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Dynamical System Design for Silicon Neurons using Phase Reduction Approach

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***Abstract*—In the present paper, we apply a computer-aided phase reduction approach to dynamical system design for silicon neurons (SiNs). Firstly, we briefly review the dynamical system design for SiNs. Secondly, we summarize the phase response properties of circuit models of previous SiNs to clarify design criteria in our approach. From a viewpoint of the phase reduction theory, as a case study, we show how to tune circuit parameters of the resonate-and-fire neuron (RFN) circuit as a hybrid type SiN. Finally, we demonstrate delay-induced synchronization in a silicon spiking neural network that consists of the RFN circuits.**

I. Introduction

In the field of neuromorphic engineering, silicon neurons (SiNs) [1]-[13] are the most fundamental element constituting neuromorphic systems for spike-based computation. Design approaches for SiNs are based on three principles. The first one is referred to as the phenomenological principle which captures some of key behaviors and functions of neurons at abstract level, leading to a compact circuitry [1]. The second one is the conductance-based principle which emulates the dynamics of ionic channels of excitable membranes to reproduce a wide variety of behaviors and functions of neurons at detailed level, and resulting circuits are complicated [6], [7]. Recently, in addition to these, the dynamical systems design principle has been proposed as the third one [10]-[13].

The dynamical system design can be characterized by the mathematical structures embedded in target circuits and devices. From this point of view, we further classify the dynamical system design approaches for nonlinear circuits and devices into three categories: (i) the phase plane and nullcline-based design [10]-[16], (ii) the potential-based design [17]-[19] and (iii) the phase response curve (PRC)-based design [20]. The distinct approaches provide us with different perspectives. For instance, the phase plane and nullcline-based design can tune the phase plane structure to reproduce complex bifurcation phenomena [10].

In this work, we apply a computer-aided phase reduction approach to dynamical system design for SiNs to enhance synchronization in an ensemble of SiNs. We clarify key criteria to optimize the design of SiNs in terms of phase response properties through analyzing various circuit models of the previous SiNs [8]-[12]. In accordance with the criteria, we show how to tune circuit parameters of a resonate-and-fire neuron (RFN) circuit [4] as a hybrid type of SiNs to obtain a desirable PRC. Furthermore, we demonstrate that the PRC of the circuit plays an important role for synchronization in a recurrent network of the RFN circuits with transmission delay.

II. Dynamical System Design for Silicon Neurons

Let us briefly review dynamical system design for nonlinear circuits and devices including SiNs as shown in Table 1.

A. Phase Plane and Nullcline-based Design

The phase plane and nullcline-based design for SiNs has been firstly established as the mathematical structure-based design [10]. In the design approach, the phase plan structure of SiNs is mapped onto silicon by qualitative modeling of the dynamics of ionic channels based on reduced system equations with a few of state variables. By tuning nullclines on the phase plane, one can control the bifurcation structure of the SiN, reproducing the functional diversity of dynamical behaviors of biological neurons, such as post-inhibitory rebound, bursting, and chaos [10], [11]. Such systematic approach can achieve the efficient SiN design. For instance, a biophysically-inspired SiN circuitry [9] can be more sophisticated and be extend to reproduce both Class I and Class II neural excitability [12]. The main advantage of the design approach is to allow us to reproduce diverse dynamics of biological neurons in a compact SiN circuitry than a conductance-based SiN circuitry.

B. Potential-based Design

The dynamical systems design based on potential of an SiN has been recently proposed [17]. In the design approach, the dynamics of an SiN are regarded as a motion of a particle on the potential with active area, and the potential can be derived from the dynamics systematically [17]. By controlling the shape of the potential, one can tune the dynamics to reproduce bursting phenomena [17].

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TABLE I
DYNAMICAL SYSTEM DESIGN FOR NONLINEAR CIRCUITS AND DEVICES INCLUDING SILICON NEURONSD

Design Approaches	Target Devices and Circuits	Main Features and Functions	Refs.
Phase Plane and Nullcline-based Design			
	Reaction Diffusion Chips	Tuning nullcline, Controlling bifurcation	[16]
	Silicon Neurons	Tuning nullcline, Controlling bifurcation, Generating chaos, Reproducing adaptation	[10]
		Tuning nullcline, Controlling bifurcation, Generating bursting, Reducing fluctuations	[11]
		Tuning nullcline, Controlling bifurcation, Reproducing adaptation	[12]
		Controlling bifurcation, Generating bursting, Reproducing adaptation	[13]
Potential-based Design			
	Silicon Neurons	Controlling bifurcation, Generating bursting	[17]
	Stochastic Memory Cells	Controlling bifurcation and state transition	[18]
	Stochastic Logic Gates	Controlling bifurcation and state transition	[19]
Phase Response Curve-based Design			
	Nonlinear Oscillator Circuits	Enhancing synchronization	[20]

C. *Phase Response Curve-based Design*

As an alternative, we consider a dynamical systems design approach based on PRC for enhancing synchronization in an ensemble of SiNs. In spite of the significance of PRCs on synchronization phenomena, those of SiNs are rarely focused on [13]. In the latter section, we describe the detail of our design approach.

III. A COMPUTER-AIDED PHASE REDUCTION APPROACH TO DYNAMICAL SYSTEM DESIGN FOR SILICON NEUONS

Let us explain the dynamical systems design based on the phase reduction theory.

A. *Phase Response Properties of Biological Neurons*

We describe perspectives from the phase reduction theory in computational neuroscience [21]. Two types of PRCs are identified in biological neurons. One is the Type I PRC that has almost all positive regions, which mean the phase advances in response to perturbations. Another one is the Type II PRC that has both positive and negative regions, which correspond to the phase advances and delays, respectively. The classification of PRCs is closely related to the classification of the neural excitability, Class 1 and Class 2, associated with bifurcation structure.

B. *Phase Response Properties of Silicon Neurons*

We summarize the phase response properties of various types of SiNs to clarify the key design criteria from a viewpoint of the phase reduction approach.

1) IFN Type SiNs: Firstly, we consider the phase response properties of the IFN type SiNs. We begin by considering the relationship between the IFN type neuron models and their PRCs. The IFN type neuron models, such as the leaky IFN model and the quadratic IFN model, have the one-dimensional subthrehold membrane dynamics with a firing threshold. Because of the saddle-node bifurcation, the IFN models have the Type I PRCs. The shape of such PRCs are determined by the nonlinearity corresponding to the positive feedback, which is a key factor in the PRC-based design. In fact, the distinct PRCs can be obtained from the different nonlinearity in the IFN circuits [13]-[15].

2) Rate Type SiNs: We here consider the phase response properties of the rate type SiNs. The rate type SiNs, such as the Wilson-Cowan neuron circuit, exhibit the Hopf bifurcation. Consequently, such SiNs have the Type II PRC.

3) Full Conductance-based Type SiNs: We consider the phase response properties of the conductance-based type SiNs [23], [24]. Since such conductance-based type SiNs have the wide variety of the conductance dynamics emulating biological ionic channels, the SiNs can have both Type I and Type II PRCs.

4) Reduced Conductance-based Type SiNs: We consider the phase response properties of the Morris-Lecar (ML) type SiNs [8]-[12]. The dynamics of the ML neuron model can be derived from the conductance-based neuron models of the Hodykin-Huxley (HH) type by model reduction. As a result, we can consider reduced dynamics with two or three state variables. Depending on the bifurcation near the equilibrium point, the reduced model can exhibit both the Type I and Type II PRCs. In a similar way, the reduced conductance-based type SiNs have both types of the PRCs depending on the gradient of the positive and negative feedbacks.

5) Hybrid Type SiNs: We consider the phase response properties of the hybrid type SiNs having both continuous subthreshold membrane dynamics and discrete firing reset mechanism. For the purpose, we compare the phase plane dynamics and phase response properties of the Izhikevich neuron model and the RFN model [21].

Figure 1 shows the phase plane portraits of the Izhikevich model. Depending on the bifurcation near the equilibrium point, the Izhikevich model can exhibit both the Type I and Type II PRCs. In the case of the saddle-node bifurcation (Fig. 1A), the Izhikevich model becomes an integrator having the Type I PRC (Fig. 2A). In contrast, in the case of the Hopf bifurcation (Fig. 1B), the Izhikevich model becomes a resonator having the Type II PRC (Fig. 2B). By changing the ratio of the time constants of the membrane dynamics (v) and the recovery dynamics (w) or the location of the reset point of the phase plane, the shape of the PRC can be modified.

Figure 3 shows the phase plane portraits of the RFN model. Depending on the parameters, the model has a stable focus and

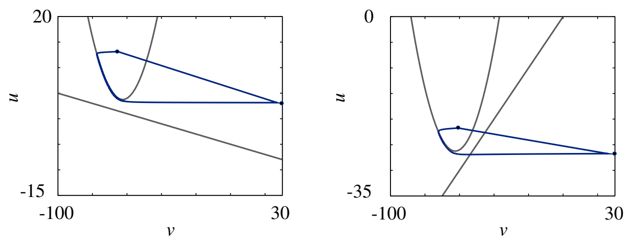

Fig. 1. Phase plane portraits of the Izhikevich model near (A) the saddle-node bifurcation point and (B) the Hopf bifurcation point.

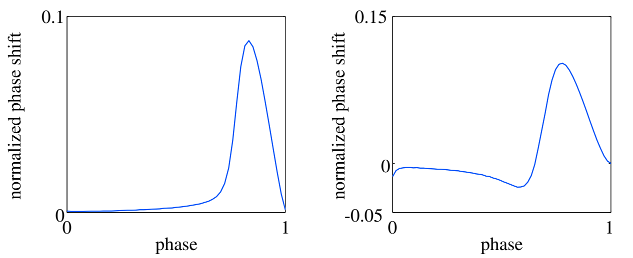

Fig. 2. PRC of the Izhikevich model. (A) Type I PRC and (B) Type II PRC.

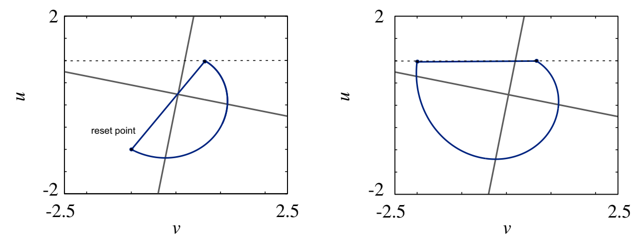

Fig. 3. Phase plane portraits of the RFN model (A) in the case of an integrator and (B) in the case of a resonator.

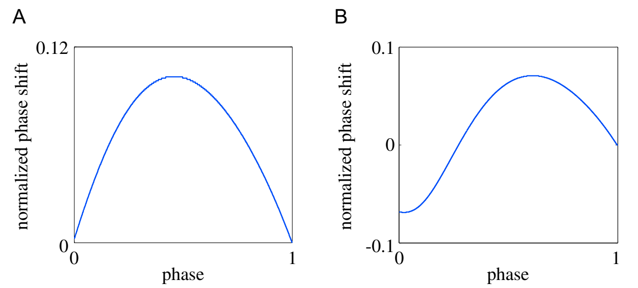

Fig. 4. PRC of the RFN model. (A) Type I PRC and (B) Type II PRC.

exhibits damping subthreshold oscillation. After the grazing bifurcation beyond a threshold, the RFN model fires a spike and the state is reset to a certain reset point. The PRCs of the RFN model depending on the location of the threshold and the reset point. The region of the negative phase shift corresponding to the Type II PRC becomes large with the increasing the angle between the threshold point and the reset point from the stable focus.

C. Design Criteria in View of Phase Reduction Approach

From the results, we clarify the criteria of the PRC-based design for SiNs: (i) controlling the gradient of the positive and negative feedbacks, (ii) tuning the ratio between the duration of action potential and the repetitive firing period, and (iii) setting the location of a reset point on the phase plane. In practice, tuning the time constant of the circuit dynamics is efficient for (i) and (ii). By tuning the phase plane structure, (iii) can be achieved.

IV. RFN Circuit and its PRC-based Design for Enhancing Synchronization

Let us show how to tune circuit parameters of the RFN circuit as a hybrid type SiN [4] in accordance with the design criteria. This is because the RFN circuit has the firing reset mechanism corresponding to precipitous negative feedback and exhibits the Class 2 excitability as a result of the grazing bifurcation.

A. Dyncamics of the RFN Circuit

Figure 5 shows the schematic diagram of the RFN circuit that consists of the membrane circuit, the threshold-and-fire circuit, the excitatory and inhibitory synaptic circuits, and the delay-and-inverter circuit. The dynamics of the RFN circuit are represented as:

$$C\frac{dV}{dt} = -gV + \acute{I}_V - I_o \exp\left(\frac{\kappa^2}{\kappa+1}\frac{W}{U_T}\right) + I \qquad (1)$$

$$C\frac{dW}{dt} = I_o \exp\left(\frac{\kappa^2}{\kappa+1}\frac{V}{U_T}\right) - \acute{I}_W \qquad (2)$$

where V and W represent the node voltages corresponding to the fast membrane dynamics and slow recovery dynamics.

The current through the current-mirrors, $\acute{I}_V$ and $\acute{I}_W$, are represented as:

$$\acute{I}_V = \alpha I_V \left(1 + \frac{VDD - V}{V_{E,P}}\right) \qquad (3)$$

$$\acute{I}_W = \beta I_W \left(1 + \frac{W}{V_{E,N}}\right) \qquad (4)$$

where I_V and I_W represents the bias currents, VDD the power-supply voltage, and $V_{E,N}$ and $V_{E,P}$ the Early voltages for the nMOS and pMOS FETs, respectively. The parameter α and β are proportional constants determined by I_V and I_W.

B. PRCs of the RFN Circuit

We consider the phase response properties of the RFN circuit. The PRCs of the RFN model as shown in Fig. 4 are theoretically derived from the model dynamics as follows [23]:

$$Z(\phi) \propto \exp(T\phi)\sin\left(2\pi\frac{T}{T_o}(1-\phi)\right) \qquad (5)$$

where ϕ represents the phase, T the repetitive firing period, and T_o the maximum firing period inversely proportional to the natural angular frequency of the RFN model.

By substituting the circuit parameters into Eq. 5, the PRC of the RFN circuit can be obtained. The natural angular frequency of the circuit can be calculated from the Jacobian near the equilibrium voltages, which is determined by the slope factor,

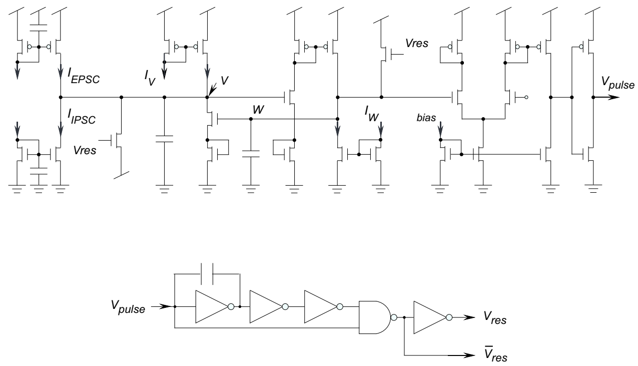

Fig. 5. Schematic diagram of the RFN circuit.

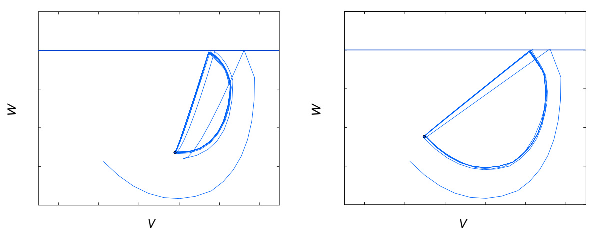

Fig. 6. Phase plane portraits of the RFN circuit.

the thermal voltage, and I_{V_o} and I_{W_o} the equilibrium currents of the circuit. Since the repetitive firing period of the circuit can be determined by setting the location of the reset voltage on the phase plane along the orbit, such as shown in Fig. 6, the ratio of T/T_o can be changed. This indicates that the type of the PRC can be tuned from the Type I to Type II.

C. Delay-induced Synchronization in the RFN Circuit Network

Through circuit simulations using SPICE, we investigate the synchronization properties of an excitatory recurrent network of the RFN circuits without self-feedback. We set the number of the circuits, $N = 5$, and the circuit parameters to obtain the Type II PRC. The device parameters were assumed to use the TSMC 0.25-μm technology.

In the case of small transmission delays, the network can exhibit an asynchronous state, as shown in Fig. 7A. In contrast, in the case of relatively large transmission delays beyond a certain value, the circuits can synchronize with each other in in-phase at a steady state, i.e., a synchronous state, as shown in Fig. 7B. The critical value for a synchronous state can be calculated by the network linear stability analysis. The results indicate that the Type II PRC can only compensate the transmission delay for synchronization at network level.

V. Conclusion

In this study, we have applied a computer-aided phase reduction approach to dynamical system design for SiNs. We clarified the design criteria by considering the phase response properties of previous SiNs. More specifically, we showed

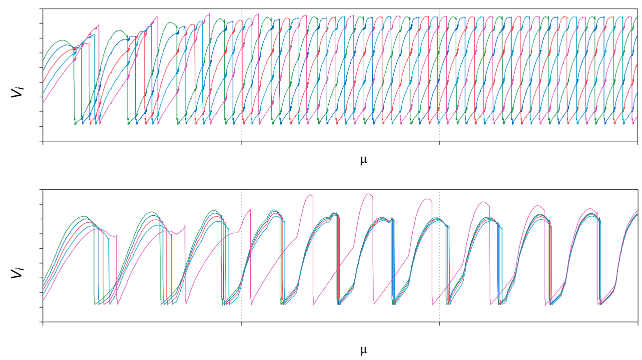

Fig. 7. Delay-induced Synchronization in the Network of the RFN Circuits.

how to tune the circuit parameters of the RFN circuit as a hybrid type SiN to obtain desirable PRCs. Furthermore, we demonstrated that the Type II PRC of the RFN circuit plays an important role for synchronization in a network of the RFN circuits with transmission delay. The results indicate the significance of our approach in practical SiN design.

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