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An extended multivariate autoregressive framework for EEG-based information flow analysis of a brain network

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***Abstract*— Recently effective connectivity studies have gained significant attention among the neuroscience community as Electroencephalography (EEG) data with a high time resolution can give us a wider understanding of the information flow within the brain. Among other tools used in effective connectivity analysis Granger Causality (GC) has found a prominent place. The GC analysis, based on strictly causal multivariate autoregressive (MVAR) models does not account for the instantaneous interactions among the sources. If instantaneous interactions are present, GC based on strictly causal MVAR will lead to erroneous conclusions on the underlying information flow. Thus, the work presented in this paper applies an extended MVAR (eMVAR) model that accounts for the zero lag interactions. We propose a constrained adaptive Kalman filter (CAKF) approach for the eMVAR model identification and demonstrate that this approach performs better than the short time windowing-based adaptive estimation when applied to information flow analysis.**

I. INTRODUCTION

Electroencephalography (EEG) data collected non-invasively using specialised modern hardware can produce high resolution information on underlying brain function. However, the volume conduction effect of the human head results in a blurred picture of this function. Thus we need special mathematical tools for functional segregation (localization) and integration (connectivity) analysis. Functional integration analysis is based on determining functional and effective connectivity, which gives us an insight to the coordinated activation among localized brain areas.

Among other popular approaches for effective connectivity analysis such as, Dynamic Causal Modeling [1] and Structural Equation Modeling, Multivariate Autoregressive Modeling (MVAR) has gained a wide number of applications due to its simplicity. MVAR modeling is derived from the definition of Granger Causality (GC) [2]. A number of estimators in the frequency domain have been proposed through MVAR modeling of the EEG time series [3], which among them partial directed coherence (PDC)[4] and directed transfer function (DTF) [5] are used in a vast number of applications.

In a conventional MVAR (cMVAR) model the linear modeling system does not include instantaneous interactions between the variables. However, instantaneous interactions can occur in neural signals [6]. If these instantaneous interactions are present among the variables and not captured by the cMVAR model, they will be transferred to the model residuals creating a correlation structure within them [7]. Most GC-based measures for analysis of the system dynamics rely on the assumption of uncorrelated model residuals. Thus, if instantaneous interactions are neglected in the model, the estimated causal structure by measures such as PDC or DTF will not reflect the true underlying network structure [8][9].

As a solution for this problem in EEG analysis, the work in [6] proposed the use of an extended MVAR (eMVAR) model, which accounts for instantaneous interactions by including the zero-lag component to the cMVAR model. eMVAR model identification is a critical step in estimating the frequency domain measures. The instantaneous effects cause lack of identifiability of the eMVAR model coefficients. Using *a priori* knowledge available on the temporal order of the variables under investigation is a viable solution to improve the identifiability of the model [6][9]. The instantaneous paths are then defined with imposed constraints, so that a path exists from $i \to j$ only when $i > j$ with no closed loops permitted. Using the non-gaussianity of the model residuals is another option for eMVAR model identification [8].

The work in [6][9] has proposed eMVAR modeling as a generalised candidate for GC-based connectivity studies. In this paper we present an extension to their work to be used in information flow analysis. The short time window (STW) approach used in their work can be extended to a overlapping sliding STW approach for the information flow analysis. Further, we propose a novel Constrained Adaptive Kalman filter (CAKF) approach for eMVAR model identification. Compared to the sliding STW approach, the proposed CAKF-based identification can resolve the underline structure better. Both approaches assume that the temporal order of the variables are known *a priori*, and thus use the same model constraints.

This paper is organised as follows. Section 2 introduces the eMVAR model and Section 3 describes the eMVAR model based connectivity measures. Section 4 presents the proposed CAKF technique. Section 5 and 6 discuss the simulated model and the results of the paper and Section 7 concludes the paper.

II. THE TIME VARYING EXTENDED MVAR MODEL

The time varying eMVAR model of k signal sources can be given as,

$$Y(n) = \sum_{i=0}^{P} A_r(i,n)Y(n-i) + E(n) \qquad (1)$$

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where $Y(n) = [y_1(n) y_2(n) \ldots y_k(n)]^T$, y_j denotes the j^{th} signal source and $n = 1, 2, \ldots, L$ denotes a time point. The superscript T refers to the transpose of a matrix throughout the paper. L is the total number of time points in the data. $A_r(i,n)$ is the $(k \times k)$ model coefficient matrix at lag $i = 0, \ldots, P$. P is the model order. $E(n) = [e_1(n), \ldots, e_k(n)]^T$ is a vector of zero mean white noise input with covariance matrix Σ_E. The matrix $A_r(0,n)$ contains the instantaneous effects.

III. Extended MVAR based connectivity measures

The time varying PDC measure [4] associated with the cMVAR is

$$PDC_{ij}(n,f) = \frac{B_{ij}(n,f)}{\sqrt{\Sigma_{m=1}^{k} |B_{mj}(n,f)|^2}} \tag{2}$$

where $B_{ij}(n,f)$ is the (i,j) element of the inverse transfer matrix, $B(n,f)$ of the frequency domain cMVAR model at frequency f. The time domain represented of cMVAR is, $Y(n) = \sum_{i=1}^{P} B_r(i,n) Y(n-i) + W(n)$, where now $B_r(i,n)$ is the $(k \times k)$ model coefficient matrix at lag i of the cMVAR model. $W(n) = [w_1(n), \ldots, w_k(n)]^T$ is a vector of zero mean white noise input with covariance matrix Σ_W. Here, $B(n,f) = \left[I - \sum_{m=1}^{P} B_r(m,n) exp\left(-2\pi i m f\right)\right]$.

In analogy to the cMVAR related PDC measure, the time varying extended PDC (EPDC) measure [9] associated with the eMVAR is formulated as

$$EPDC_{ij}(n,f) = \frac{\frac{1}{\sigma_i} A_{ij}(n,f)}{\sqrt{\sum_{m=1}^{k} \frac{1}{\sigma_m^2} |A_{mj}(n,f)|^2}} \tag{3}$$

where $A_{ij}(n,f)$ is the (i,j) element of the inverse transfer matrix, $A(n,f)$ of the frequency domain eMVAR model at frequency f. $A(n,f) = \left[I - \sum_{m=0}^{P} A_r(m,n) exp\left(-2\pi i m f\right)\right]$.

The EPDC measure will show significant values at both the instantaneous and lagged transfer paths. However in an information flow analysis we are more focused towards the lagged flow pertaining to the definition of GC. For this purpose we use the time varying lagged PDC (LPDC) measure is defined in [9]

$$LPDC_{ij}(n,f) = \frac{\frac{1}{\sigma_i} \bar{A}_{ij}(n,f)}{\sqrt{\sum_{m=1}^{k} \frac{1}{\sigma_m^2} |\bar{A}_{mj}(n,f)|^2}} \tag{4}$$

where $\bar{A}(n,f) = \left[I - \sum_{m=1}^{P} A_r(m,n) exp\left(-2\pi i m f\right)\right]$.

IV. Constrained Adaptive Kalman Filtering

In order to apply a Kalman filtering approach to estimate the time-varying MVAR coefficients (1) should be represented in a state space formulation

$$\begin{aligned} x(n) &= x(n-1) + V(n) \\ Y(n) &= C(n)x(n) + E(n) \end{aligned} \tag{5}$$

constrained by

$$Dx(n) = d \tag{6}$$

where $x(n) = \begin{pmatrix} vec[B_r^T(0,n)]^T \\ \vdots \\ vec[B_r^T(P,n)]^T \end{pmatrix}$ is the concatenation of vectorised $B_r(i,n)$ of size $(kk(P+1) \times 1)$ and $C(n) = I_k \otimes \begin{bmatrix} Y^T(n) \\ \vdots \\ Y^T(n-P) \end{bmatrix}$ is the concatenated matrix of the past measurements of size $k \times kk(P+1)$. $\otimes$ is the Kronecker-product and I_k is the $k \times k$ identity matrix.

The vector carrying the model coefficients, $x(n)$ are the states that has to be estimated. As no *a priori* knowledge is available about the states we use the most common approach of using a random walk model [10]. $V(t)$ is the state noise, which is a multivariate Gaussian noise with zero mean and covariance matrix Σ_V.

Due to the non stationarity of the EEG signals Σ_E and Σ_V will be time varying parameters and with no *a priori* knowledge available should be recursively estimated within the standard Kalman filter iterations. Among the many different adaptive estimation techniques [11], we update Σ_E and Σ_V using

$$\begin{aligned} \Sigma_E(n) &= \alpha \Sigma_E(n-1) + (1-\alpha)\tilde{Y}\tilde{Y}^T \\ \Sigma_V(n) &= I(1-\alpha) trace(P(n|n))/P \end{aligned} \tag{7}$$

where α is the forgetting factor controlling the adaptation speed, $\tilde{Y}(n)$ measurement residual and $P(n|n)$ is the *a posteriori* estimation error covariance matrix. The value of α lies in the interval $[0,1]$ and is chosen to be near to 1 [12].

Additionally we include the state constraints on the state matrix to reflect the condition of directional influence of instantaneous connections pertaining to the temporal order of the time series. This step is in analogy to the Cholesky decomposition step in the STW identification approach [9], which naturally sets $B_r(0,n)$ to be a zero diagonal lower triangular matrix. The matrix D is selected such that the entries of $x(n)$, corresponding to upper diagonal and the diagonal entries of $B_r(0,n)$ is set to zero. Thus the number of constraints for this application is $s = k(k+1)/2$, resulting in a known $s \times kk(P+1)$ constraint matrix D and d is a known equality matrix, in our application $d = zeros(s,1)$.

In this paper the constrained estimates $\tilde{x}(n|n)$ are estimated by directly projecting the unconstrained state estimates $\hat{x}(n|n)$ onto the constrained surface [13]. The maximum probability method gives the constrained estimation as

$$\begin{aligned} \tilde{x}(n|n) &= \hat{x}(n|n) - P(n|n)^{-1} D^T (D P(n|n)^{-1} D^T)^{-1} \\ &\quad .(D\hat{x}(n|n) - d). \end{aligned} \tag{8}$$

Here the *a priori* estimate of the filter recursions is chosen to be the constrained estimate $\tilde{x}(n|n)$ [14]. With an identity state transition matrix I_k. The Constrained Adaptive Kalman filter (CAKF) recursions are given by

- Time Update

$$\begin{aligned} \hat{x}(n|n-1) &= \tilde{x}(n-1|n-1) \\ P(n|n-1) &= P(n-1|n-1) + \Sigma_V(n) \\ \tilde{Y}(n) &= Y(n) - C(n)\hat{x}(n|n-1) \end{aligned} \tag{9}$$

- Measurement Update

$$
\begin{aligned}
\Sigma_E(n) &= \alpha\Sigma_E(n-1) + (1-\alpha)\tilde{Y}\tilde{Y}^T \\
S(n) &= C(n)P(n|n-1)C(n)^T \\
&\quad +\Sigma_E(n) \qquad (10)\\
K(n) &= P(n|n-1)C(n)^T S(n)^{-1} \\
\hat{x}(n|n) &= \hat{x}(n|n-1) + K(n)\tilde{Y}(n) \\
\tilde{x}(n|n) &= \hat{x}(n|n) - P(n|n)D^T \\
&\quad (DP(n|n)D^T)^{-1} \\
&\quad .D\hat{x}(n|n) \\
P(n|n) &= [I - K(n)C(n)]P(n|n-1) \\
\Sigma_V(n) &= I(1-\alpha)trace(P(n|n))/P
\end{aligned}
$$

where $\hat{x}(n|n-1), P(n|n-1)$ are the *a priori* state estimate and the estimation error covariance matrix respectively,$K(n)$ is the Kalman filter gain and $S(n)$ the residual covariance matrix.

V. Simulation Study

A three dimensional eMVAR process of order two with imposed lagged and instantaneous interactions together with a time varying parameter is used to investigate the ST windowing approach and the proposed CAKF approach in eMVAR based information flow analysis. The process is generated using the equations:

$$
\begin{aligned}
y_1(n) &= 0.5y_1(n-1) - 0.7y_1(n-2) \\
&\quad +c12(n)y_2(n-1) + e_1(n) \\
y_2(n) &= 0.7y_2(n-1) - 0.5y_2(n-2) \qquad (11)\\
&\quad +0.2y_1(n-1) + c23(n)y_3(n-1) + e_2(n) \\
y_3(n) &= 0.8y_3(n-1) + 0.5y_1(n) + e_3(n)
\end{aligned}
$$

with

$$
c_{12}(n) = \begin{cases} n/L & n \leq L/2 \\ (L-n)/L & n > L/2 \end{cases} \qquad (12)
$$

and with

$$
c_{23}(n) = \begin{cases} 0.4 & n \leq 0.7L \\ 0 & n > 0.7L \end{cases} \qquad (13)
$$

This is a modified version of the model used in a previous study [15] to evaluate time varying directed interactions in EEG data. We modify this model structure by imposing an instantaneous interaction from $y_1 \rightarrow y_3$. A graphical representation of the model (11) is given in Fig. 1 in a directed graph. $A_p(i,j)$ refer to the magnitude of influence from j^{th} region to the i^{th} region at the p^{th} lag. $e_i \sim N(0,1)$, where $N(\mu,\sigma^2)$ denotes a normal distribution with mean μ and variance σ^2.

We generate data points $L = 10000$, however assume a sampling frequency (F_s) of 256Hz for the purpose of connectivity measure calculations. This F_s results in the number of frequency bins used for the LPDC and EPDC calculations as 128, with a maximum frequency of 128Hz $(= F_s/2)$. The value of α was set to 0.999 for the CAKF estimations. The window length (W) of the STW approach

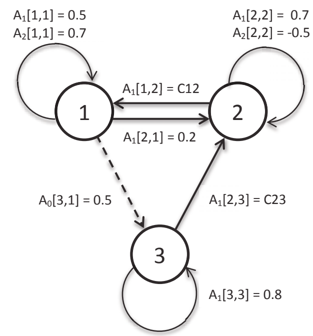

Fig. 1. Graphical representation of the Simulated model as a directed graph

was selected as 256 points, in order to fulfill the requirement of sufficient amount of data.

VI. Results And Discussion

Fig. 2 to Fig. 5 illustrate the most significant values of the estimated PDC, LPDC and EPDC after applying a random permutations surrogate data method. The most significant values are calculated by setting a 99% level of significance. Fig. 2 shows the PDC measure based information flow of the model when a cMVAR model fitted to the simulated model (11). An unconstrained adaptive Kalman filter (AKF) is used for the adaptive estimation of Σ_V and Σ_E. It can be seen that the PDC measure shows a information flow from $y_1 \rightarrow y_3$, which is actually an instantaneous interaction and a flow from $y_2 \rightarrow y_3$, which is a spurious connection. Thus it can be seen that by using a cMVAR model when zero lag interactions are present can heavily misinterpret the underline information flow.

Fig. 3 and Fig. 4 illustrates the estimated LPDC and EPDC measures using the eMVAR model and the CAKF. It can be observed that Fig. 3 evidently represents the lagged structure ($y_1 \rightarrow y_2$, $y_2 \rightarrow y_1$ and $y_3 \rightarrow y_2$) in model (11) and EPDC contains both the lagged and instantaneous ($y_1 \rightarrow y_3$) interactions. In contrast to the PDC measure in Fig. 2, the LPDC in Fig. 3 represents the correct information flow. Similar to the CAKF approach the STW based LPCD (Fig. 5) measures also resolve the lagged and mixed interactions fairly well, in comparison to the STW based PDC measure in Fig. 2.

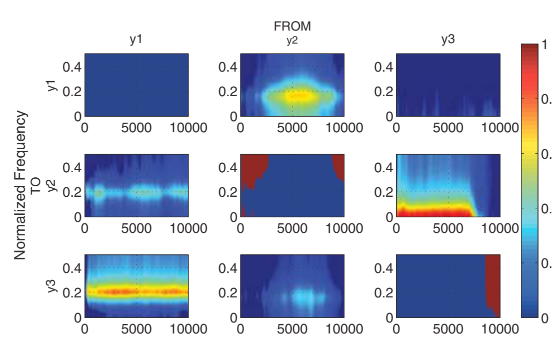

Fig. 2. PDC for model using AKF cMVAR

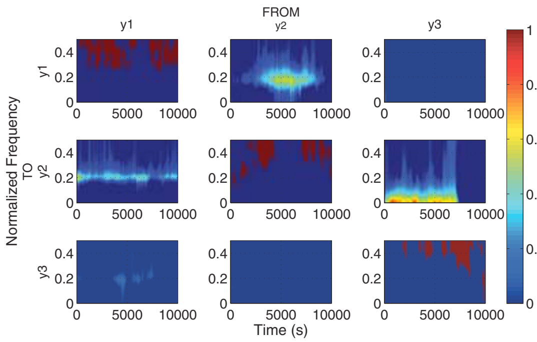

Fig. 3. LPDC for model using CAKF eMVAR

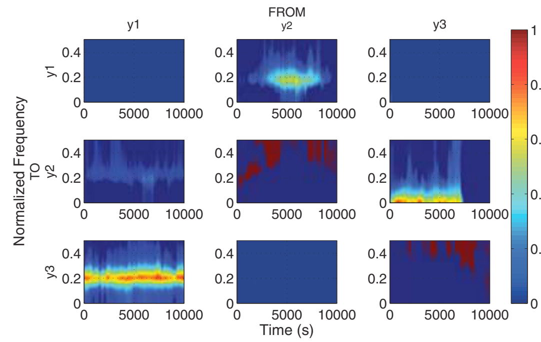

Fig. 4. EPDC for model using CAKF eMVAR

The STW based LPDC measure illustrated in Fig. 5 has a poor time frequency resolution compared to Fig. 3 based of CAKF. Nevertheless, the sudden changes in the time varying information flow from $y_3 \to y_2$ is captured accurately when in both STW and CAKF based LPDC measures. However, faint spurious information flow patterns can be observed in Fig. 5 showing the LPDC measure.

Thus, overall the proposed CAKF can be concluded as a better candidate over STW for eMVAR coefficient estimation. Within the proposed framework, the zero lag coefficient matrix is constrained to a lower triangular form, which corresponds to force these effects to be present over pre-determined directions determined by the order of the time courses. In real data analysis the estimation would fail if the actual instantaneous effects are not precisely pre-determined. Ordering of the source time courses in the temporal order of source activation is one method to overcome this issue.

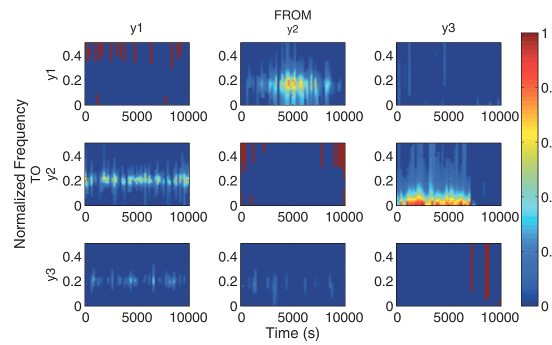

Fig. 5. LPDC for model using STW eMVAR

VII. CONCLUSION

The paper presents a state space framework for adaptive estimation of eMVAR model parameters that can be applied for information flow analysis of non-stationary EEG data. We discuss an extension to the work presented in [9]. The main contribution in the paper is to propose an CAKF algorithm for the estimation of a eMVAR model. While eMVAR provides a generalised model for GC-based frequency domain connectivity analysis in presence of instantaneous interactions, the proposed CAKF estimation framework improves the usability of the eMVAR model in information flow studies, offering advantages over the STW approach in terms of improved image resolution and the occurrence of spurious patterns compared to the STW approach.

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