moreover, we would apply to more realistic head model in near future.

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