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A Novel Algorithm for Linear Parameter Varying Identification of Hammerstein Systems with Time-Varying Nonlinearities

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Abstract— **This paper describes a novel method for the identification of Hammerstein systems with time-varying (TV) static nonlinearities and time invariant (TI) linear elements. This paper develops a linear parameter varying (LPV) state-space representation for such systems and presents a subspace identification technique that gives individual estimates of the Hammerstein components. The identification method is validated using simulated data of a TV model of ankle joint reflex stiffness where the threshold and gain of the model change as nonlinear functions of an exogenous signal. Pilot experiment of TV reflex EMG response identification in normal ankle joint during an imposed walking task demonstrate systematic changes in the reflex nonlinearity with the trajectory of joint position.**

I. INTRODUCTION

Hammerstein systems are a class of block-oriented nonlinear systems consisting of a cascade of a static nonlinearity and a linear dynamic element. Many physiological systems including ankle joint reflex pathway can be represented by a Hammerstein model [1], [2]. Moreover, many physiological systems show time-varying/non-stationary behavior [3], [4]. Consequently, it is important to develop reliable algorithms that can accurately identify TV Hammerstein systems.

It seems likely that under many conditions, time-varying behavior is not explicitly time-dependent but is implicitly dependent on variables within the system that vary with time [5], [6]. Linear parameter varying (LPV) models are very good candidates for representing such TV systems [7], [8]. LPV models have a model structure resembling that of a linear system but have parameters that change as a function of a time-dependent signal called scheduling variable (SV). The TV parameters of the LPV model can be the coefficients of its transfer function or its state-space matrices [7], [8].

This paper addresses the identification of a class of LPV Hammerstein systems whose static nonlinear element is time varying and the linear element is time invariant. The motivation for focusing on this specific class of TV Hammerstein models is the dynamics of the ankle reflex pathway. Ankle reflex stiffness is defined as the dynamic relationship between a joint position and the torque that originates from changes in muscle activation induced by the stretch reflex arc. Previous studies have shown that reflex stiffness varies with joint position and/or neural activation to counteract external perturbations and control voluntary movement [9], [10]. Therefore, the joint position and muscle activation are suitable for representing the SV of an LPV Hammerstein model of reflex stiffness. Experimental results have shown that the characteristics of the nonlinearity such as the reflex gain [11] and threshold [12] are modulated by the joint operating point defined by joint position and level of activation.

Identification of TV systems has been the subject of many works in the literature. Ensemble-based TV methods have been used for identification of joint stiffness during movement [13], [14], [15]. These methods have some shortcomings: (a) they require trials with exactly the same TV behavior which limits their applicability mainly to movements that can be repeated; and (b) they cannot predict the response to a new trajectory since TV changes are identified as explicit functions of time without describing the functional relationships underlying the modulation.

On the other hand, various LPV identification methods have been developed which address the shortcomings of the ensemble-based approaches using state-space [16] or transfer function representations [17]. However, the major disadvantages of the works to date are: (a) they estimate a discrete-time LPV model of the system; converting these to a continuous-time representation needed for physiological interpretation can be difficult; and (b) they do not map the identified discrete-time LPV model to individual elements of the TV Hammerstein representation (i.e., static nonlinearity and linear dynamics). Consequently, existing LPV identification methods cannot provide useful physiological information.

This paper presents a novel LPV algorithm that estimates the TV nonlinearity and TI linear dynamics of the Hammerstein cascade. The method allows straightforward extraction of the continuous-time representation of the system elements. Furthermore, the resulting LPV model can predict system response to novel trajectories of the SV. This work is an extension of the TI subspace Hammerstein identification algorithm developed in [18].

This paper is organized as follows: Section II formulates the problem and gives the theoretical development of the algorithm. Section III demonstrates the validity of the method using simulated data. Pilot experiment presented in Section IV describes identification of ankle reflex EMG response, revealing its functional variations with respect to ankle position during an imposed walking task. Section V

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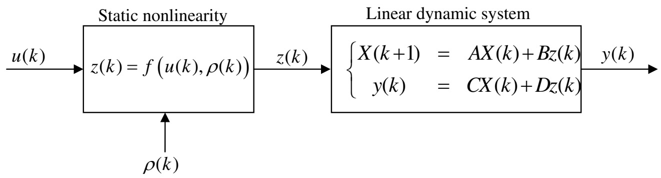

Fig. 1. LPV Hammerstein system model schematic.

provides concluding remarks and a discussion.

II. Theory

A. Problem Formulation

Throughout this paper, vectors, matrices and scalars are indicated by bold-face uppercase, uppercase and lowercase letters, respectively. The objective is to identify a TV Hammerstein system consisting of a TV nonlinearity followed by LTI dynamics. As shown in Fig. 1, let the input of the Hammerstein cascade be $u(k)$, the output of the nonlinearity be $z(k)$, the system output be $y(k)$, and the SV be $\rho(k)$. Assume that N samples of input-output data are recorded, i.e. $k \in \{0 \cdots N-1\}$. The output of the nonlinearity is described with the LPV model:

$$z(k) = f\left(u(k), \rho(k)\right) \simeq \sum_{i=1}^{n} \omega_i\left(\rho(k)\right) g_i\left(u(k)\right) \tag{1}$$

where ω_i is:

$$\omega_i = \sum_{j=1}^{n_\rho} \omega_{ij} g_j\left(\rho(k)\right) \tag{2}$$

where ω_{ij} is the i, j^{th} coefficient for the i^{th} basis expansion of the input, $g_i\left(u(k)\right)$, and j^{th} basis expansion of the SV, $g_j\left(\rho(k)\right)$. The basis expansion can be selected as Chebychev, Hermite, etc. The objective is to estimate ω_{ij}, $\forall i \in \{1, \cdots, n\}$ and $\forall j \in \{1, \cdots, n_\rho\}$.

It will be assumed that the LTI component is stable and can be represented by a state-space model:

$$\begin{cases} \boldsymbol{X}(k+1) & = A\boldsymbol{X}(k) + \boldsymbol{B}z(k) \\ \tilde{y}(k) & = C\boldsymbol{X}(k) + Dz(k) + n(k) \end{cases} \tag{3}$$

where, $\boldsymbol{X}(k)$ is a $m \times 1$ state vector; $A_{m\times m}$, $\boldsymbol{B}_{m\times 1}$, $C_{1\times m}$ and $D_{1\times 1}$ are the state-space matrices; and $\tilde{y}(k)$ is the measured system output contaminated with additive noise, $n(k)$ that is zero mean and uncorrelated with the input $u(k)$. Denote the elements of $\boldsymbol{B}$ and D by:

$$\begin{aligned} \boldsymbol{B} &= [b_1, \cdots, b_m]^T \\ D &= [d] \end{aligned} \tag{4}$$

Define the vectors:

$$\begin{aligned} \boldsymbol{\Omega_i} &= \left[\omega_{i1}, \cdots, \omega_{in_\rho}\right]^T \\ \boldsymbol{\Omega} &= \left[\boldsymbol{\Omega_1} \cdots \boldsymbol{\Omega_n}\right]^T \\ \boldsymbol{U_i}(k) &= \left[g_i\left(u(k)\right) g_1\left(\rho(k)\right), \cdots, g_i\left(u(k)\right) g_{n_\rho}\left(\rho(k)\right)\right]^T \\ \boldsymbol{U}(k) &= \left[\boldsymbol{U_1}(k) \cdots \boldsymbol{U_n}(k)\right]^T \end{aligned} \tag{5}$$

Substitute (5) in (3) to yield:

$$\begin{cases} \boldsymbol{X}(k+1) & = A\boldsymbol{X}(k) + B_\Omega \boldsymbol{U}(k) \\ \tilde{y}(k) & = C\boldsymbol{X}(k) + D_\Omega \boldsymbol{U}(k) + n(k) \end{cases} \tag{6}$$

where:

$$B_\Omega = \boldsymbol{B} \otimes \boldsymbol{\Omega} = \begin{bmatrix} b_1\Omega_1^T & \cdots & b_1\Omega_n^T \\ \vdots & \ddots & \vdots \\ b_m\Omega_1^T & \cdots & b_m\Omega_n^T \end{bmatrix} \tag{7}$$

$$D_\Omega = D \otimes \boldsymbol{\Omega} = \begin{bmatrix} d\Omega_1^T & \cdots & d\Omega_n^T \end{bmatrix} \tag{8}$$

where $\otimes$ is the Kronceker product.

Note that this parameterization is not unique since for any arbitrary scalar β, the vectors $\beta\boldsymbol{B}$, βD and $\beta^{-1}\boldsymbol{\Omega}$ generate the same matrices B_Ω and D_Ω. Consequently, to provide a unique solution we set the first non-zero element of the vector $\boldsymbol{\Omega}$ to be positive and $||\boldsymbol{\Omega}|| = 1$, where $||\cdot||$ is the 2-norm.

B. Identification Algorithm

The objective is to identify 1) the vectors $\boldsymbol{\Omega}$ which represent the shape of the nonlinearity as a function of the SV and system input and 2) the linear state-space matrices A, C, $\boldsymbol{B}$ and d which can be used to build the *impulse response function* (IRF) model of the LTI system.

We use *multivariable output error state-space* (MOESP), a class of subspace identification algorithms, to estimate the state-space matrices $\hat{A}$ and $\hat{C}$ based on constructed input $\boldsymbol{U}(k)$ and measured output $\tilde{y}(k)$ [19]. Once these matrices are estimated, we can form the following data equation [20]:

$$\boldsymbol{Y} = \Psi\boldsymbol{\theta} \tag{9}$$

where $Y = [\tilde{y}(0) \cdots \tilde{y}(N-1)]$ and Ψ is the regressor matrix constructed based on known elements:

$$\Psi = \begin{bmatrix} 0 & \boldsymbol{U}^T(0) \\ \vdots & \vdots \\ \sum_{\tau=0}^{N-2} \boldsymbol{U}^T(\tau) \otimes \hat{C}\hat{A}^{N-2-\tau} & \boldsymbol{U}^T(N-1) \end{bmatrix} \tag{10}$$

Define:

$$\boldsymbol{bd} = \left[\boldsymbol{B}^T d\right]^T \tag{11}$$

The vector $\boldsymbol{\theta}$ of unknown parameters is:

$$\boldsymbol{\theta} = \boldsymbol{bd} \otimes \boldsymbol{\Omega} \tag{12}$$

Equation (12) shows that the elements of the unknown parameter vector $\boldsymbol{\theta}$ are the product of the elements of $\boldsymbol{bd}$ and $\boldsymbol{\Omega}$. To estimate the individual parameters of $\boldsymbol{bd}$ and $\boldsymbol{\Omega}$, we transform (9) into two standard least-squares problems.

If we hold $\boldsymbol{\Omega}$ fixed in (12), the output Y is a linear function of the parameters of $\boldsymbol{bd}$ as follows:

$$Y = \Psi_{\boldsymbol{\Omega}} \boldsymbol{bd} \tag{13}$$

where the new regressor $\Psi_{\boldsymbol{\Omega}}$ is:

$$\Psi_{\boldsymbol{\Omega}} = \Psi \begin{bmatrix} D_{\boldsymbol{\Omega_1}} & 0 \\ \vdots & 0 \\ D_{\boldsymbol{\Omega_n}} & 0 \\ 0 & \boldsymbol{\Omega} \end{bmatrix}, \; D_{\boldsymbol{\Omega_i}} = \begin{bmatrix} \boldsymbol{\Omega_i} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \boldsymbol{\Omega_i} \end{bmatrix} \tag{14}$$

Similarly, if $\boldsymbol{\Omega}$ is held fixed, the output Y is a linear function of $\boldsymbol{bd}$:

$$Y = \Psi_{\boldsymbol{bd}}\boldsymbol{\Omega} \tag{15}$$

where the new regressor $\Psi_{\boldsymbol{bd}}$ is:

$$\Psi_{\boldsymbol{bd}} = \begin{bmatrix} D_{\boldsymbol{bd}} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & D_{\boldsymbol{bd}} \end{bmatrix}, \; D_{\boldsymbol{bd}} = \begin{bmatrix} b_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & b_1 \\ \vdots & \vdots & \vdots \\ b_m & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & b_m \\ d & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & d \end{bmatrix} \tag{16}$$

Algorithm: The following algorithm uses (13) and (15) to estimate the unknown parameters $\boldsymbol{bd}$ and $\boldsymbol{\Omega}$. The algorithm is iterative so variables are indexed according to the iteration number j.

(i) Initialization:
Let $j = 1$ and $\hat{\boldsymbol{\Omega}}(0) = [1, \cdots, 1]^T_{n\times 1}$.

(ii) Construct the matrix $\Psi_{\hat{\Omega}(j-1)}$ using (14).

(iii) Estimate $\boldsymbol{bd}$ by solving the least-squares problem in (13):

$$\hat{bd}(j) = \left(\Psi_{\hat{\Omega}(j)}\right)^{\dagger} \boldsymbol{Y} \tag{17}$$

where $\dagger$ is the pseudo inverse.

(iv) Construct the matrix $\Psi_{\hat{bd}(j)}$ using (16).

(v) Estimate $[\omega_1, \cdots, \omega_n]^T$ by solving the least-squares problem in (15):

$$\hat{\boldsymbol{\Omega}}(j) = \left(\Psi_{\hat{bd}(j)}\right)^{\dagger} \boldsymbol{Y} \tag{18}$$

(vi) Let s be the sign of first non-zero element of $\hat{\boldsymbol{\Omega}}(j)$:

$$s = \operatorname{sgn}\left(\hat{\omega}_1(j)\right) \tag{19}$$

Then, perform the normalization:

$$\begin{aligned} \hat{bd}(j) &\leftarrow \hat{bd}(j)s\left\|\hat{\boldsymbol{\Omega}}(j)\right\| \\ \hat{\boldsymbol{\Omega}}(j) &\leftarrow \frac{\hat{\boldsymbol{\Omega}}(j)s}{\left\|\hat{\boldsymbol{\Omega}}(j)\right\|} \end{aligned} \tag{20}$$

(vii) Compute the *sum of squared error* (SSE) between the predicted output and the measured output.

(viii) Terminate if SSE satisfies the following condition; otherwise replace j by $j + 1$ and go to step (ii).

$$\frac{\mathrm{SSE}(j) - \mathrm{SSE}(j-1)}{\mathrm{SSE}(j-1)} \leq \text{threshold} \tag{21}$$

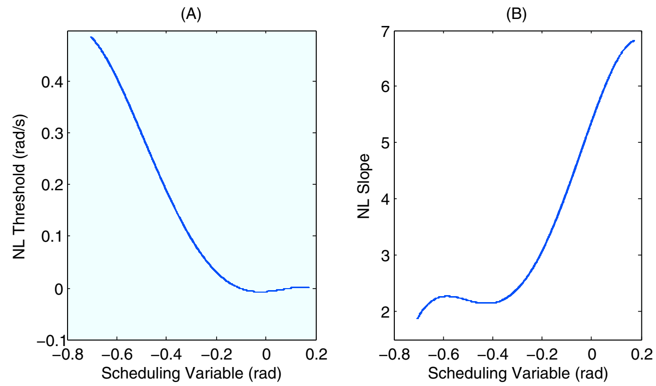

Fig. 2. Properties of the nonlinearity used in the simulation studies as a function of scheduling variable. (A) Threshold (B) Slope.

III. Simulation Results

We simulated a TV Hammerstein model of reflex stiffness during an imposed walking task. The model is an extension of TI small signal model of ankle reflex stiffness described in [11], which was a cascade of a nonlinearity and a second-order low-pass filter. The nonlinearity was represented by a threshold and a gain. Mirbagheri et al. [11] showed that the reflex gain was highly modulated with the joint position whereas damping and natural frequency of the linear system were almost constant. Jalaleddini and Kearney [12] showed that the threshold is also modulated with operating point. Therefore, in the TV model, we simulated a fixed linear component, and a static nonlinearity where the threshold and gain change as a function of ankle trajectory during walking. Consequently, the SV ($\rho(k)$) was set to the walking trajectory recorded from a subject walking on a treadmill at 3 km/h [21]. We formulated the simulated nonlinearity as follows:

$$\begin{aligned} z(k) &= G_r(\rho(k))\frac{\Delta u(k) + \Delta u(k)\operatorname{sgn}\left(\Delta u(k)\right)}{2} \\ \Delta u(k) &= (u(k) - \epsilon(\rho(k))) \end{aligned} \tag{22}$$

where ϵ is the TV threshold and G_r is the TV reflex gain. Fig. 2 shows variation of the threshold and gain as a function of SV.

The linear system had TI second-order low-pass dynamics:

$$H(s) = \frac{w^2}{s^2 + 2s\zeta w + w^2} \tag{23}$$

where $w = 25$ (rad/s) is the natural frequency and $\zeta = 0.9$ is the damping.

We simulated this model in MATLAB Simulink for 60s with sampling frequency of 1kHz. Joint velocity was the input and reflex torque was the output. The input perturbation was a pseudo random multilevel sequence low-pass filtered at 30Hz to mimic the dynamics of the hydraulic actuator that is used to drive the ankle joint in real experiments. We computed the joint velocity, the input of the Hammerstein system, by numerical differentiation of the position. A realization of a white Gaussian noise was added to the output

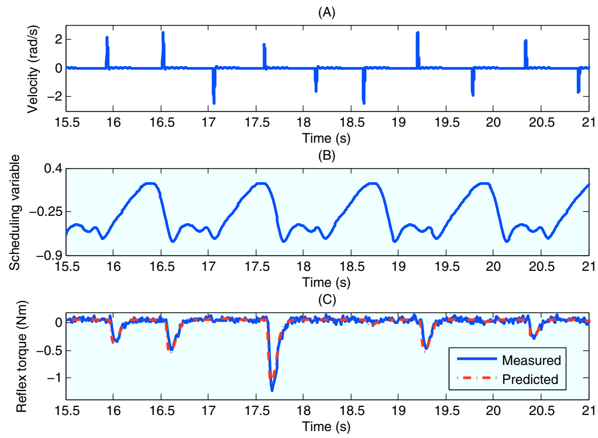

Fig. 3. Simulated signals: (A) perturbation velocity: input signal, (B) walking cycle: scheduling variable, (C) measured and predicted output.

and scaled to give signal to noise ratio of 5dB. We used Chebychev polynomials for basis function expansions.

Fig. 3 shows the input and output signals, the SV and the output predicted by the LPV model identified using the method developed in this paper. Inspection of Fig. 3 reveals that the reflex torque is modulated by the SV where the response to a perturbation at a plantarflexed segment of the walking trajectory (around 16s) is considerably smaller than that observed at a dorsiflexed segment (around 17.5s). The LPV model accurately predicted 97% of the variations of the true system's output.

The identification results are summarized in Fig. 4. Panel (A) shows the estimated static nonlinearity as a function of the SV and system input. The amplitude of the estimated nonlinearity significantly increased as the ankle was dorsiflexed. This can be associated with an increase in the gain of the nonlinearity and/or a decrease in its threshold towards dorsiflexion. This is consistent with the simulated gain and threshold of the nonlinearity depicted in Fig. 2. Furthermore, panel (B) shows that the estimation error of the 2-dimensional (2D) nonlinearity was relatively small. Panel (C) shows the estimated and true IRF of the LTI component of the TV Hammerstein system. There is an excellent agreement between the two.

IV. Experimental Validation

We performed a pilot experiment with a male subject with no history of ankle pathologies. He gave informed consent to the experimental procedures which had been reviewed and approved by McGill University's institutional review board. The subject lay supine with the left foot attached by a custom built fiberglass boot to the pedal of an electro-hydraulic actuator operating as a very stiff position servo [11]. We used a perturbed imposed walking paradigm where the actuator moved the ankle along a trajectory similar to that observed during normal gait superimposed by *Pseudo Random Binary Sequence* (PRBS) perturbations. The peak-to-peak amplitude of PRBS was set to 0.02 rad. The perturbed imposed walking trial lasted 70 seconds. The subject was instructed to maintain a constant voluntary plantarflexion contraction (i.e., push on the pedal) aided by visual feedback of a linear combination of the *processed* EMG of the three TS muscle heads (Medial and Lateral Gastrocnemius, and Soleus).

Joint angular position was measured using a potentiometer mounted in parallel with the actuator; the mid position of the ankle was taken as zero, dorsiflexed displacement taken as positive and plantarflexed as negative. Joint velocity was numerically computed from the recorded position. EMG signals were pre-amplified by a gain of 1000 and high-pass filtered at 20 Hz to remove artifacts. Signals were anti-alias filtered and then sampled at 1kHz. We applied the developed LPV identification method to experimental data where joint velocity was the input, Soleus EMG was the output and the walking trajectory was the SV.

Fig. 5 shows the results of the identification. Panel (A) depicts the estimated 2D nonlinearity which shows unidirectional sensitivity to joint velocity which is consistent with small signal identification results [1]. Moreover, it shows that the gain is modulated by the walking trajectory. Specifically, it is bigger in the dorsiflexed position compared to the plantarflexed by a factor of almost 4. This trend in the reflex gain is also consistent with that of small signal identification at multiple operating points (OP) [11]. However, estimating a nonlinearity such as the one in Fig. 5 using OP-based identifications, requires a large number of trials. Moreover, it is not always analytically valid to interpolate local models at each operating point to represent the global TV behavior.

V. Discussion

We presented a novel method for LPV identification of a class of TV Hammerstein systems with TV nonlinearity followed by an LTI dynamic component. We demonstrated that the developed method is very accurate and robust to measurement noise using simulations of a TV Hammerstein model of reflex stiffness. Proof-of-principle experiments verified the utility of the method for identification of a TV Hammerstein model of the stretch reflex EMG response modulated by changes in joint position during an imposed walking task. Experimental validation was performed for reflex EMG rather than reflex stiffness since reflex torque is not directly available for measurements (i.e., it is shadowed by voluntary and intrinsic torques that are presumably time-varying as well).

The LPV model structure developed in this paper can be used to represent TV Hammaerstein systems with static nonlinearity that is functionally related to a TV signal. This corresponds to a relatively wide set of physiological systems and functional tasks such as biomechanics of musculoskeletal system during movement (e.g., joint dynamics during posture and gait, upper arm end-point stiffness in reaching and pointing tasks). Moreover, the proposed identification method estimates the elements of a TV Hammerstein system using data from only a *single* trial. This is a significant practical advantage over the OP-based methods.

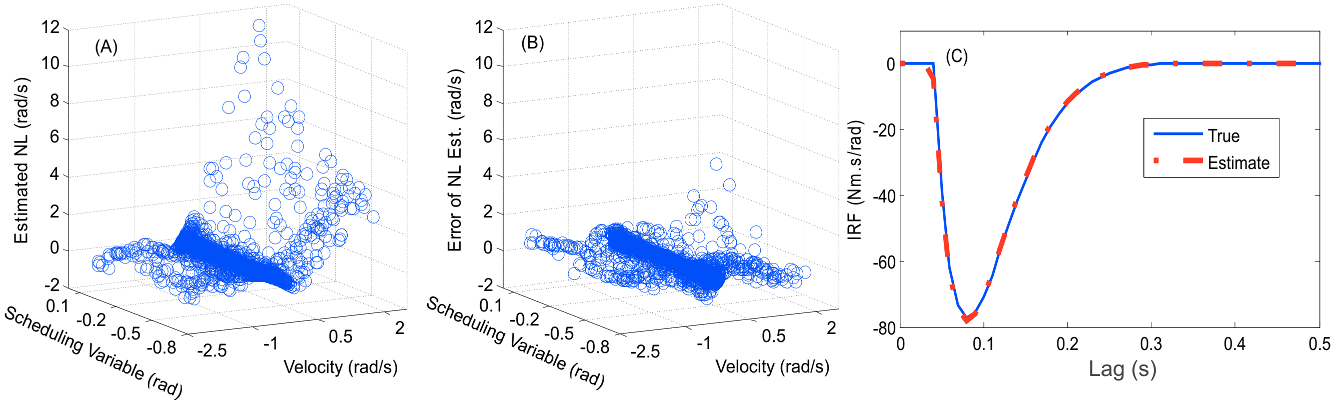

Fig. 4. Identified LPV Hammerstein model of reflex stiffness from simulated data: (A) estimated static nonlinearity, (B) error in the estimation of the nonlinearity, (C) identified and true IRF.

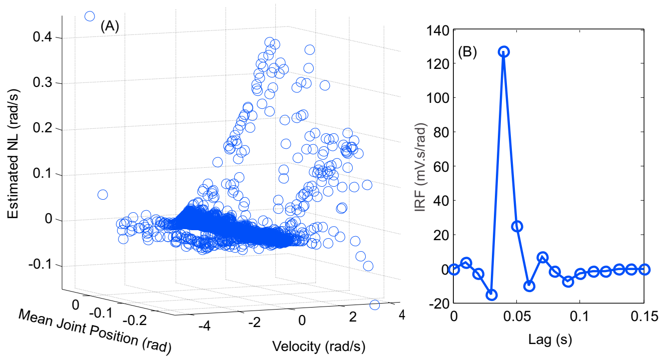

Fig. 5. Identified LPV Hammerstein model of Soleus reflex EMG from experimental data: (A) estimated 2D static nonlinearity and (B) identified IRF.

To identify TV behavior of dynamic joint stiffness defined as the summation of intrinsic and reflex stiffness, we need to extend the developed methodology to a TV parallel-cascade (TVPC) structure. TVPC consists of a parallel combination of a TV Hammerstein system for reflex pathway and TV linear dynamics for intrinsic stiffness. This is a subject of future work.

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