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Local Binary Fitting Energy Solution by Graph Cuts for MRI Segmentation

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***Abstract*— This paper proposes a new solution for local binary fitting energy minimization based on graph cuts for automatic brain structure segmentation on magnetic resonance images. The approach establishes an effective way to embed the energy formulation into a directed graph, such that the energy is minimized by maximizing the graph flow. Proposed and conventional solutions are compared by segmenting the well-known *BrainWeb* synthetic brain Magnetic Resonance Imaging database. Achieved results show an improvement on the computational cost (about 10 times shorter) while maintaining the segmentation accuracy (96%).**

***Index Terms*— Implicit active contours, Local binary fitting, Level set, Graph cuts, MRI segmentation.**

I. Introduction

Magnetic Resonance Imaging (MRI) from brain can provide important information for diagnosis, therapy planning and execution, and monitoring the progress of diseases or treatments. Such assessment depends on the proper detection of boundaries between brain structures. MRI is prone to suffer from Intensity Non-Uniformity (INU) across the images. INU manifests itself as a smooth intensity variation, and it is caused by radio frequency (RF) pulse attenuation in tissue, non-uniform RF coil transmission and sensitivity, non-uniformity in the scanner's magnetic field, gradient-induced eddy currents, RF standing waves, magnetic susceptibility of tissue, and inter-slice cross talk [1].

Active contour models (ACM) are commonly used for dealing with INU in the segmentation stage. Specifically, geometric active contours are implicit level set functions defined on a higher dimension, which evolve according to a partial differential equation (PDE). Usually, the evolution equation is the minimization solution of an energy formulation, obtained by variational calculus. Aiming to overcome the INU, local energy formulations have been proposed.

ACM based on Local Binary Fitting (LBF) energy is one of such local formulations, where the energy is computed by means local weighted average operators [2]. Segmentation results using LBF have been demonstrated to deal effectively with bias illumination issues for many kinds of images. Nevertheless, the number of operations required to compute the energy implies a higher computational cost. Furthermore, the algorithms for solving accurately PDEs need a large number of iterations to converge.

Bearing the above in mind, this work discusses the minimization of the LBF to be solved by embedding the energy into a directed graph and, then, maximizing the flow between the two terminals of the graph. Since the maximum flow algorithm allows to minimize the LBF over the whole image domain, a considerably smaller number of iterations is required. Performance of both, traditional and proposed, approaches are compared by segmenting the well-known *BrainWeb* synthetic brain MRI database. Achieved results show an improvement, regarding the solution by PDE, on the computational cost (as much as 10 times shorter) while maintaining the high segmentation accuracy (96%).

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II. Background

A. Implicit active contour as level set functions

Let $\Omega \subset \mathbb{R}^2$ be the image domain, where each pixel is denoted as $\boldsymbol{x} \in \Omega$, and f be a given image that is a mapping of the form: $f : \Omega \to \mathbb{R}$, $\boldsymbol{x} \mapsto f(\boldsymbol{x})$. Besides, defined on Ω, a level set function ϕ is considered that separates the image domain into two subregions (Ω^+ and Ω^-) by a boundary $\Gamma \in \Omega$, such that:

$$\Omega^+ = \{\boldsymbol{x} : \phi(\boldsymbol{x}) > 0\} \tag{1a}$$

$$\Omega^- = \{\boldsymbol{x} : \phi(\boldsymbol{x}) < 0\} \tag{1b}$$

$$\Gamma = \{\boldsymbol{x} : \phi(\boldsymbol{x}) = 0\} \tag{1c}$$

Grounded on the piecewise constant energy $\varepsilon^{CV}(\Lambda) \in \mathbb{R}^+$, which is given in terms of the parameter set Λ, a basic formulation of implicit active contours is as follows [3]:

$$\varepsilon^{CV}(\Lambda) = \lambda_1 \int_{\Omega^+} \|f(\boldsymbol{x}) - f_1\|^2 d\boldsymbol{x} + \lambda_2 \int_{\Omega^-} \|f(\boldsymbol{x}) - f_2\|^2 d\boldsymbol{x} + r(\boldsymbol{x}) \tag{2}$$

where f_1 and f_2 correspond to the average value of the image f on the subregions Ω^+ and Ω^-, respectively; operator $\|\cdot\|$ stands for the Euclidean norm; parameters $\lambda_1, \lambda_2 \in \mathbb{R}^+$ are energy weighting factors for each region, which are usually set heuristically; and $r \in \mathbb{R}^+$ stands for an introduced regularization term, which is usually employed to avoid algorithm reinitialization and spurious solutions [4].

Nevertheless, since the main assumption of such approach is that the image aims to be constant along each region, solutions using Equation (2) are not able to deal with illumination bias, where image properties inside each subregion can vary smoothly [2].

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B. Implicit active contours driven by local binary fitting energy

Aiming to avoid such a problem, [2] proposes a level set formulation based on the estimation of an energy function around each pixel $\boldsymbol{x}$ in the image, known as local binary fitting energy, which is defined as:

$$\varepsilon^{LBF}(\Lambda, \boldsymbol{x}) = \lambda_1 \int_{\Omega^-} K(\boldsymbol{x}-\boldsymbol{y})\|f(\boldsymbol{y}) - f_1(\boldsymbol{x})\|^2 d\boldsymbol{y} + \lambda_2 \int_{\Omega^+} K(\boldsymbol{x}-\boldsymbol{y})\|f(\boldsymbol{y}) - f_2(\boldsymbol{x})\|^2 d\boldsymbol{y} \quad (3)$$

where $K(\cdot) \in \mathbb{R}^+$ is an introduced kernel function that makes functions f_1 and f_2 behave as local averaging operators (or prototypes) over each pixel for both regions, which are computed as:

$$f_1(\boldsymbol{x}) = \frac{K(\boldsymbol{x}) * (H(\phi(\boldsymbol{x}))f(\boldsymbol{x}))}{K(\boldsymbol{x}) * H(\phi(\boldsymbol{x}))} \quad (4a)$$

$$f_2(\boldsymbol{x}) = \frac{K(\boldsymbol{x}) * (1 - H(\phi(\boldsymbol{x}))f(\boldsymbol{x}))}{K(\boldsymbol{x}) * (1 - H(\phi(\boldsymbol{x})))} \quad (4b)$$

being $H(\cdot)$ the Heaviside function. Notation $*$ stands for the convolution operator.

By marginalizing out the variable $\boldsymbol{x}$ in Equation (3) and including two regularization terms that depend on the contour Γ length, $l(\Gamma)$, as well as the region Ω^- area, $a(\Omega^-)$, the total energy function can be recomputed as:

$$\varepsilon^{LBF}(\Lambda) = \int_{\Omega} \varepsilon^{LBF}(\Lambda, \boldsymbol{x}) d\boldsymbol{x} + \mu l(\Gamma) + \nu a(\Omega^-) \quad (5)$$

where $\mu \in \mathbb{R}^+$ and $\nu \in \mathbb{R}^+$ are the given weighting factors for the contour length and region area, respectively.

The implicit active contour solution for Equation (5) using gradient descent optimization and variational calculus is given by:

$$\frac{\partial \phi}{\partial t} = -\delta(\phi)(\lambda_1 d_1(\boldsymbol{x}) - \lambda_2 d_2(\boldsymbol{x})) + \nu\delta(\phi)\text{div}\left(\frac{\nabla\phi}{|\nabla\phi|}\right) + \mu\left(\nabla^2\phi - \text{div}\left(\frac{\nabla\phi}{|\nabla\phi|}\right)\right) \quad (6)$$

where $\text{div}(\cdot)$ and ∇ stand for divergence and gradient operators, respectively; $\delta(\cdot)$ is the smooth Dirac function; functions $d_1, d_2 \in \mathbb{R}^+$ representing distance for each pixel $\boldsymbol{x}$ to regions Ω^- and Ω^+, respectively, are computed as:

$$d_1(\boldsymbol{x}) = \int_{\Omega} K(\boldsymbol{y}-\boldsymbol{x})\|f(\boldsymbol{x}) - f_1(\boldsymbol{y})\|^2 d\boldsymbol{y}$$

$$d_2(\boldsymbol{x}) = \int_{\Omega} K(\boldsymbol{y}-\boldsymbol{x})\|f(\boldsymbol{x}) - f_2(\boldsymbol{y})\|^2 d\boldsymbol{y}$$

C. Local binary fitting solution by graph cuts

The energy minimization of some variational methods can be performed by embedding effectively the energy function into a directed graph [5]–[7]. The graph set $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$ is composed of a subset of nodes $\mathcal{V}$ and directed edges $\mathcal{E}$ connecting them. In general, it is assumed that each pixel $\boldsymbol{x}_i$ corresponds to a node $v_i \in \mathcal{V}$. Two additional nodes, denoted as terminals, are included: a *source* and a *sink*, labelled as s and t, respectively. Such nodes are related to the labels of both regions (Ω^- and Ω^+, respectively). Regarding the edges, two kind of connections can be considered: n-links, connecting pairs of neighboring pixels ($e_{ij} \in \mathbb{R}^+$) and t-links, connecting each pixel with both terminals ($e_{si} \in \mathbb{R}^+$ and $e_{it} \in \mathbb{R}^+$).

Since terms in Equation (3) represent the affinity or linkage from each pixel to regions Ω^- and Ω^+, this work proposes the LBF energy to be further embedded into the graph, as follows:

$$e_{ij} := 0$$

$$e_{si} := \lambda_1 \int_{\Omega^-} K(\boldsymbol{x}_i - \boldsymbol{y})\|f(\boldsymbol{y}) - f_1(\boldsymbol{x}_i)\|^2 d\boldsymbol{y}$$

$$e_{it} := \lambda_2 \int_{\Omega^+} K(\boldsymbol{x}_i - \boldsymbol{y})\|f(\boldsymbol{y}) - f_2(\boldsymbol{x}_i)\|^2 d\boldsymbol{y}$$

Therefore, the binary segmentation can be reformulated as a task searching for the s-t-cut $C = \{\mathcal{S}, \mathcal{T}\}$ with the maximum flow $\varphi(C)$ from s to t. A cut C is a partition of the vertices $\mathcal{V}$ into two sets $\mathcal{S}, \mathcal{T} \subset \mathcal{V}$, subject to $s \in \mathcal{S}$, $t \in \mathcal{T}$, $\mathcal{S} \cup \mathcal{T} = \mathcal{V}$ and $\mathcal{S} \cap \mathcal{T} = \emptyset$. The flow value in a graph $\mathcal{G}$ given the cut C is computed as:

$$|\varphi(C)| = \sum\nolimits_{v \in \mathcal{V}} e_{sv} \quad (7)$$

As a result, the optimal vertex sets $\mathcal{S}$ and $\mathcal{T}$ are obtained corresponding to the regions Ω^- and Ω^+, respectively.

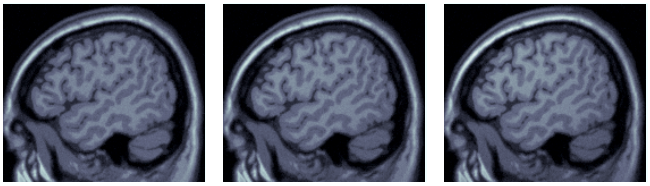
III. Experimental Setup

A. Database description

Proposed method is tested in a common MRI brain segmentation task consisting of separating white matter (WM) tissue from gray matter tissue (GM). Simulated MRI data set, generated with the Internet connected MRI Simulator at the McConnell Brain Imaging Centre in Montreal[1], is considered for evaluating the approach performance [8]. The precomputed simulated MRI volumes for normal brain database was employed with the following parameters: $T1$ image modality, 1mm×1mm×1mm voxel size, 3% noise, intensity non-uniformity (INU) values of 0%, 20%, 40%, coronal axis slices and slice size 512×512 pixels. An example from MRI database is shown in Figure 1. In this paper, the assessed task is the well-known brain segmentation, i.e., white matter/gray matter segmentation. Since the MRI are simulated, the tissue label is known and employed as the golden standard for comparing the approaches.

B. Parameter selection and implementation

The proposed energy minimization, based on graph cut (GC), is compared to the traditional solution by variational equations and partial derivatives (PD). Since both approaches are based on the local binary fitting energy formulation, parameter values employed for the experiments, shown in Table I, are chosen as suggested in [2] and kernel function $K(\cdot)$ is assumed as Gaussian. However, it is worth noting



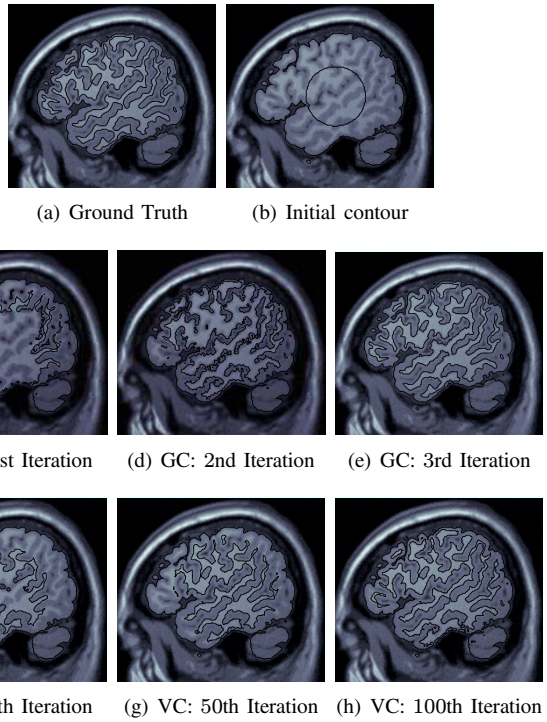
(a) 0% (b) 20% (c) 40%

Fig. 1. Simulated MRI sample for the considered INU values for a sagittal slice.

Parameter		*Solution method*	
		PD	GC
τ	Time step	0.1	–
σ	Gaussian kernel scale	3.0	3.0
λ_1	Weighting of the inside region energy	1.0	1.0
λ_2	Weighting of the outside region energy	1.0	1.0
μ	Curve length regularization	1.0	–
ν	Inside region area regularization	0.003×255^2	–

TABLE I

EMPLOYED PARAMETER VALUES IN THE EXPERIMENTAL SETUP FOR BOTH ENERGY MINIMIZATION APPROACHES.



(a) Ground Truth (b) Initial contour

(c) GC: 1st Iteration (d) GC: 2nd Iteration (e) GC: 3rd Iteration

(f) VC: 5th Iteration (g) VC: 50th Iteration (h) VC: 100th Iteration

Fig. 2. Comparison of the proposed method with conventional LBF model solution on 40% INU MRI slice. The ground truth and initial contour are plotted on top. The contour evolution for the proposed approach is plotted on the middle row. Results for the conventional LBF solution are depicted on the bottom.

that no regularization or time step parameters are required when using the proposed approach.

Furthermore, convolution operations in Equations (4a) and (4b) are efficiently computed by the Fast Fourier Transform, while term $\lambda_1 d_1(\boldsymbol{x}) - \lambda_2 d_2(\boldsymbol{x})$ is solved as a weighted sum of three convolutions. Additionally, derivatives in Equation (6) are estimated by finite differences, as discussed in [9]. Finally, regarding the segmentation using graph cuts, the optimal cut is computed by using the Boykov-Kolmogorov algorithm for max-flow/min-cut problems [7].

C. MRI segmentation performance

In the present work, the proposed approach for LBF energy minimization by Graph Cuts is compared to the baseline minimization by Partial Derivative Equations. In the considered MRI segmentation task, the performance of both approaches is measured in terms of their computational cost and their segmentation accuracy. The former is carried out by means of the algorithm time consumption until its convergence to any solution, while the latter is achieved by using the dice overlap coefficient, defined as:

$$DOC = \frac{2t_P}{2t_P + f_P + f_N} \tag{8}$$

where t_P, f_P, and f_N represent the *true positive*, *false positive* and *false negative* pixels, respectively.

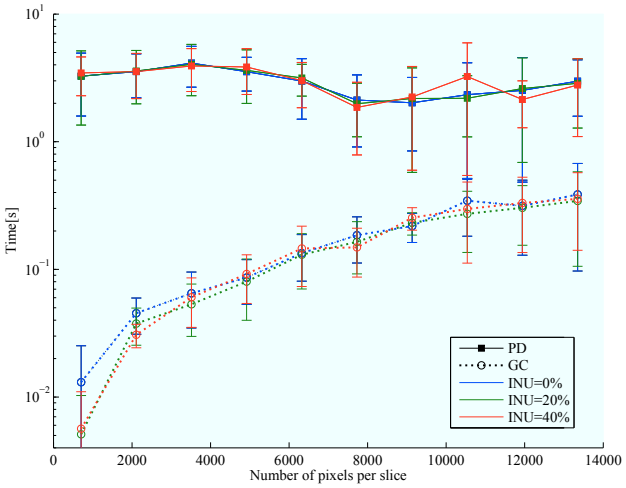
Considered regions in the database are white and gray matter, while the phantom of the simulated images is taken as the ground truth. The proposed approach is compared to the standard LBF solved by variational calculus and partial derivatives. Figure 2 depicts a concrete result for an MRI slice for both tested approaches. As seen, the proposed approach segments successfully gray matter after three iterations, while the baseline takes a hundred iterations to converge. Achieved performance results for time consumption versus the number of pixels to segment as well as the segmentation accuracy versus INU are shown in Figures 3(a) and 3(b), respectively.

[1] http://brainweb.bic.mni.mcgill.ca/brainweb/

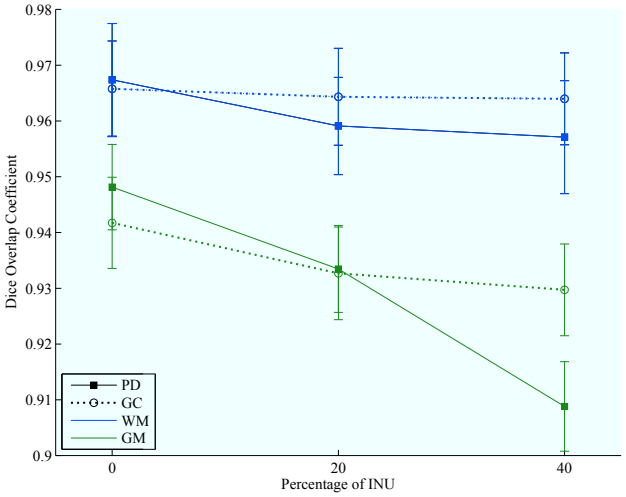
IV. DISCUSSION

Figure 3(a) shows the time consumption versus the number of pixels to segment for three levels of INU using both considered approaches. For the proposed approach, the larger the number of pixels, the larger the time spent to segment the image. Such behavior is expected, since the computational cost of the maximum-flow algorithm depends on the number of nodes in the graph. Nevertheless, it is important to highlight that overall computational cost of the proposal is about ten times less than the baseline. Additionally, it is worth noting that for both approaches the time consumption is statistically equivalent along the INU. Therefore, it can be inferred that the INU does not influence the computational cost for the considered image segmentation task.

Results regarding the segmentation accuracy (Figure 3(b)) show that for both approaches the larger the INU, the worse the performance, which is expected since the INU phenomenon implies a more complex segmentation task. Nevertheless, the achieved accuracy (96%) reaches the state-of-the art results for the considered database. Moreover, it



(a) Computational cost



(b) Accuracy

Fig. 3. Performed computational cost and segmentation accuracy for both considered approaches: the LBF model solved by partial derivatives (continuous line) and graph cut (dashed line). a) CPU time (in seconds) versus the number of pixels per slice, b) Dice overlap coefficient versus INU value for brain white matter and gray matter

can be seen that the accuracy decay rate is higher for the baseline than for the proposed approach.

Finally, results in Figure 2 show faster convergence of the proposed approach than the conventional one. One of the reasons for such behavior is the time step limitation in the solution by variational calculus. Small values of the time step are required to achieve a stable solution, but yields to small changes in the level set function and slows down the convergence time. On the other hand, large evolution movements are performed with large time step values, but the convergence is not guaranteed. All of this issues are avoided by the graph cut method, since it can see the whole picture of the problem without requiring any time step value. Nevertheless, it has to be highlighted that no regularization terms were included in the proposed approach; but for some problems, regularization can be required to control not only the size, but also the shape of the solution. Dealing with such kind of terms are let as future work, since not all the energy formulations can be embedded into a graph cut problem [6].

V. Conclusions and Future Work

In this paper, a new solution for local binary fitting energy minimization towards MRI segmentation is introduced. The proposed approach embeds the energy into a graph and minimizes it by maximizing the flow in the graph. In this way, the properties of the LBF to deal with INU are exploited while reducing the time consumption. Additionally, achieved results prove an accurate segmentation.

Two main tasks are let as future work: Firstly, performance of the methodology has to be evaluated in other kind of medical images and body structures; and secondly, other implicit active contour models have to be formulated as a graph cut problem, such that they include shape constraints and need less computational cost.

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