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Magnetic Resonance Image Restoration via Dictionary Learning under Spatially Adaptive Constraints

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Abstract— **This paper proposes a spatially adaptive constrained dictionary learning (SAC-DL) algorithm for Rician noise removal in magnitude magnetic resonance (MR) images. This algorithm explores both the strength of dictionary learning to preserve image structures and the robustness of local variance estimation to remove signal-dependent Rician noise. The magnitude image is first separated into a number of partly overlapping image patches. The statistics of each patch are collected and analyzed to obtain a local noise variance. To better adapt to Rician noise, a correction factor is formulated with the local signal-to-noise ratio (SNR). Finally, the trained dictionary is used to denoise each image patch under spatially adaptive constraints. The proposed algorithm has been compared to the popular nonlocal means (NLM) filtering and unbiased NLM (UNLM) algorithm on simulated T1-weighted, T2-weighted and PD-weighted MR images. Our results suggest that the SAC-DL algorithm preserves more image structures while effectively removing the noise than NLM and it is also superior to UNLM at low noise levels.**

I. INTRODUCTION

Magnetic resonance (MR) imaging has been widely used in diagnoses, follow-up, and in-vivo research, due to its non-invasive and non-radiation nature and its ability to produce high quality images. However, MR imaging generally suffers from several degradation factors, such as the bias filed, partial volume effect and noise. Particularly, a major issue accompanying MR imaging is its limitation in practical scanning time, since increased scan duration may introduce problems like patient discomfort and physiological motion artifacts. This limitation leads to a trade-off between the signal-to-noise ratio (SNR) and image resolution. As reported in [1], [2], [3], due to random fluctuations in the patient body or receiving coil electronics, thermal noise is one main source that competes with the nuclei MR signal. The corruption of MR images may lead to a major hurdle on the path from the acquisition to the interpretation of MR images. Therefore, noise reduction is a necessity for obtaining MR images of the required quality.

Commonly, MR imaging obtains structural and functional information via the inverse Fourier transform of the K-space data. The signal component of measurements thus usually consists of real and imaginary channels, each of which is corrupted with zero mean additive Gaussian noise. Since the magnitude of the reconstructed MR images is widely used in clinical practice and research settings, in this research we restrict our attention to reducing the noise in MR magnitude images. Computing a magnitude image from the corresponding real and imaginary parts maps the Gaussian noise into the Rician noise [4]. The Rician noise is signal-dependent and follows a Rayleigh distribution in low intensity regions and a Gaussian distribution in high intensity regions [2]. Reduced image contrast is the practical consequence.

To overcome the deficiencies introduced by this noise, a number of denoising methods have been proposed in recent years, including wavelet-based methods [2], [3], [5] and nonlocal filtering methods [6], [7], [8]. Most wavelet-based denoising methods consist of three steps: (a) applying the discrete wavelet transform (DWT) to the noisy MR image; (b) filtering the wavelet coefficients to reduce the noise; and (c) reconstructing the denoised signal using the inverse wavelet transform. The nonlocal means (NLM) filtering technique explores the similarity between noisy patches and utilizes the redundancy in the image for denoising. One of the most state-of-the-art Rician noise removal techniques is the unbiased NLM (UNLM) algorithm [9]. As observed by Nowak [10], the noise bias in squared magnitude image is no longer signal-dependent and equals to $2\sigma^2$, where σ^2 is the noise variance in each channel (i.e. real or imaginary channel). The UNLM algorithm estimates each pixel value in the squared magnitude space and then uses a simple bias subtraction scheme to correct the bias. Traditional Rician noise removal algorithms have been proved to be capable of removing noise effectively. However, they are less successful in simultaneous noise reduction and structure preservation, due to their inherent lack of the ability to capture detailed image information.

Motivated by the fact that dictionary learning techniques can robustly capture the local structures in each image patch, we propose in this paper a spatially adaptive constrained dictionary learning (SAC-DL) algorithm for noise reduction

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in MR magnitude images. This algorithm consists of four major steps. First, the magnitude image is separated into a number of partly overlapping image patches. Second, the statistics of these patches are collected and analyzed to obtain a local noise variance for each patch. Third, a correction factor is formulated according to the local SNR to make our algorithm better adapt to the Rician noise. Fourth, the trained dictionary is used to denoise each image patch under spatially adaptive constraints. The proposed SAC-DL algorithm has been compared to the popular NLM and UNLM algorithms on simulated T1-weighted, T2-weighted and PD-weighted MR images.

II. THEORY

A. Rician Noise Model

The thermal noise N in MR images is commonly modeled as a complex random process, whose real and imaginary parts are independent zero mean Gaussian random variables with a standard deviation of σ. Thus, the complex MR image $\widetilde{I}_c$ acquired by the MR imaging system can be modeled as

$$\widetilde{I}_c(i,j) = I_c(i,j) + N(i,j) \tag{1}$$

where $I_c(i,j)$ is the magnetization distribution and $N(i,j)$ is the noisy component at pixel (i,j). The corresponding magnitude image is

$$\begin{aligned} X(i,j) &= \left|\widetilde{I}_c(i,j)\right| \\ &= \sqrt{\left(I_r(i,j)+N_r(i,j)\right)^2 + \left(I_i(i,j)+N_i(i,j)\right)^2} \end{aligned} \tag{2}$$

where the subscripts r and i highlight the real and imaginary parts, respectively. Generally, $N_r(i,j)$ and $N_i(i,j)$ are assumed to be independent and identically distributed with zero mean and standard deviation σ. The intensity of each pixel in the magnitude image $|\widetilde{I}_c(i,j)|$, or equally $X(i,j)$, has the following probability density function

$$f_{X(i,j)}(X) = \frac{X}{\sigma^2} \exp\left(-\frac{X^2 + |I_c(i,j)|^2}{2\sigma^2}\right) I_0\left(\frac{|I_c(i,j)|X}{\sigma^2}\right) \tag{3}$$

where

$$|I_c(i,j)| = \sqrt{I_r(i,j)^2 + I_i(i,j)^2} \tag{4}$$

denotes the noise-free image and I_0 denotes the Bessel function of the first kind at zero-th order. Therefore, the Rician-noise-corrupted MR image can also be formulated as follows in the magnitude image space [7]

$$X(i,j) = \sqrt{\left(|I_c(i,j)| + N_1(i,j)\right)^2 + N_2(i,j)^2} \tag{5}$$

where $N_1(i,j)$ and $N_2(i,j)$ are random numbers generated from two sets of Gaussian distributions, which have zero mean and the identical standard deviation of σ.

B. Dictionary Learning

We aim to estimate the noise-free magnitude image $|I_c(i,j)|$ via denoising the observed magnitude image $X(i,j)$. To this end, we first separate the magnitude image into a number of L partly overlapping image patches. Suppose R_l is an extraction matrix, the l-th extracted patch can be formally written as $R_l X$. Let D represent the overcomplete dictionary and α_l denote the coefficient vector for sparse representation of the patch $x_l = R_l X$. Then, we have

$$x_l \approx D\alpha_l \quad \text{s.t. } \|\alpha_l\|_0 \leq \mathrm{T}_1,\ l = 1,2,...,L \tag{6}$$

where $\|\cdot\|_0$ counts the number of non-zero elements and T_1 is a positive constant. With all image patches being considered, we obtain the following objective function

$$\min_{D,\Gamma} \sum_l \|x_l - D\alpha_l\|_2^2 + \lambda\|\alpha_l\|_0,\ l = 1,2,...,L \tag{7}$$

where Γ is the collection of the sparse coefficients. The first term enforces the data fidelity between the extracted patch and the sparse representation. The second term emphasizes the sparsity of the representation of each image patch. The parameter λ balances the contribution of these two terms.

As shown in Eq. (3) , the Rician noise is signal-dependent and not additive noise. Traditional dictionary learning approaches, such as the classical K-SVD algorithm [11] and augmented Lagrangian algorithms [12], [13], [14] were developed mainly for removing the Gaussian noise, and hence may not be suitable for solving this problem. Therefore, particular constraints should be incorporated into the dictionary learning technique to make it adaptive to Rician noise removal.

C. Rician Noise Adaptation Scheme

Since the Rician noise follows a Rayleigh distribution in low intensity regions and a Gaussian distribution in high intensity regions [2], the objective given in Eq. (7) should be optimized with different constraints, which adapt to the intensity levels of each image patch. To this end, the local noise variance in each patch is formulated for the constraint, namely

$$\|x_l - D\alpha_l\|_2^2 \leq \mathrm{T}_2 {\sigma_l}^2 \tag{8}$$

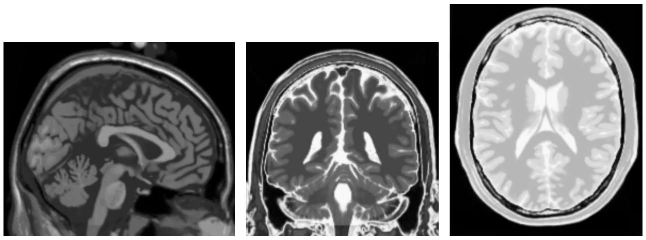
where T_2 is a positive weighting constant and σ_l is the local noise standard deviation in the l-th image patch. To estimate the local noise variance, we caculate the expectation of the squared Euclidean distance of two noisy patches x_l and $x_{l'}$ as follows

$$E(d(x_l, x_{l'})) = E\big(\|x_l - x_{l'}\|_2^2\big) = \|x_{lo} - x_{l'o}\|_2^2 + 2\sigma^2 \tag{9}$$

where the subscript o denotes the noise-free image. Therefore, $E(d(x_l, x_{l'})) = 2\sigma^2$ if $x_{lo} = x_{l'o}$. With the assumption that each patch in the image has another equal image patch [15], the noise variance can be given as follows

$$\sigma_l^2 = \min\left(d(x_l, x_{l'})\right)/2. \tag{10}$$

However, in case of the Rician noise, the noise variance may be underestimated in regions with low intensities. To



(a) (b) (c)

Fig. 1. Three brain MR images from the BrainWeb database: (a) T1-weighted MR image; (b) T2-weighted MR image, (c) PD-weighted MR image.

attack this problem, the noise variance is corrected [16] according to the following rule

$$\widehat{\sigma}_l^2 = \sigma_l^2/\xi(SNR) \tag{11}$$

where $\widehat{\sigma}_l^2$ is the corrected local noise variance in the l-th image patch. The SNR is counted as $SNR = \mu/\sigma$, μ is the mean value of each patch, and

$$\begin{aligned} &\xi(SNR) \\ =&2 + SNR^2 - \frac{\pi}{8}\exp\left(-\frac{SNR^2}{2}\right) \\ &\times\left((2+SNR^2)I_0\left(\frac{SNR^2}{4}\right) + SNR^2 I_1\left(\frac{SNR^2}{4}\right)\right)^2 \end{aligned} \tag{12}$$

where I_0 and I_1 are the zero-th and first order modified Bessel functions of the first kind, respectively. The stopping criterion is now updated as

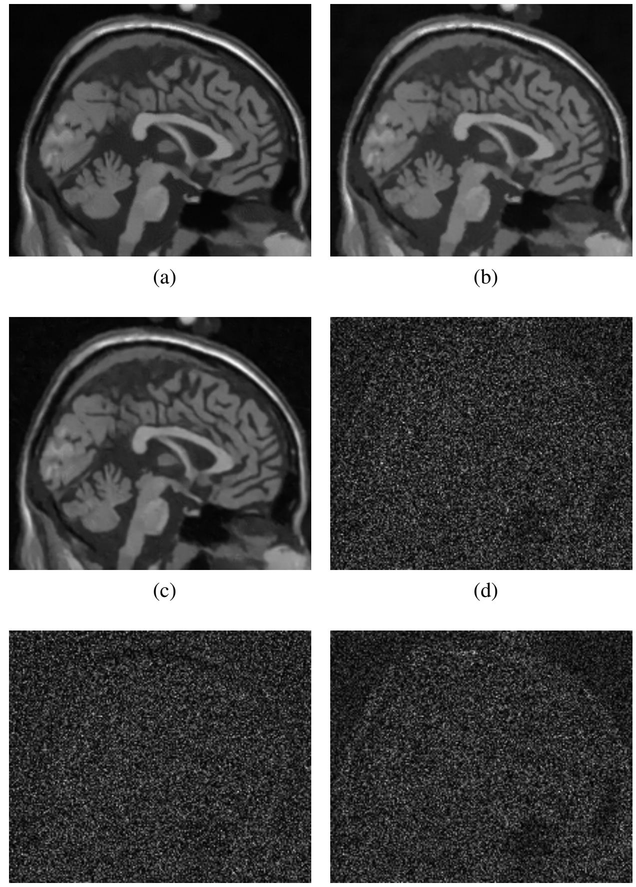
$$\|x_l - D\alpha_l\|_2^2 \le \mathrm{T}_2\widehat{\sigma}_l^2 \quad \text{s.t. } \|\alpha_l\|_0 \le \mathrm{T}_1,\ l = 1, 2, ..., L. \tag{13}$$

After obtaining the sparse representation $D\alpha_l$ for each patch x_l, each denoised pixel value is calculated as the average of restored values at the corresponding location in all related patches.

III. EXPERIMENTS AND RESULTS

The proposed SAC-DL algorithm has been compared to two widely-used denoising techniques, i.e. the NLM and unbiased UNLM [9], on simulated brain MR studies obtained from the BrainWeb database at the McConnell Brain Imaging Center of the Montreal Neurological Institute, McGill University, Montreal, Canada [17]. Each study has a dimension of $181 \times 217 \times 181$ and a voxel size of $1 \times 1 \times 1$ mm^3. Three reference 2D MR slices were shown in Fig. 1. The T1-weighted MR slice (left column) has a dimension of 217×181, the T2-weighted MR slice (middle column) has a dimension of 181×181, and the PD-weighted MR slice (right column) has a dimension of 181×217. The test images were corrupted with five levels of noise, namely, $1\%, 3\%, 5\%, 7\%$ and 9%.

Applying the SAC-DL algorithm to an $H \times$W MR image restoration consists of five steps: (1) extract $(W-4)(H-4)$ image patches from the corrupted images by gliding a 5×5 window with a sliding distance of one pixel over the entire image; (2) calculate and remove the direct current of each patch using a low-pass filter; (3) calculate the corrected local noise variance according to Eq. (11); (4) train the dictionary with the adaptive constraints for denoising each patch; and (5) reconstruct the intensity value of each pixel and add the corresponding direct current back to it. The NLM algorithm and unbiased UNLM algorithm were implemented with their default parameters.



(a) (b)

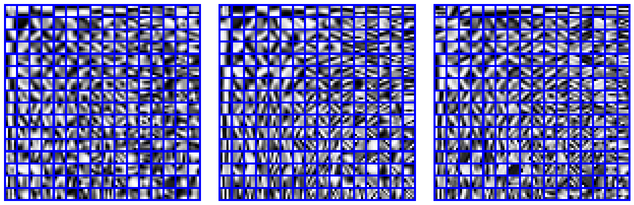
(c) (d)

(e) (f)

Fig. 2. The ability of the three algorithms in restoring image details. The test image used is the T1-weighted image which is contaminated by 9% Rician noise. The restored image gained by (a) NLM algorithm; (b) UNLM algorithm and (c) the proposed SAC-DL algorithm. The absolute difference between the denoised image and the noisy image, gained by (d) NLM algorithm; (e) UNLM algorithm and (f) the proposed SAC-DL algorithm.

First, we applied each of these three algorithms to the T1-weighted brain MR image that was corrupted with 9% Rician noise. The noise level is calculated relative to the brightest tissue. The restored image obtained by each algorithm and the absolute difference between the denoised image and the noise-corrupted image were displayed in Fig. 2. It reveals that there exists somewhat blurring artifacts on the edges in the result of the NLM algorithm. The absolute difference maps show that both the UNLM algorithm and SAC-DL algorithm removed more noise than the NLM algorithm did.

Next, we perfomed the experiment on both the PD-



(a) (b) (c)

Fig. 3. The final trained dictionaries by SAC-DL from 1% noise corrupted (a) T1-weighted image, (2) T2-weighted image, (3) PD-weighted image.

TABLE I
A SUMMARY OF THE DENOISED RESULTS IN STRUCTURAL SIMILARITY (SSIM) GAINED BY THE THREE METHODS.

Image	PD-weighted image				
Noise level	1	3	5	7	9
NLM	0.9376	0.8298	0.7591	0.6976	0.6460
UNLM	0.9367	0.8305	0.7755	0.7244	0.6807
SAC-DL	0.9381	0.8496	0.7925	0.7211	0.6737
Image	T2-weighted image				
Noise level	1	3	5	7	9
NLM	0.9647	0.8888	0.8304	0.7760	0.7306
UNLM	0.9697	0.8942	0.8430	0.7954	0.7559
SAC-DL	0.9610	0.8964	0.8407	0.7879	0.7377

weighted and T2-weighted brain MR images corrupted by different levels of noise. We quantitatively assess the quality of restored images using the structural similarity (SSIM) index, which has been shown to outperform the widely-used peak SNR (PSNR) in measuring the quality of natural images across a wide variety of distortions [18]. To measure the quality of a distorted image, the SSIM index compares the correlations in luminance, contrast and structure, locally. These differences between the reference and distorted images are averaged over the entire image. It ranges from 0 to 1, with higher score representing better image quality. The denoising performances of three algorithms were compared in Table 1. In accordance with the previous visual comparison, this experiment demonstrates again that the UNLM algorithm and SAC-DL algorithm produced better results than the NLM algorithm. Furthermore, Table 1 also shows that in case of heavy noise the UNLM algorithm outperforms the proposed algorithm; whereas, when the noise level is low, our SAC-DL algorithm performs better than the UNLM algorithm.

Finally, the final dictionaries trained by our algorithm for denoising three test images (i.e. T1-weighted, T2-weighted and PD-weighted) with 1% Rician noise were displayed in Fig. 3. It shows that a lot of edges and structure information have been captured in the learned dictionary. It can also be observed that the dictionaries trained from the three images vary a bit due to their distinctive anatomical information.

IV. CONCLUSIONS

This paper proposes the SAC-DL algorithm for Rician noise removal in magnitude magnetic resonance images. This algorithm combines the strength of dictionary learning technique and local noise estimation scheme. The dictionary learning technique enables it to capture local image details from each patch and thus preserve image features. The local noise estimation scheme makes the denoising process adaptive to the Rician noise. The experimental results demonstrated that the proposed SAC-DL algorithm can effectively remove noise while keeping the important image details. In our future research, we will further investigate more effective noise estimation and the optimal parameter setting that can adapt to distinctive SNR regions and multi-component MR image structures.

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