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Isotropic Anomalous Filtering in Diffusion-Weighted Magnetic Resonance Imaging

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***Abstract*— Noise is inherent to Diffusion-Weighted Magnetic Resonance Imaging (DWI) and noise reduction methods are necessary. Although process based on classical diffusion is one of the most used approaches for digital image, anomalous diffusion has the potential for image enhancement and it has not been tested for DWI noise reduction. This study evaluates Anomalous Diffusion (AD) filter as DWI enhancement method. The proposed method was applied to magnetic resonance diffusion weighted images (DW-MRI) with different noise levels. Results show better performance for anomalous diffusion when compared to classical diffusion approach. The proposed method has shown potential in DWI enhancement and can be an important process to improve quality in DWI for neuroimage-based diagnosis.**

I. INTRODUCTION

The level of noise found in medical images are often harmful for the diagnosis. One solution for improving the signal is apply spatial filters that are widely used in several procedures of analysis and digital image processing. One of the most well known smoothing filters is based on Gaussian function and can be generated by solving the isotropic diffusion equation given as

$$\frac{\partial I(x,y,t)}{\partial t} = D.\nabla^2 I(x,y,t) \qquad (1)$$

Where ∂t is the partial derivative in time, D is the diffusion coefficient, ∇ is the Laplacian operator and $I(x,y,t)$ is the image function. The classical diffusion paradigm assumes that the transport medium is homogeneous and isotropic, i.e. non exist preferential smoothing direction, all orientations has the same filtering intensity. However, in a more restrictive approach, we consider the local inhomogeneities present in many structures of nature such as the geometric composition of the brain and myelination level of neurons in white matter. For this reason we can not continue with a isotropic filter that has no assumptions for the neighborhood properties. The anomalous distribution model, with q-Gaussian curves, may be useful to add the inhomogeneity information in spatial filtering. Here we are using the anomalous equation (AD) given by (2).

$$\frac{\partial I(x,y,t)}{\partial t} = D_q.\nabla^2 I(x,y,t)^{2-q} \qquad (2)$$

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Where q is known as anomalous parameter. Note that for $q = 1$ we return to the classical paradigm given in (1), so the equation (2) is a generalization of the classical paradigm.[1] The anomalous distributions has the your variance proportional with time that follows a power law of type t^μ ($\sigma^2 \propto t^\mu$). When we correlate the time power law with the anomalous parameter q, we obtain $\mu = 2/(3-q)$.[2] For $\mu = 1$ we return to the classical Brownian motion that explain the classical behavior of Gaussian probability distribution. There are several applications of the equation (2) such as fluid transport in porous media [3], MRI relaxometry in liquids [4] and many other applications in medicine.[5], [6], [7]

The properties of q-Gaussian probability distribuition, which arise from the solution of equation (2), has an interesting behavior when we study the influence of each pixel at all filtering process. We can assume that the information of the neighborhood has more or less restricted influence that depends the value of q parameter. These restrictions help in preserving boundary edges, i.e. the AD filter itself can smooth the noise while preserving the edges of objects without any anisotropic adoption. See Figure 1 for more details of q-Gaussian probability distribuitions.

The diffusion weighted images in MRI (DW-MRI) have larger noise level at acquisition. The use of techniques for rapid acquisition (EPI), intense field gradients for prolonged periods, external field inhomogeneity and patient movement are the most important components of noise in DW-MRI. It is needed smoothing processes to improve diagnosis. It is often used to mean successive noise attenuation to improve quality in DW-MRI images, i.e. to achieve a higher quality image should make repeated aquisitions from the same region and, soon after, make the average of these images. However, a high number of averages (N) results in a long acquisition time and causing discomfort to the patient.

DW-MRI is widely used for studies in neuropathology and neurological disorders such as Alzheimer's, Parkinson's, Epilepsy and other diseases.[8], [9], [10] Usually it is used to quantification methods which is obtained the measure of apparent diffusion coefficient (ADC). The ADC is sensitive to noise in acquiring process therefore smoothing methods can be useful. Others methods have been proposed to improve the DWI diagnostic images. The various filtering algorithms use statistical, morphological or multi modalities approaches [11], [12], [13], [14], [15], [16]. In all methods there is the concern to maintaining the edges with the maximum noise smoothing.

In this study we applied the anomalous spatial filters as

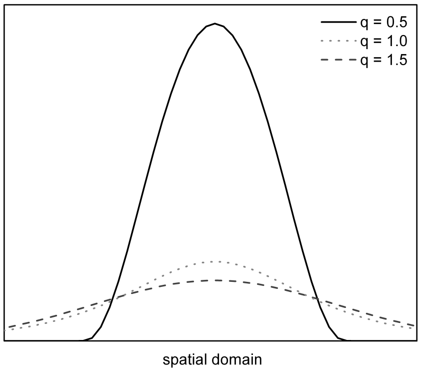

Fig. 1. q-Gaussian generated with the equation (3). With $q < 1$ we obtain a q-Gaussian distribution with finite support, so the q-Gaussian curve has delimited values in the spatial domain. For $1 < q < 2$ we obtain a q-Gaussian with infinite support that has distribution with all the spatial domain. The function discretization, given by equation (3), we must take care for $q < 1$ because of the numerical precision. For the range of $1 < q < 2$ we can simulate the q-Gaussian distribuition without numerical precision problem. All the curves are normalized.

enhancement in DW-MRI imaging with the objective of improving images with lower acquisition times. We propose that the anomalous spatial filtering can be used with a DW-MRI image with a small number of aquisitions.

II. METHODS

A. Spatial Filter

The numerical solution of equation (2) arises with the use of differential operators by finite difference. It is used the first and second numerical order derivative for the time and space discretization, respectively. The equation (3) illustrated the discretization of partial differential equation given in (2)

$$I_{t+1,\beta} = I_{t,\beta} + D_q.(I^{2-q}_{t,\beta+1} - 2.I^{2-q}_{t,\beta} + I^{2-q}_{t,\beta-1}) \qquad (3)$$

For simplicity, $I^{2-q}_{t,\beta}$ represents the image at time t according locating a 3×3 neighborhood of the center pixel with anomalous parameter q. The parameter β informs the spatial position of the neighbor relative with the central pixel (North, South, Noth-West, etc). Anomalous diffusion applied to impulse function results in q-Gaussian, generalizing Gaussian function [17]. A representation of the q-Gaussian that can be generated with our method should be seen in the Figure 1.

The discretization used here is similar to that proposed by Perona e Malik with the anisotropic filtering.[18] It is used eight differential operators with a central pixel. Each derivative operator, both in space and in time, are made with first order finite difference (Truncation error of order $O(h)$). Of course, the fundamental difference between the AD filter proposed here and anisotropic filtering proposed by Perona and Malik is the not using the edge stop function $g(|\nabla I(x, y)|)$ as anisotropic maps for edges preservation.[18] In this study all the smoothing process is done with a fixed D_q value, which characterizes the isotropic diffusion.

B. DW-MRI imaging protocol

Images used in this study were acquired by an 3 Tesla (3T) MRI tomograph, echo planar image (EPI) technique, resolution of 0.90×0.90 mm per pixel and 5 mm spacing between planes. All images were acquired with a healthy volunteer. Were acquired 4 volumes with different aquisition time (N = 1,2,4 and 8). The images with $N = 8$ were used as reference for the quantitative methods and analysis with the other images.

C. Analysis methods

Signal-noise ratio (SNR) is used as a measure of quality for the anomalous spatial filters. The parameters of time t and diffusion coefficient D_q were fixed with $t = 5$ and $D_q = 1$. These values were chosen to generate a q-Gaussian with standard deviation suitable for smoothing with minimal distortion in the images.

Another method used to analyze the images was the structural similarity index (SSIM) [19]. This index is widely used in image processing to estimate the visual quality of digital images.

III. RESULTS AND DISCUSSION

When we apply the anomalous diffusion experessed by equation (3) in image processing must be made a cut-off in the q range. The discretization given in (3) limited to q values to $0 < q < 2$, due to instability of the algorithm and limitations of the q-Gaussian generated iteratively. For $q < 1$ we have an finite range for spatial domain, i.e. the q-Gaussian has a finite support, for $1 < q < 2$ we have a q-Gaussian with infinite support therefore a better algorithm stability. Even with instability for $q < 1$, we can obtain values of the diffusion coefficient that support the numerical equilibrium. Thus the method is well behaved for $0 < q < 2$.[1]

The characteristic separation between regions of q which have finite support or infinite support is useful for studying the effect of information from short- or long-range, respectively. If we take the curves for $q < 1$, where there a finite support, we are taking the weighting values for the central pixel in a neighborhood more restricted than for $q > 1$. These changes results in the rise effects seen in the graph of Figure 2.

The results suggest that for $q > 1$ we obtain an improvement of smoothing on all images. For $N = 4$ we find the greatest contrast between the results obtained with the Gaussian filter ($q = 1$) and the AD filter ($q = 1.5$). Similar results are found for other N values, with less contrast between the reference image and the test image. Even if the absolute difference between the values of SNR is relatively low, the visual effect reveals an enhancement easy to notice. See the images of Figure 3 to compare the results of the classic filter with the anomalous filter both with the same control parameters.

[1]Remember that all the q-Gaussian probability distribuition function are normalized, i.e. $\int_{-\infty}^{\infty} G_q(x)dx = 1$

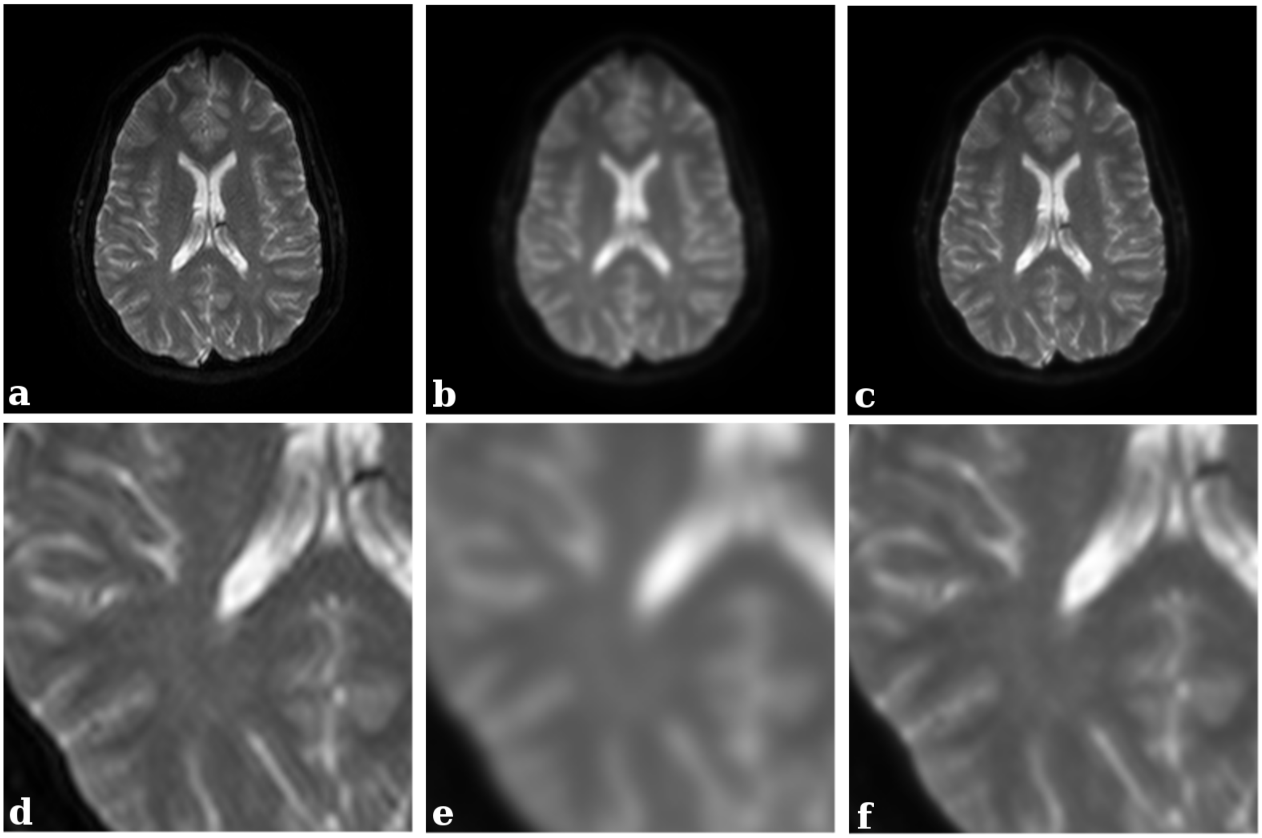

Fig. 3. Smoothing results from the AD filter. In a) DW-MRI image with N = 1, b) Result with q = 1, t = 20, D = 1, c) Results for q = 1.5, t = 20 and D = 1. The other images found in d), e) and f) are magnifications of the same a), b) and c) images, respectively. It is not difficult to notice that the Gaussian filter (q = 1) show an intense blurring on entire image. The opposite occurs for q = 1.5 that we can easily notice the edges preservation and an efficient smoothing effect. The improvement of the images tend to be better as N increases, see the graph in Figure 2 to compare the gain between images N = 1 and N = 4. Still, in all noise levels that the AD filter is suitable to preserve edges and smoothing images.

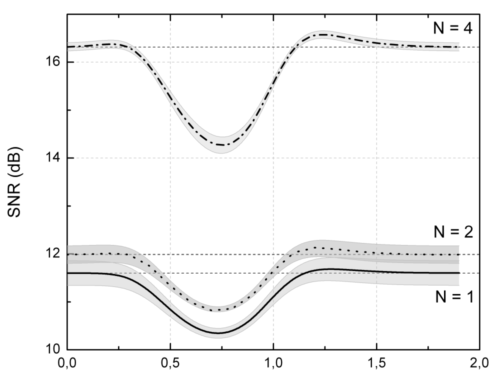

Fig. 2. Effect of the AD filtering on the DW-MRI images with N = 1, 2 and 4. The image with N = 8 was used as a reference for all other values of N and appears as a dotted line in each curve. Each curve is made with the mean value of the entire volume image acquired and the standard deviation is represented as a shaded region. Note that for values of $q = 1$ we obtain the results for the classical Gaussian filter. For $q \neq 1$ we obtain the q-Gaussian smoothing results. It is found for $q > 1$ a peak above the values found for $q = 1$ and the for the value found in the reference value (N = 8).

Image quality visually evaluated shown an improvement when the SSIM is measured. SSIM measure indicates that the anomalous filtering process keeps the structures shapes of the image, thereby the edges are preserved with the anomalous filtering process. Even for N = 4 the improvement is considerable when compared with the successive averages image (N = 8). Compare the results of SSIM with SNR curves of Figure 2. For N larger SSIM is also higher and further enhance the quality when using the filter anomalous. All the measure found in the Table I were made with $t = 20$ and $D = 1$. Compare the results found in the Table I with the classical filtering ($q = 1$) and the anomalous filtering ($q = 1.5$).

TABLE I

SSIM FOR THE DW-MRI VOLUMETRIC IMAGE.

		N=1	N=2	N=4
$q = 1$	$SSIM_{mean}$	0.7657	0.8006	0.8706
	$SSIM_{max}$	0.9044	0.9317	0.9899
$q = 1.5$	$SSIM_{mean}$	0.8634	0.8860	0.9777
	$SSIM_{max}$	0.9771	0.9894	0.9998

IV. CONCLUSION

AD filter applied to DWI shows that it is possible to improve the quality of images. With a simple algorithm of low computational cost and few control parameters was seen that the quality of AD processed images is higher than that of the classical Gaussian filter. The edges preservation found in AD method is useful to guarantee a better post processing visualization and analysis, which was not found in Gaussian filters. In many procedures of MRI analysis, smoothing filters are used to reduce the noise level. The AD filter proposed here shows robustness in smoothing and preserving edges. SNR gains in AD method may allow reductions in DWI acquisition time, preserving SNR levels. Its application in DW-MRI analysis is useful for diagnosis of neurodegenerative diseases such as Alzheimer's and Parkinson's disease.

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REFERENCES

[1] V. Schwammle, F. D. Nobre, and C. Tsallis, "q-Gaussians in the porous-medium equation: stability and time evolution," *European Physical Journal B*, vol. 66, no. 4, pp. 537–546, 2008.

[2] C. Tsallis, "Nonextensive statistical mechanics, anomalous diffusion and central limit theorems," *Journal of Mathematics*, pp. 145–176, 2005.

[3] B. Y. Li and W. W. Sun, "Heat-sweat flow in three-dimensional porous textile media," *Nonlinearity*, vol. 25, no. 2, pp. 421–447, Feb. 2012.

[4] C. Tsallis and D. J. Bukman, "Anomalous diffusion in the presence of external forces: Exact time-dependent solutions and their thermostatistical basis," *Physical review. E, Statistical physics, plasmas, fluids, and related interdisciplinary topics*, vol. 54, no. 3, pp. R2197—-R2200, 1996.

[5] H. T. Amaral-Silva, L. O. Murta, L. Wichert-Ana, A. C. Sakamoto, and P. M. Azevedo-Marques, "Medical image registration using TSallis Entropy in Statistical Parametric Mapping (SPM)." *Conference proceedings : ... Annual International Conference of the IEEE Engineering in Medicine and Biology Society. IEEE Engineering in Medicine and Biology Society. Conference*, vol. 2010, pp. 6276–9, Jan. 2010.

[6] P. S. Rodrigues and G. A. Giraldi, "Improving the non-extensive medical image segmentation based on Tsallis entropy," *Pattern Analysis and Applications*, vol. 14, no. 4, pp. 369–379, July 2011.

[7] P. R. B. Diniz, L. O. Murta-Junior, D. G. Brum, and A. C. Santos, "Brain tissue segmentation using q-entropy in multiple sclerosis magnetic resonance images." *Brazilian journal of medical and biological research = Revista brasileira de pesquisas médicas e biológicas / Sociedade Brasileira de Biofísica ... [et al.]*, vol. 43, no. 1, pp. 77–84, Jan. 2010.

[8] C. R. Jack, "Alzheimer Disease: New Concepts on Its Neurobiology and the Clinical Role Imaging Will Play," *Radiology*, vol. 263, no. 2, pp. 343–360, 2012.

[9] M. F. H. Schocke, K. Seppi, R. Esterhammer, C. Kremser, W. Jaschke, W. Poewe, and G. K. Wenning, "Diffusion-weighted MRI differentiates the Parkinson variant of multiple system atrophy from PD," *Neurology*, vol. 58, no. 4, pp. 575–580, 2002.

[10] S. Lehericy, M. A. Sharman, C. L. Dos Santos, R. Paquin, C. Gallea, and S. Lehéricy, "Magnetic resonance imaging of the substantia nigra in Parkinson's disease," *Movement Disorders*, vol. 27, no. 7, pp. 822–830, June 2012.

[11] S. Aja-Fernandez, M. Niethammer, M. Kubicki, M. E. Shenton, and C. F. Westin, "Restoration of DWI data using a Rician LMMSE estimator," *Ieee Transactions On Medical Imaging*, vol. 27, no. 10, pp. 1389–1403, 2008.

[12] M. Martin-Fernandez, E. Munoz-Moreno, L. Cammoun, J. P. Thiran, C. F. Westin, and C. Alberola-Lopez, "Sequential anisotropic multichannel Wiener filtering with Rician bias correction applied to 3D regularization of DWI data," *Medical Image Analysis*, vol. 13, no. 1, pp. 19–35, 2009.

[13] H. Bartsch, P. Maechler, and J. Annese, "Automated Determination of Axonal Orientation in the Deep White Matter of the Human Brain," *Brain connectivity*, p. undefined, 2012.

[14] K. M. Hasan, I. S. Walimuni, H. Abid, and K. R. Hahn, "A review of diffusion tensor magnetic resonance imaging computational methods and software tools," *Computers In Biology And Medicine*, vol. 41, no. 12 Special Issue: SI, pp. 1062–1072, 2011.

[15] A. W. A. Zhaohua Ding, Z. Ding, J. C. Gore, and A. W. Anderson, "Reduction of Noise in Diffusion Tensor Images Using Anisotropic Smoothing," *Magnetic Resonance in Medicine*, vol. 53, no. 2, pp. 485–490, Feb. 2005.

[16] M. E. Shenton, H. M. Hamoda, J. S. Schneiderman, S. Bouix, O. Pasternak, Y. Rathi, M. A. Vu, M. P. Purohit, K. Helmer, I. Koerte, A. P. Lin, C. F. Westin, R. Kikinis, M. Kubicki, R. A. Stern, and R. Zafonte, "A review of magnetic resonance imaging and diffusion tensor imaging findings in mild traumatic brain injury." *Brain imaging and behavior*, 2012.

[17] S. Umarov, C. Tsallis, and S. Steinberg, "On a q-Central Limit Theorem Consistent with Nonextensive Statistical Mechanics," *Milan Journal of Mathematics*, vol. 76, no. 1, pp. 307–328, Mar. 2008.

[18] P. PERONA and J. MALIK, "SCALE-SPACE AND EDGE-DETECTION USING ANISOTROPIC DIFFUSION," *Ieee Transactions On Pattern Analysis And Machine Intelligence*, vol. 12, no. 7, pp. 629–639, 1990.

[19] Q. A. Wang, L. Nivanen, A. Le Mehaute, and M. Pezeril, "Fractal geometry, information growth and nonextensive thermodynamics," *Physica A-statistical Mechanics And Its Applications*, vol. 340, no. 1-3, pp. 117–125, 2004.