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Computing the Electrical Activity of the Heart with a Dynamic Inverse Monodomain Operator

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Abstract— **The inverse problem in electrocardiography is to reconstruct the voltage in the surface of the heart, using a high density electrocardiogram. This problem is usually solved using regularization techniques, which tend to give the minimum energy response in a static scheme. In our work, we propose to calculate a dynamic inverse solution using the Monodomain as a model of electrical heart activity, thus constraining the family of solutions to one that satisfies the model.**

I. Introduction

A leading cause of death in the world is Sudden Cardiac Arrest (SCA), claiming an estimated 2,000 lives each day in the United States, and Europe. SCA is not a random event, and can be predicted [1]. Our motivation is to develop new tools to explain the behavior of the heart. The non-invasive electrocardiography imaging (ECGI) calculates the voltage distribution in the heart's surface by body surface maps. This is known as the inverse problem in electrocardiography [2]. There have been many works which solve this static inverse problem [2,3,4], using different mathematical methods. The problem is that the electrical activity of the heart is not described by a static model, but a dynamic one. Our approach is to develop a method, and the necessary tools to reconstruct the potential on the heart's surface, using a global dynamic model.

The body is considered as a volume conductor, with boundary conditions in heart, and thorax surfaces (1-3).

$$-\nabla \cdot (M_t \nabla u) = 0 \; in \; \Omega, \tag{1}$$

$$u = u_h \; on \; \Gamma_H, \tag{2}$$

$$(M_t u) \cdot \eta = 0 \; on \; \Gamma_T. \tag{3}$$

The boundary values on the heart's surface (Γ_H) are unknown, and the measures and null flux conditions are given in the thorax's surface (Γ_T). The conductivity in the torso is M_t. Using the geometry, and the volume conductor model an operator is created, which gives the relationship between the potential on the two surfaces [5]. Then, for each step of the time, the solution is calculated in a static scheme.

The inverse problem in electrocardiography is an ill-posed problem; therefore regularization techniques are necessary, like Tikhonov regularization. Tikhonov regularization gives the minimum energy solution, which could have no physical meaning [6].In this work we propose to calculate the inverse solution in the heart's surface using the Monodomain which describes the electrical activity in the heart dynamically. This limits the possible solutions to a calculated solution given by a model that describes the electrical activity.

The heart is composed with excitable cells connected between each other, which react to an electrical stimulus. When an excitable cell receives a stimulus, the potential inside the cell (intracellular) and outside the cell (extracellular) changes, and propagates to the neighboring cells. The intracellular and extracellular domains are separated by the cell membrane. The cycle of depolarization and repolarization of the cell is known as action potential. To consider the potential difference across the membrane the Bidomain [7] is used

$$\nabla \cdot (M_i \nabla v) + \nabla \cdot (M_i \nabla u_e) = xC_m \frac{\partial v}{\partial t} + xI_{ion}(v, s) + xI_{app}, \tag{4}$$

$$\nabla \cdot (M_i \nabla v) + \nabla \cdot ((M_i + M_e) \nabla u_e) = 0, \tag{5}$$

$$\frac{\partial s}{\partial t} = f(v, s, t), \tag{6}$$

$$(M_i \nabla v + M_i \nabla u_e) \cdot \eta = 0, \tag{7}$$

$$(M_e \nabla u_e) \cdot \eta = 0. \tag{8}$$

The transmembrane and extracellular potential respectively are v, u_e. The membrane area to volume ratio is x. The capacity of the cell membrane is C_m. The tensor quantities for the conductivity in the two domains are M_i and M_e,. The conductive properties of heart are strongly Anisotropic, so M_i and M_e, depend on the direction of the fibers of the heart muscle, σ_l,σ_t,σ_n are the conductivities along the fibers, tangent to the fibers and normal respectively. The ionic current per unit cell membrane is I_{ion}, and is given by a heart cell electrophysiology model, e.g. : FitzHugh-Nagumo Model, Hodgkin-Huxley, Fenton-Karma. The variables given by the ionic model are *s*(voltage, and gating channels). The external stimulus is I_{app}.The time is described by *t*.

If we consider [7]

$$M_e = \lambda M_i, \tag{9}$$

Where

$$M_i = DM^*D^t, \tag{10}$$

and

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$$M^* = \begin{bmatrix} \sigma_l & & \\ & \sigma_t & \\ & & \sigma_n \end{bmatrix}, \quad (11)$$

and D is a matrix having the unit vectors of the fiber directions as columns. With λ as a scalar, then the system will simplify to

$$\frac{\lambda}{1+\lambda}\nabla\cdot(M_i\nabla v) = xC_m\frac{\partial v}{\partial t} + xI_{ion}(v,s) + xI_{app}, \quad (12)$$

$$\frac{\partial s}{\partial t} = f(v,s,t), \quad (13)$$

$$(M_i v)\cdot\eta = 0, \quad (14)$$

$$\nabla\cdot(M_i\nabla v) + \nabla\cdot((1+\lambda)(M_i)\nabla u_e) = 0, \quad (15)$$

which is the monodomain model. In this paper we work with the monodomain model, by considering an isolated heart. For the ionic heart cell model, we use the Fenton-Karma 1998 ionic model [12].

II. Creating the Inverse Operator

A. Transfer Matrix from Heart to Thorax

To create the transfer matrix, we use the finite element method (FEM) approximation, and discretize the volume in linear tetrahedra. Considering (1-3), we will have

$$\begin{pmatrix} K_{vv} & K_{vt} \\ K_{tv} & K_{tt} \end{pmatrix} * \begin{pmatrix} u_v \\ u_t \end{pmatrix} = \begin{pmatrix} -K_{vh} \\ -K_{th} \end{pmatrix} * u_h, \quad (16)$$

where the indexes *h, v, t* mean heart, thorax and volume respectively. Next, $K_{vv}, K_{vt}, K_{tv}, K_{vh}, K_{vh}$ and K_{tt} are matrix created with the relationships given by the stiffness matrix using FEM [5]. The vectors with the nodal values for the voltage in the nodes in the heart surface, thorax surface an volume in between are u_h, u_t, u_v. Solving for u_v, and substituting the transfer matrix will be,

$$T = (K_{tt} - K_{tv}K_{vv}^{-1}K_{vt})^{-1}K_{tv}K_{vv}^{-1}K_{vh}. \quad (17)$$

This gives us the relationship

$$Tu_h = u_{t.} \quad (18)$$

B. Creating the Monodomain Inverse Operator

To solve the Monodomain numerically we use the operator splitting technique

$$\frac{\partial v}{\partial t} = -I_{ion}(v_n, s_n) - I_{app}, \quad (19)$$

$$\frac{\partial v}{\partial t} = \frac{\lambda}{1+\lambda}\nabla\cdot(M_i\nabla v). \quad (20)$$

This means that we solve the system by steps. If we discretize (19) in time we obtain

$$\frac{v_{n+1}-v_n}{\Delta t} = -I_{ion}(v_n, s_n) - I_{app}. \quad (21)$$

This implies

$$v_{n+1=} - \Delta t I_{ion}(v_n, s_n) - \Delta t I_{app} + v_n. \quad (22)$$

Using the θ rule for the discretization in (20), we get

$$\frac{v_{n+2}-v_{n+1}}{\Delta t} = \theta\left(\frac{\lambda}{1+\lambda}\nabla\cdot(M_i\nabla v_{n+2})\right) + ((1-\theta)\frac{\lambda}{1+\lambda}\nabla\cdot(M_i\nabla v_{n+1})). \quad (23)$$

Applying the Green's Lemma and the weak formulation, the result is

$$\int \mathrm{v_{n+2}}\varphi\mathrm{dx} + \theta\frac{\Delta\mathrm{t}\lambda}{1+\lambda}\int \mathrm{M_i}\nabla\mathrm{v_{n+2}}\nabla\varphi\mathrm{dx} = \int \mathrm{v_{n+1}}\varphi\mathrm{dx} - (1-\theta)\left(\frac{\Delta\mathrm{t}\lambda}{1+\lambda}\right)\int \mathrm{M_i}\nabla\mathrm{v_{n+1}}\nabla\varphi\mathrm{dx} \quad (24)$$

Using the definition

$$v_{n+2} = \sum_{j=1}^{N} v_j\emptyset_j. \quad (25)$$

The equation (24) becomes

$$\sum_{j=1}^{N} v_j\int \emptyset_i\emptyset_j dx + \theta\frac{\Delta t\lambda}{1+\lambda}\sum_{j=1}^{N} v_j\int M_i\nabla\emptyset_j\nabla\emptyset_i dx = \int v_{n+1}\emptyset_i dx - (1-\theta)\frac{\Delta t\lambda}{1+\lambda}\int M_i\nabla v_{n+1}\nabla\emptyset_i dx, \quad (26)$$

where N is the number of nodes in the heart volume. Using this we obtain the system Ax=b (being x=v_{n+2}), where A and b have the following form

$$A_{ij} = \int \emptyset_i\emptyset_j dx + \theta\frac{\Delta t\lambda}{1+\lambda}\int M_i\nabla\emptyset_j\nabla\emptyset_i dx\,, \quad (27)$$

$$b_i = \int v_{n+1}\emptyset_i dx - (1-\theta)\frac{\Delta t\lambda}{1+\lambda}\int M_i\nabla v_{n+1}\nabla\emptyset_i dx\,. \quad (28)$$

If the substitution (25) is used in (28) it becomes

$$B_{ij} = \int \emptyset_j\emptyset_i dx - (1-\theta)\frac{\Delta t\lambda}{1+\lambda}\int M_i\nabla\emptyset_j\nabla\emptyset_i dx\,. \quad (29)$$

This system is equivalent to

$$Av_{n+2} = Bv_{n+1}. \quad (30)$$

In a similar way, equation (15) gives the following matrix relationship

$$Rv_{n+2} = Qu_e. \quad (31)$$

Using (31-32) we have

$$u_e = Q^{-1}RA^{-1}Bv_{n+1}. \quad (32)$$

If $u_e = u_h$ we get

$$TQ^{-1}RA^{-1}Bv_{n+1} = u_t, \quad (33)$$

$$TQ^{-1}RA^{-1}B(-\Delta t I_{ion}(v_n, s_n) - \Delta t I_{app} + v_n) = u_t. \quad (34)$$

For simplicity, we let

$$P = TQ^{-1}RA^{-1}B, \quad (35)$$

$$-P\Delta t I_{ion}(v_n, s_n) - P\Delta t I_{app} + Pv_n = u_t. \quad (36)$$

For the inverse problem, we consider that v_n, u_t, s_n are given, and we can calculate $I_{ion}(v_n, s_n)$. To solve the inverse

problem we need to calculate I_{app}. To obtain I_{app} we need to minimize the functional

$$\min_{I_{app}} \left\| -P\Delta t(I_{app}) - (u_t + P\Delta t I_{ion}(v_n, s_n) - Pv_n) \right\|, \quad (37)$$

once we have I_{app} we can retrieve v_{n+2} solving (31-32).

III. Tests

For the tests we use the geometry from Fig. 1, of the thorax and ventricles. The fiber directions necessary for the Monodomain were created analytically with the sketch-based interface [12]. The values for the monodomain parameters appear in table 1 [7].

TABLE I. MONODOMAIN PARAMETERS

σ_l	3.0 mS/cm
σ_t	1.0 mS/cm
σ_n	0.31525 mS/cm
λ	0.8
Δt	0.5 ms
x	800 cm^{-1}
θ	0.5
C_m	1 uF/cm^2

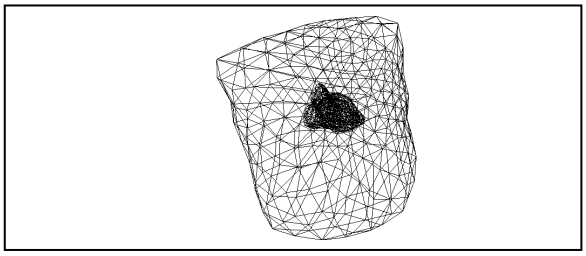

Figure 1. Geometry of Heart and Thoras Surfaces

The summary of the overall process is the following: First using the surfaces of the heart and thorax, create the transfer matrix. Then, using the volume of the heart create the inverse operator, and analytically create the fiber directions for simulating the Monodomain. Next, we use the measures in the thorax for each time step and the values from the last time step calculate I_{app} (I_{app} is calculated using Tikhonov regularization). The used regularization parameter is 0.001. Finally, using the calculated I_{app}, we solve the forward problem and we calculate the voltage distribution in heart and thorax. The simulation was made for 0.49 ms for the QRS complex.

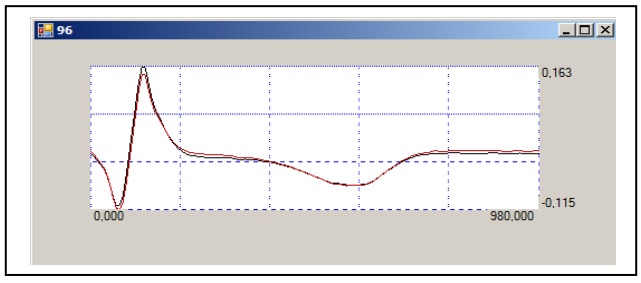

Figure 2. Comparison in one lead in the thorax, between the calculated voltage using the direct (black), and inverse problem (red).

The used measured values in the thorax came from ECGSIM [8]. In Fig. 2 there is the comparison in one lead on the thorax, with the original measured value (black), and the calculated by the direct problem with the inverse solution (red).

In Fig. 3, we can see the calculated values in the heart for different time steps (37.5 ms, 65 ms, 112.5 ms, 150 ms, and 275 ms) for the measured values in the thorax. In the left column there are the voltage distributions in the thorax, and in the right the inverse solutions in the heart for each time step. In Fig. 4 we can see the comparison between the calculated solution using the Monodomain inverse approach and the calculated solution using Tikhonov regularization in a static scheme.

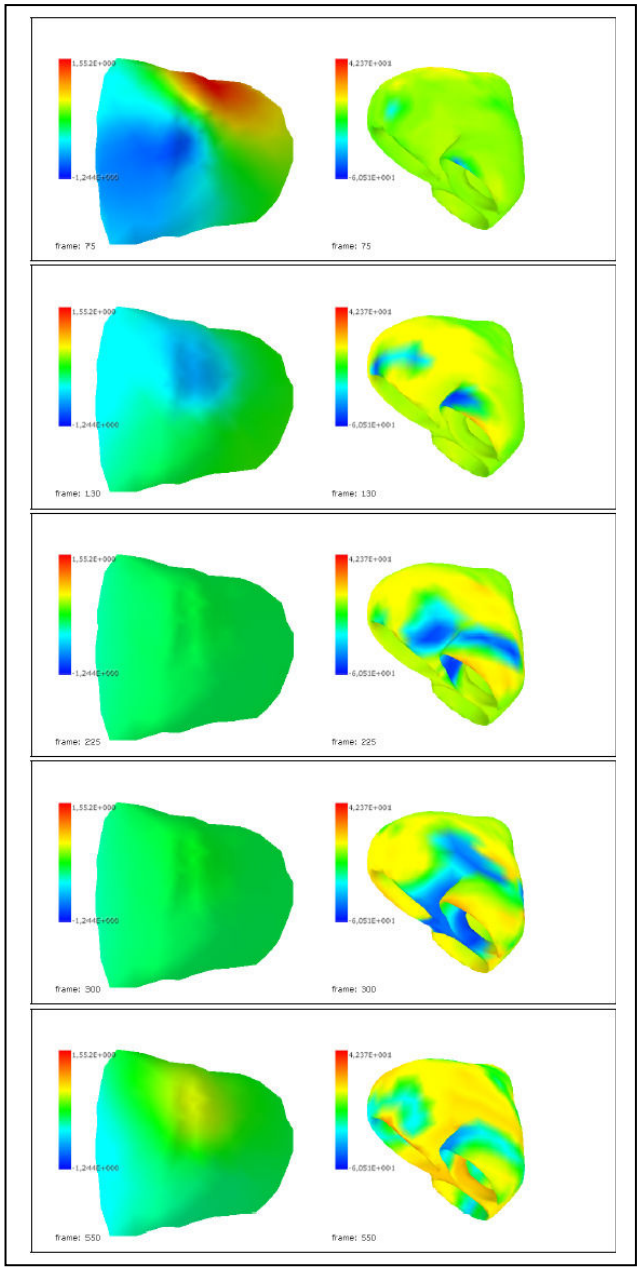

Figure 3. Values measured in the thorax (left), and values calculated in the heart by the dynamic solution for extracellulaire potential.

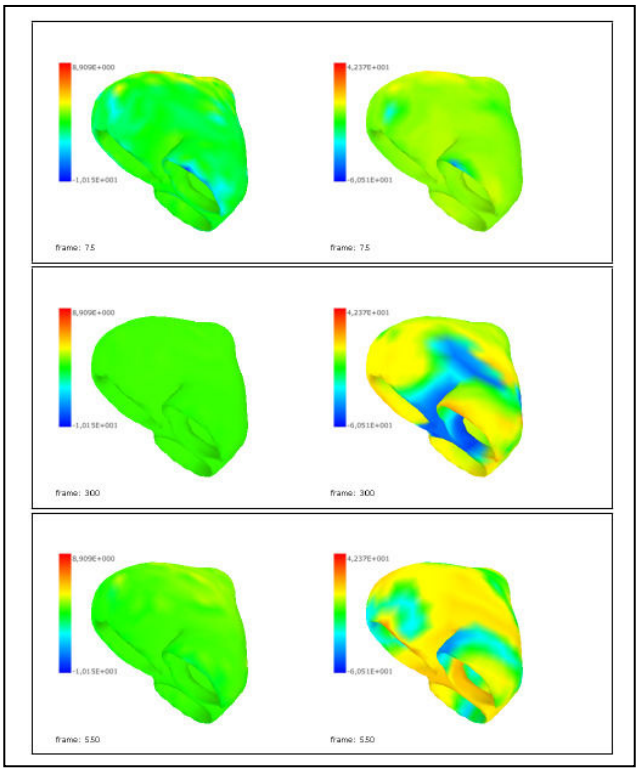

Figure 4. Comparison between the two methods; minimum energy (left column), and dynamic solution (right column).

IV. Conclusion and Future Work

As mentioned before the inverse problem in electrocardiography is an ill-posed problem. Either solution in the heart, whether only using the transfer matrix or the Monodomain inverse operator, reconstructs the same values in the thorax. The problem is to choose the correct solution. If we compare the two, the response from the Monodomain inverse operator is closer to how the heart really works.

With this method it is possible to find at the same time, the extracellular potential, the transmembrane potential, and the current in the heart. In Figure 5 there is the comparison for voltage (left) and current (right) for two different time steps: at 7.5 ms, and 115 ms. Finding the minimum current instead of the voltage, allows having a solution closer to reality. One of the applications of finding the current instead of the voltage using a dynamic model could allow controlling the electrical activity in the heart, as the artificial cardiac pacer. For example, if we calculate the current of a patient having a heart pathology, and we calculate the necessary current to have a normal electrocardiograph, we can apply the difference through evaluating the points where it is need to be applied and how much. One other application is to use it to find the epicenters of Ischemia in patients.

It needs to be considered that when using this method, the solution takes longer to calculate. It is slower because for each time step it is necessary to calculate the inverse solution, and then the direct solution of the Monodomain for the next step. Also the preprocessing is longer: with the static solution, we only need the surface of the heart and thorax. For the Monodomain inverse operator method the volume of the heart, and the fiber directions are necessary.

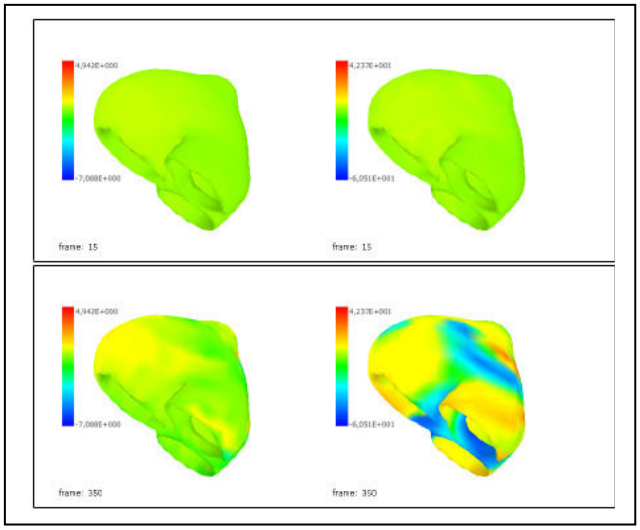

Figure 5. Current (left column), and extracellular potential (right) calculated.

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