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Non-Euclidean Basis Function Based Level Set Segmentation With Statistical Shape Prior

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***Abstract*— We present a new framework for image segmentation with statistical shape model enhanced level sets represented as a linear combination of non-Euclidean radial basis functions (RBFs). The shape prior for the level set is represented as a probabilistic map created from the training data and registered with the target image. The new framework has the following advantages: 1) the explicit RBF representation of the level set allows the level set evolution to be represented as ordinary differential equations and reinitialization is no longer required. 2) The non-Euclidean distance RBFs makes it possible to incorporate image information into the basis functions, which results in more accurate and topologically more flexible solutions. Experimental results are presented to demonstrate the advantages of the method, as well as critical analysis of level sets versus the combination of both methods.**

I. INTRODUCTION

Image segmentation using the level set method involves the maintenance of an implicit surface through a distance function. The surface evolution is governed by a set of partial differential equations (PDEs), commonly solved using the finite difference scheme on sampled grid points. The surface evolution however may develop step or flat gradients, hence periodical reinitialization may be required to maintain numerical stability. In recent years, radial basis functions (RBFs) have been used in conjunction with level sets for image segmentation.

The RBFs were originally used as a primary tool for interpolation of multivariate scattered data because it does not require any underlying mesh for interpolation. In 1990, RBFs were extended by Kansa to approximate parabolic, hyperbolic and elliptic PDEs systems in the field of computational fluid dynamics [1]. Recently, RBFs have received much attention for solving PDE systems [2]-[3] as well as for image segmentation in combination with the level sets. With this approach, instead of partial differential equations, surface evolution is governed by a set of ordinary differential equations, which is much easier to solve, reinitialization is no longer necessary, and more complex topological changes are readily achievable [4]. Several methods combining level sets with RBFs have recently been published in the image segmentation field. For example, Wimmer et al. used RBFs to reconstruct a surface to initialize a level set algorithm [5]. Turk et al. [6] introduced constraint points to model the implicit level set surface using RBFs, which were applied to implicit active contour modeling by Morse et al. [7]. Gelas et al. [8] applied compactly supported RBFs to image segmentation and introduced prior knowledge of shape by placing the RBF centers quasi-uniformly over an uncertainty area. Bernard et al. [9] formulated the segmentation problem in a Maximum Likelihood framework using the Generalized Gaussian as a priori distribution and minimizing the resulting functional using a multiphase level set and RBF model. Slabaugh et al. [10] proposed to use anisotropic Gaussian kernels and optimized their orientation as well as their weight, position and scales. Mory et al. [11] proposed to build RBFs according to image features using non-Euclidean distance and incorporated prior information by casting inside/outside labels as linear inequality constraints.

In spite of the potential advantages, the combination of RBFs and level sets is relatively new in image segmentation. Implementation of the approach can be complex and it is still unclear what benefit and drawback this combination can potentially bring for an application. In this paper we detail a new framework for biomedical image segmentation by combining the level set method with shape prior and non-Euclidean RBFs to arrive at an accurate and efficient image segmentation method. Existing approaches to integrating shape prior into RBFs include RBFs center placement approaches [8], which are quite complex. We propose a new approach to integrating shape prior to RBFs using a statistical shape prior and introduce the non-Euclidean RBF within the optimization framework of Gelas et al. [8]. Additionally, we report experimental results as well as critical analysis of the combination.

II. LEVEL SET WITH RADIAL BASIS FUNCTIONS

A RBF is a circularly-symmetric function centered in a particular point. The sum of RBFs is typically used to approximate functions. The level set function Φ can be approximated by a linear combination of translated and scaled RBFs centered around N points x_i traditionally called collocation points:

$$\Phi(x) = \sum_{i=1}^{N} \lambda_i \varphi\left(||x - x_i||\right) \quad (1)$$

where φ is the radially-symmetric non-negative kernel, x_i is the position of the known values h_i in the interpolation and λ_i is the weight of the RBF positioned at that point. Euclidean distance is typically used as a distance function.

Eq.1 is used to calculate the unknown weights for the collocation points x_j satisfying the known values h_i: $\Phi(x_i) = h_i = \sum_{j=1}^{N} \lambda_i \varphi\left(||x_i - x_j||\right)$.

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The level set propagation can be achieved by considering the front evolving along the normal direction according to a localized speed function. It can be expressed as follows:

$$\frac{\partial \Phi(x,t)}{\partial t} = V(x,t) \cdot \delta(\Phi(x,t)) \tag{2}$$

where δ is a regularized version of the Dirac function and V is a velocity function. Decomposing RBF as an implicit function $\Phi(x,t) = \varphi \cdot \lambda(t)$ by assuming that time and space are separable and replacing this function into the Eq. 2, we get the following expression:

$$H \cdot \frac{\partial \lambda(t)}{\partial t} = B(\lambda(t), t) \tag{3}$$

where $H_{ij} = \varphi(||x_i - x_j||)$, $B = V(x_i,t) \cdot \delta(\varphi(x_i) \cdot \lambda(t))$ and λ are the scalar weights.

To solve the ordinary differential equation, Gelas et al. [8] use a first order forward Euler method which lead them to $\lambda^n = \lambda^{n-1} - \tau \cdot H^{-1} \cdot B^{n-1}(\lambda^{n-1})$. As H^{-1} is not sparse, the following evolution equation of the level set is defined for optimization:

$$\begin{cases} H = L \cdot L^\top \\ L \cdot u^n = B^n(\tilde{\lambda}^{n-1}) \\ L^\top \cdot v^n = u^n \\ \lambda^n = \tilde{\lambda}^{n-1} - \tau \cdot v^n \\ \tilde{\lambda}^n = \frac{\alpha}{||\lambda^n||_1} \cdot \lambda^n \end{cases}$$

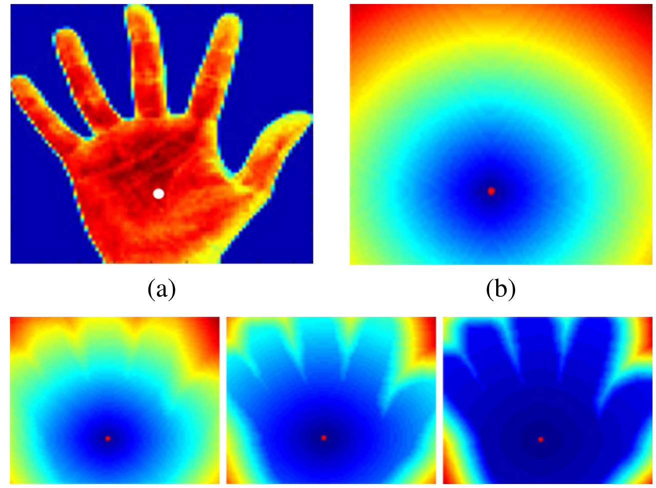
where H is decomposed by Cholesky decomposition, n indicates the iteration, α is a positive constant and τ is the time step.

III. NON-EUCLIDEAN RADIAL BASIS

The Euclidean distance is commonly used in RBFs. As a consequence RBF kernels are of spherical shape, resulting over-smoothed shape representation. In order to improve the segmentation, Mory et al. [11] proposed to use an image-dependent non-Euclidean distance to build the RBF kernel. In so doing, the RBFs are no longer spherical but determined by the image features. The new formulation is as follows:

$$\varphi_i(x) = \varphi\left(\frac{||x - x_i||_{g_i}}{\sigma_i}\right) \tag{4}$$

where σ_i are the scales and g_i is the metric function chosen. The authors define the non-Euclidean distance from a physical interpretation of fronts propagating from the center points x_i with the image-dependent speed function $1/g_i$. In case of $g_i = 1$, the Euclidean case is re-obtained. The metric g_i recommended for general cases is the local image intensity distribution P_{x_i}, estimated in the neighborhood of x_i: $g_i(x) = 1 - \beta log P_{x_i}(I(x))$ where $\beta > 0$ controls the non-Euclidean part of the metric. The effect of the metric is illustrated in Fig.1 where we show that bigger β is the better the basis function will adapt to the image features. A fast marching method is used to calculate the geodesic distances between points [11].



(a) (b)

(c)

Fig. 1. The effect of the non-Euclidean distance metric. (a) Original Image with a RBF center in the middle. (b) Spherical-shaped RBF with $\beta = 0$. (c) Increasing the non-Euclidean part of the metric from left to right with $\beta > 0$.

IV. REGION AND SHAPE PRIOR DEFINITION

Having defined the methods for explicitly representing the level set surface using RBFs, we need to incorporate it into the level set evolution to guide the segmentation towards the object of interest. The Chan-Vese active contour model [12] is a popular method for region-based level set segmentation which aims at partitioning an image into regions with piecewise constant intensity. The following level set evolution equation minimizes the Chan-Vese model: $\frac{\partial \phi}{\partial t} = \delta_\epsilon[\gamma \cdot \kappa - (I - c_1)^2 + (I - c_2)^2]$ where I is the original image, $c1$ and $c2$ are the average values of pixels inside and outside the curve respectively, and κ is the curvature term which makes the curve smooth weighted by γ. Due to the intrinsic smoothness of the RBF formulation, the smoothing term is omitted, and the velocity term referred in the Eq.2 is simplified as follows: $V(x,t) = -(I(x) - c_1)^2 + (I(x) - c_2)^2$. The Chan Vese method is suitable for piecewise-constant images but for more general cases, it can be replaced by a maximum-likelihood criterion: $V(x,t) = r_1 - r_2$ where $r_i(I(x)) = -log P_i(I(x))$ and P_1 and P_2 are the intensity distributions [11]. In Fig.2, we show an example of minimization of a region-based functional with non-Euclidean distance basis.

(a) (b) (c)

Fig. 2. Abdominal fat segmentation. (a) Original Image. (b). Segmentation using non-Euclidean RBFs. (c). The level set function.

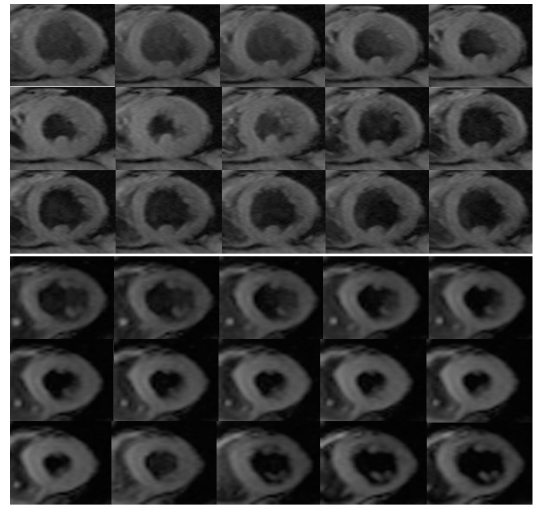
In addition to the region term, we require prior information to guide the evolution of the level set surface to a certain shape. To do so we compute the average shape of a training set and then align it using registration to the target image [13]. As a result, we obtain a probabilistic map which is mapped onto the level set framework as follows:

$$V(x) = \mu_1(-(I(x) - c_1)^2 + (I(x) - c_2)^2) - \mu_2 \left(log \frac{P_{in}(x)}{P_{out}(x)} \right)$$

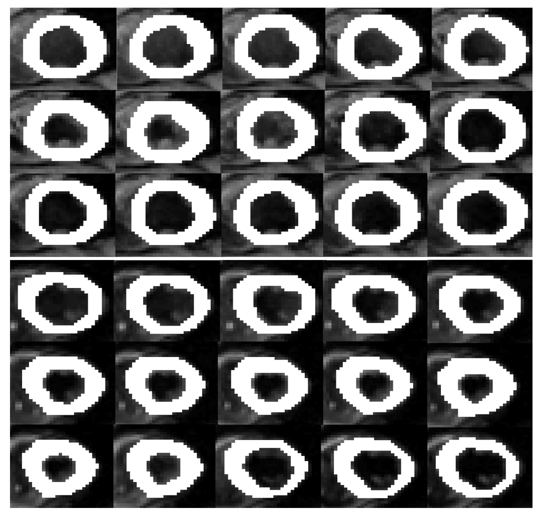
where the first term is the Chan-Vese model that can be replaced by any other suitable model according to the image features [11] and the second term is the shape prior term where P_{in} and P_{out} are the probability of the region inside and outside the contour respectively, obtained from the probabilistic map. μ_1 and μ_2 are the weights for the region and prior terms respectively and are chosen empirically.

V. EXPERIMENTS

We test our method for segmentation of the myocardium using fifteen datasets of different patients. Each dataset contains among 26 to 57 frames. The ground truth is manually segmented by an expert. Experimental results with our method are compared with the common level set technique and that obtained using Euclidean RBFs. All methods use the same energy terms and a single circle as an initial contour. In order to make the level set more flexible topologically, we did not implement the narrow band, the computation of a band around the front instead of the whole image, to cope not only with the endocardium but also the epicardium. This causes an increase of the computational complexity of the algorithm, whereas the radial basis representation allows more flexible topologies without adding additional cost. According to the experiments, the level set method produced more outliers due to image noise. The level set re-initialization also caused problems in some cases. On the other hand, the level set gave sharper segmentations than the Euclidean RBFs in some images. However, this trend is reversed when more RBFs are added, or when non-Euclidean RBFs are used. Non-Euclidean RBFs follow the image features better, allowing closer initialization and therefore better convergence. Some examples using the non-Euclidean basis function with shape prior are shown in Fig.3.



(a)



(b)

Fig. 3. Segmentation of the myocardium using our approach with two different patients: a) Original data sets. b) Segmentation in white.

To quantify the segmentation quality assessment, we compute different performance measures: point to mesh distance which indicates the distance between the segmentation and the ground truth and overlap, sensitivity, specificity and similarity [1].

Table I summarizes the average performance for all methods showing a better performance with our approach. The distance between the ground truth and the segmentation is smaller and the metric values for similarity, overlap and sensitivity (fraction of pixels belonging to the myocardium correctly detected) are higher. And for specificity (fraction of pixels not belonging to the myocardium correctly detected), according to our experiments, the results remain the same for all methods.

	Myocardium		
	New	Euclid. RBFs	Level Sets
Distance (voxel)	0.76	0.77	0.79
Overlap	0.76	0.74	0.72
Sensitivity	0.80	0.77	0.75
Specificity	0.98	0.98	0.98
Similarity	0.87	0.85	0.84

TABLE I

AVERAGE PERFORMANCE MEASURES FOR CARDIAC SEQUENCES, FOR THE PROPOSED APPROACH (NEW), THE EUCLIDEAN RADIAL BASIS APPROACH, AND THE LEVEL SET METHOD.

[1]The performance measures are defined as [14]: $Overlap = \frac{TP}{TP+FN+FP}$, $Sensitivity = \frac{TP}{TP+FN}$, $Specificity = \frac{TN}{TN+FP}$ and $Similarity = \frac{2TP}{2TP+FN+FP}$ where TP and FP stand for true positive and false positive and TN and FN for true negative and false negative.

VI. CONCLUSION

We have implemented a new framework for medical image segmentation using a statistical shape-based level set method represented as a combination of non-Euclidean RBFs. As described in [11], by using non-Euclidean distance, basis functions can incorporate image features giving more accurate results. To guide the segmentation to the object of interest, we use a probabilistic map obtained as an average shape of training data. The experiments suggest that our method is robust and accurate even for noisy and low contrast images.

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