

Iterative Image Reconstruction for Ultra-low-dose CT with a Combined Low-mAs and Sparse-view Protocol

Yunwan Zhang, Jing Huang*, Jianhua Ma, Hua Zhang, Zhaoying Bian, Dong Zeng, Yang Gao, and Wufan Chen

Abstract—Ultra-low-dose x-ray computed tomography (CT) imaging is needed in CT fields. Through a scan protocol by lowering the milliampere-seconds (mAs) and reducing the number of projections per rotation around the body, we can realize low-dose CT imaging. However, the resulting noisy and insufficient measurements will unavoidably cause the degradation of desired-image. To solve this problem, iterative image reconstruction is a promising choice for achieving high-quality image with a low-dose scan. In this study, we are focusing on ultra-low-dose CT image reconstruction by using penalized weighted least-square (PWLS) criteria with a combined low-mAs and sparse-view protocol. Specifically, the sinogram data acquired with a combined low-mAs and sparse-view protocol is first restored by using a PWLS based sinogram restoration method. Then, the restored sinogram data is hereafter used to reconstruct image by using a PWLS based total variation (PWLS-TV) method. Qualitative and quantitative evaluations by simulations were carried out to validate the present method.

I. INTRODUCTION

Ultra-low-dose x-ray computed tomography (CT) imaging is needed in CT fields due to the concern of radiation exposure induced cancerous, genetic, and other diseases. For reducing radiation dose, many investigations have been performed including both the hardware-based optimal imaging protocols [1] and the software-based image reconstruction techniques [2]. Through lowering the milliampere-seconds (mAs) [3] and reducing the number of projections per rotation around the body [4], we can realize low-dose CT imaging. However, the resulting noisy and insufficient measurements will unavoidably cause the degradation of desired-image if no adequate data noise controlling is applied during image reconstruction. Up to now, various approaches for low-mAs or sparse-view CT image reconstruction have been proposed respectively [5-12].

For low-mAs image reconstruction, statistic-based sinogram restoration and image reconstruction algorithms have shown their advantages in noise reduction [5-7]. The idea of this strategy is to retain the benefits of statistical modeling

within the objective function. Usually, the associative objective function consists two terms, *i.e.*, “data-fidelity term” which is developed by incorporating the statistical measurement model and “penalty term” which is often designed by considering the properties of the desired-image itself. One typical example is the penalized weighted least-square (PWLS) approach proposed by Wang *et al.* By modeling the signal-dependent noise properties, the PWLS method has shown very promising results in either sinogram space or image domain [5].

For sparse-view image reconstruction, Sidky *et al* proposed a total variation (TV) based projection onto convex sets (POCS) algorithm, which was called “TV-POCS” . With the assumption that the desired-image satisfies a piecewise constant distribution, this algorithm can adapt TV minimization of the desired-image [9,10]. As an updating algorithm of TV-POCS, an adaptive-steepest-descent based POCS (ASD-POCS) algorithm was proposed with improved performance against the cone beam artifacts from sparse-view projection data [11]. But it is worth to note that the related algorithms often suffer the over-smoothing effect on the reconstructed image due to the assumption of isotropic edge property in TV minimization. To solve this problem, the weighted-TV_s as an extension of the conventional TV were proposed [12].

Although these studies have shown their advantages for achieving high-quality image, to the knowledge of authors, up to now, there are no special methods for addressing the problem of image reconstruction with a combined low-mAs and sparse-view protocol. In this study, we are focusing on ultra-low-dose CT image reconstruction by using the PWLS criteria with a combined low-mAs and sparse-view protocol. Specifically, the sinogram data acquired with a combined low-mAs and sparse-view protocol is first restored by using a PWLS based sinogram restoration method. Then, the restored sinogram data is hereafter used to reconstruct image using a PWLS-TV based method.

II. METHODOLOGY

A. CT Imaging Model

Without loss of generality, under the assumption of monoenergetic beam, the x-ray CT measurement can be approximately expressed as a discrete linear system:

$$y = H\mu \quad (1)$$

This work was partially supported by the National Natural Science Foundation of China under grand (No. 81101046), the Science and Technology Program of Guangdong Province of China under grant (No. 2011A030300005), and the 973 Program of China under grant (No. 2010CB732503). Asterisk indicates corresponding author.

Y. Zhang, J. Ma, H. Zhang, Z. Bian, D. Zeng, Y. Gao, and W. Chen are with the School of Biomedical Engineering, Southern Medical University, Guangzhou 510515 China.

*J. Huang is with the School of Biomedical Engineering the Southern Medical University, Guangzhou 510515 China (telephone: 020-616-48285; e-mail: hjing@smu.edu.cn).

where y represents the obtained sinogram data (projections after system calibration and logarithm transformation), *i.e.*, $y = (y_1, y_2, \dots, y_M)^T$, μ is the vector of attenuation coefficients to be estimated, *i.e.*, $\mu = (\mu_1, \mu_2, \dots, \mu_N)^T$, where ‘ T ’ denotes the matrix transpose. The operator H represents the system or projection matrix with the size of $M \times N$. In our implementation, the associated element was pre-calculated by a fast ray-tracing technique stored as a file. The goal for CT image reconstruction is to estimate the attenuation coefficients μ from the measurement y with H .

B. Proposed PWLS Criteria for Ultra-low-dose CT Image Reconstruction

The proposed iterative image reconstruction algorithm for ultra-low-dose protocol contains two major steps: (1) the sinogram restoration by using a PWLS based sinogram restoration method; (2) image reconstruction from the restored sinogram using a PWLS-TV based method. Each step is described in detail as follows.

1) *PWLS based sinogram restoration*: The sinogram data acquired with a combined low-mAs and sparse-view protocol was first restored by using a PWLS based sinogram restoration method. The PWLS statistical approach in sinogram domain can be described by [5]:

$$p^* = \arg \min_{p \geq 0} \{ (y - p)' \Sigma^{-1} (y - p) + \beta R(p) \} \quad (2)$$

where y represents the obtained sinogram data and p represents the vector of ideal projection to be estimated. β is a hyper-parameter to balance the fidelity term (*i.e.*, first term of equation (2)) and the priori/penalty term (*i.e.*, second term of equation (2)). Σ is a diagonal matrix with the i th element of σ_i^2 which is the variance of sinogram data y . $R(p)$ is the penalty term. In this paper, a quadratic penalty form was used, *i.e.*,

$$R(p) = \frac{1}{2} \sum_j \sum_{k \in M_j} w_{j,k} (p_j - p_k)^2 \quad (3)$$

where M_j indicates the set of four nearest neighbors of the j th voxel in the sinogram. The parameter $w_{j,k}$ is the directional weighting coefficients.

In the implementation, the variance of σ_i^2 was determined by the following mean-variance relationship proposed by Ma *et al* [13]:

$$\sigma_i^2 = \frac{1}{I_0} \exp(\bar{p}_i) \left(1 + \frac{1}{I_0} \exp(\bar{p}_i) (\sigma_e^2 - 1.25) \right) \quad (4)$$

where I_0 denotes the incident x-ray intensity, \bar{p}_i is the mean of the sinogram data at bin i and σ_e^2 is the background electronic noise variance. The PWLS sinogram restoration as a preprocessing step is named as ‘‘SR’’ for simplicity hereinafter.

2) *PWLS-TV based image reconstruction*: In solving μ from the equation (1), to invert (1) directly is difficult because the system matrix dimension is huge in current CT system and degraded seriously for image reconstruction from the measures noisy sinogram data. In this paper, we are using the PWLS based criterion. The associated mathematical formula can be expressed as follows:

$$\mu^* = \arg \min_{\mu \geq 0} \{ (p - H\mu)' \Sigma^{-1} (p - H\mu) + \beta R(\mu) \} \quad (5)$$

where H represents the system of projection matrix, p represents the restored sinogram data from equation (2), β is a hyper-parameter to balance the fidelity term and the penalty term. Σ is a diagonal matrix with the i th element of σ_i^2 which is described in (4). In this paper, a TV-based penalty term is used and can be written as [9]:

$$R(\mu) = \sum_{s,t} \sqrt{(\mu_{s,t} - \mu_{s-1,t})^2 + (\mu_{s,t} - \mu_{s,t-1})^2} + \delta \quad (6)$$

where s and t are the indices of the location of the attenuation coefficients of the desired image. δ is a small constant used for keeping differentiable with respect to the voxel value.



Fig. 1. XCAT phantom.

C. Experimental Data Acquisitions

To validate and evaluate the performance of the proposed method, a digital XCAT phantom [14] as shown in Fig. 1 is used for simulation of ultra-low-dose image with different levels of mAs and different number of views. This phantom is composed by 512×512 square pixels with the intensity value from 0 to 130. The size of each pixel is $1.25 \text{ mm} \times 1.25 \text{ mm}$. We chose a geometry that is representative for a mono-energetic fan-beam CT scanner setup with a circular orbit to acquire 1,160 views over 2π . The number of channels per view is 672. The distance from the rotation center to the curved detector is 570 mm and the distance from the X-ray source to the detector is 1,140 mm. Each projection datum along a X-ray through the sectional image is computed based on the known densities and intersection areas of the ray with the geometric shapes of the objects in the sectional image.

D. Performance Evaluation

For quantitative evaluation of the reconstruction accuracy, the relative root mean squared error (RRMSE) metrics is calculated over a region of interest (ROI). The RRMSE is defined as:

$$\text{RRMSE} = \sqrt{\frac{\sum_{m=1}^Q (\mu(m) - \mu_{\text{xtrue}}(m))^2}{\sum_{m=1}^Q (\mu_{\text{xtrue}}(m))^2}} \quad (7)$$

The mean per cent absolute error (MPAE) metrics is also utilized to evaluate the noise reduction for the proposed method:

$$\text{MPAE} = \frac{100}{Q} \sum_{m=1}^Q \left| \frac{\mu(m)}{\mu_{\text{xtrue}}(m)} - 1 \right| \quad (8)$$

where $\mu(m)$ denoted the voxel value of low-dose image, $\mu_{\text{xtrue}}(m)$ denotes the ground truth image. Q is the number of voxels in the ROI as indicated by a square in Fig. 1.

III. RESULTS

A. Simulation Setup

In this study, ultra-low-dose CT images with different levels of mAs and different numbers of views were simulated. For the simulation of low-mAs projection data, similar to the study [15], after calculating the noise-free line integral y as a direct projection operation based on model (1), the noisy measurement b_i at each bin i was generated according to the statistical model of pre-logarithm projection data:

$$b_i = \text{Poisson}(I_0 \exp(-y_i)) + \text{Normal}(0, \sigma_e^2) \quad (9)$$

where I_0 denotes the incident X-ray intensity and σ_e^2 is the background electronic noise variance which was set to 10. In the present study, the X-ray exposure level I_0 was set at three different levels, *i.e.*, 1.0×10^4 , 3.0×10^4 , 5.0×10^4 . The noisy measurement was calculated by the logarithm transform of b_i . For the simulation of sparse-view projection data, original 1,160 views were undersampled to five different levels, *i.e.*, 25, 40, 58, 80 and 116 views, respectively.

B. XCAT Phantom Studies

Fig. 2 shows the images reconstructed from ultra-low-dose sinogram data acquired with different sparse-view levels under a fixed $I_0 = 3 \times 10^4$. From top to bottom, the number of views is 25, 40, 58, 80 and 116. From left to right, the results are from the filtered back-projection (FBP), PWLS-TV without SR and PWLS-TV with SR, respectively. The parameters β for the PWLS sinogram restoration and PWLS-TV reconstruction were 3×10^{-3} and 5×10^{-2} , respectively. Serious streak artifacts can be observed from the images reconstructed by the FBP. Images reconstructed by the PWLS-TV without SR shows fewer artifacts but with noticeable noise. It can be seen that the PWLS-TV with SR achieves better image recovery than other methods in terms of the

noise and artifacts suppressions. The results demonstrate that the PWLS sinogram restoration as a preprocessing step is useful for the ultra-low-dose CT image reconstruction with a combined low-mAs and sparse-view protocol.

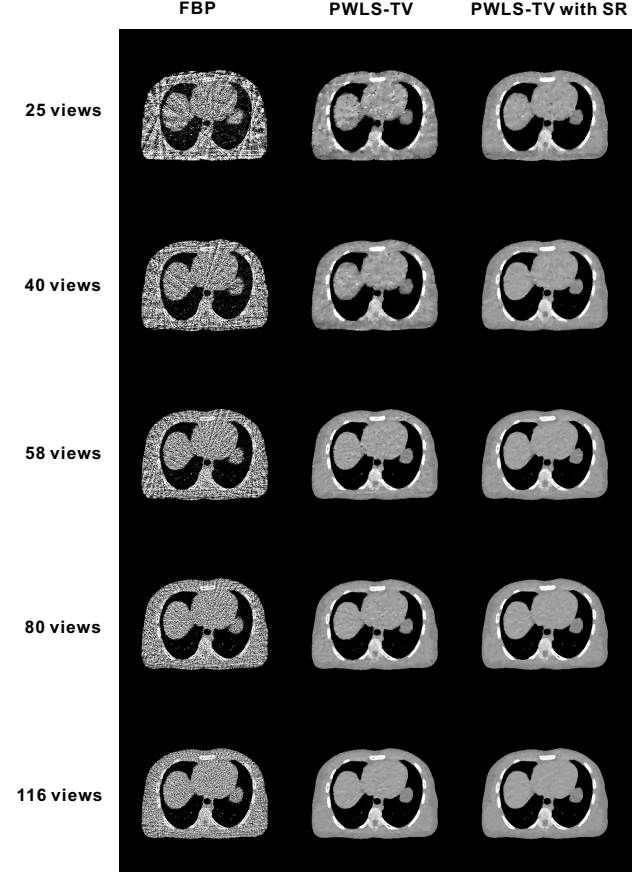


Fig. 2. XCAT phantom reconstructions by different methods with mAs level of $I_0 = 3 \times 10^4$ and views of 25, 40, 58, 80 and 116, respectively.

To evaluate the PWLS-TV approach with and without SR at different levels of mAs and different numbers of views quantitatively, the image quality metrics RRMSE and MPAAE on the ROI indicated by the squares in Fig. 1 were measured. Besides the mAs level in Fig. 1, the X-ray exposure level I_0 was set with other two different values, *i.e.*, 1.0×10^4 , 5.0×10^4 . The number of views were 25, 40, 58, 80 and 116, respectively. Results are listed in Table 1. Fig. 3 is the corresponding curves of Table 1. It can be seen that the PWLS-TV method with SR performed better than the PWLS-TV method without SR in the most protocols especially in these protocols when dose is lower. In the last protocol described in Table 1, *i.e.*, I_0 is 5.0×10^4 and the number of views is 116, the PWLS-TV without SR seems to have a better performance than PWLS-TV with SR. One possible reason is that the PWLS sinogram restoration method produced an over-smoothing effect when the used dose is not ultra-lower than the standard one.

TABLE I

IMAGE QUALITY METRICS ON THE ROI INDICATED BY THE SQUARE IN FIG. 1.

I_0	views	PWLS-TV		PWLS-TV with SR	
		RRMSE	MPAE	RRMSE	MPAE
1×10^4	25	0.113	6.672	0.048	2.875
	40	0.107	6.158	0.028	2.121
	58	0.096	6.540	0.028	2.127
	80	0.084	5.463	0.027	2.204
	116	0.064	4.156	0.023	1.688
3×10^4	25	0.047	2.602	0.040	2.268
	40	0.035	2.114	0.025	1.664
	58	0.029	1.936	0.015	1.224
	80	0.022	1.612	0.014	1.114
	116	0.014	1.066	0.012	0.931
5×10^4	25	0.020	1.491	0.016	1.195
	40	0.015	1.204	0.014	1.168
	58	0.013	1.050	0.012	1.014
	80	0.012	0.891	0.011	0.882
	116	0.010	0.802	0.011	0.813

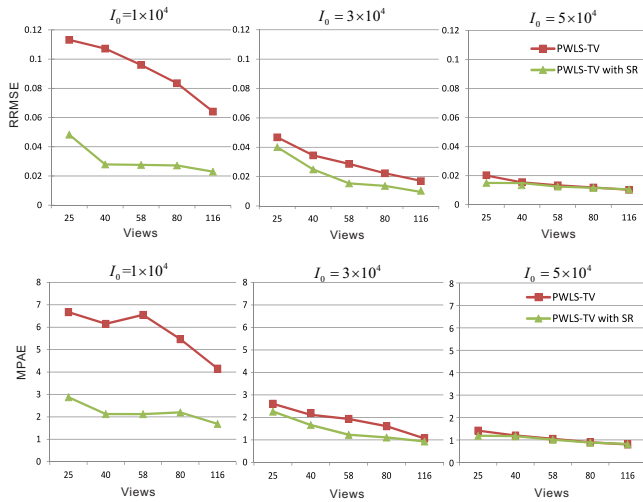


Fig. 3. Corresponding curves of the data from Table. 1.

IV. DISCUSSION AND CONCLUSION

Statistical iterative reconstruction (SIR) for x-ray CT has been explored long time ago for radiation dose reduction in CT fields. In this work, we presented a PWLS-TV with PWLS sinogram restoration as a preprocessing step for ultra-low-dose image reconstruction from the sinogram data acquired with a combined low-mAs and sparse-view protocol. First, the PWLS sinogram restoration was carried out to reduce the noise of the original sinogram data. Then, the PWLS-TV method was performed for the image reconstruction from the restored sinogram data.

In the implementation, the weights of the PWLS term were estimated by using the nonlinear relationship between the variance and mean values of the sinogram data. The hyper-parameter β for the PWLS sinogram restoration and PWLS-TV image reconstruction was selected by hand with

noise reduction, respectively. The preliminary experimental results demonstrate that the PWLS sinogram restoration as a preprocessing step is useful for the ultra-low-dose CT image reconstruction with a combined low-mAs and sparse-view protocol. But, it is worth to note that in the protocol when the level of mAs and number of views are not very low, the result obtained from the PWLS-TV with sinogram restoration method seems to suffer an over-smoothing effect. And the resolution of the reconstructed image descended. This demonstrates that the result of the sinogram restoration as a preprocessing step is very important and has a significant influence on the final reconstruction. How to keep the resolution of the image when reducing the noise is an interesting topic for the ultra-low-dose image reconstruction. More studies are going on in our group.

REFERENCES

- [1] M. K. Kalra, C. Wittram, M. M. Maher, A. Sharma, G. B. Avinash, K. Karau, T. L. Halpern, S. Saini, and J. A. Shepard, "Can noise reduction filters improve low-radiation-dose chest CT images?" *Pilot study Radiology*, vol. 228, pp. 257-264, 2003.
- [2] C. H. McCollough, M. R. Bruesewitz, and J. M. Kofler, "CT dose reduction and dose management tools: Overview of available options," *Radiographics*, vol. 26, pp. 503-512, 2006.
- [3] D. J. Brenner and E. J. Hall, "CT-an increasing source of radiation exposure," *New England J. Med.*, vol. 357, pp. 2277-2284, 2007.
- [4] M. Persson, D. Bone, and H. Elmqvist, "Total variation norm for three-dimensional iterative reconstruction in limited view angle tomography," *Phys. Med. Biol.*, vol. 46, pp. 853-866, 2001.
- [5] J. Wang, T. Li, H. Lu, and Z. Liang, "Penalized weighted least-squares approach to sinogram noise reduction and image reconstruction for low-dose X-ray CT," *IEEE Trans. Med. Imaging*, vol. 25, pp. 1272-1283, 2006.
- [6] J. Ma, H. Zhang, Y. Gao, J. Huang, Z. Liang, Q. Feng, and W. Chen, "Iterative image reconstruction for cerebral perfusion CT using a pre-contrast scan induced edge-preserving prior," *Phys. Med. Biol.*, vol. 57, pp. 7519-7542, 2012.
- [7] D. F. Yu and J. A. Fessler, "Edge-preserving tomographic reconstruction with nonlocal regularization," *IEEE Trans. Med. Imaging*, vol. 21, pp. 159-73, 2002.
- [8] J. Ma, J. Huang, Q. Feng, H. Zhang, H. Lu, Z. Liang, and W. Chen, "Low-dose computed tomography image restoration using previous normal-dose scan," *Med. Phys.*, vol. 38, pp. 5714-5731, 2011.
- [9] E. Sidky, C. Kao, and X. Pan, "Accurate image reconstruction from few-views and limited-angle data in divergent beam CT," *J. X-Ray Sci. Technol.*, vol. 14, pp. 119-39, 2006.
- [10] J. Huang, J. Ma, N. Liu, H. Zhang, Z. Bian, Y. Feng, Q. Feng, and W. Chen, "Sparse angular CT reconstruction using non-local means based iterative-correction POCS," *Comput Biol Med.*, vol. 41, pp. 195-205, 2011.
- [11] E. Sidky and X. Pan, "Image reconstruction in circular cone-beam CT by constrained, total-variation minimization," *Phys. Med. Biol.*, vol. 53, pp. 4777-4807, 2008.
- [12] Y. Liu, J. Ma, Y. Fan, and Z. Liang, "Adaptive-weighted total variation minimization for sparse data toward low-dose x-ray computed tomography image reconstruction," *Phys. Med. Biol.*, vol. 57, pp. 7923-7956, 2012.
- [13] J. Ma, Z. Liang, Y. Fan, Y. Liu, J. Huang, W. Chen, and H. Lu, "Variance analysis of x-ray CT sinograms in the presence of electronic noise background," *Med. Phys.*, vol. 39, pp. 4051-4065, 2012.
- [14] W. P. Segars, G. Sturgeon, S. Mendonca, J. Grimes, and B. M. Tsui, "4D XCAT phantom for multimodality imaging research," *Med. Phys.*, vol. 37, pp. 4902-4915, 2010.
- [15] P. J. La Rivière and D. M. Billmire, "Reduction of noise-induced streak artifacts in x-ray computed tomography through spline-based penalized-likelihood sinogram smoothing," *IEEE Trans. Med. Imaging*, vol. 24, pp. 105-111, 2005.