

# A Basic Study on Variable-Gain Kalman Filter based on Angle Error Calculated from Acceleration Signals for Lower Limb Angle Measurement with Inertial Sensors

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**Abstract—** In this study, development of wearable motion measurement system using inertial sensors has been focused with the aim of rehabilitation support. For measurement of lower limb joint angles with inertial sensors, Kalman-filtering-based angle measurement method was developed. However, it was required to reduce variation of measurement errors that depended on movement speeds or subjects. In this report, variable-gain Kalman filter based on the difference between the estimated angle by the Kalman filter and the angle calculated from acceleration signals was tested. From angle measurement during treadmill walking with healthy subjects, it was shown that measurement accuracy of the foot inclination angle was significantly improved with the proposed method compared to the method of fixed parameter value.

## I. INTRODUCTION

Motion measurement with optical 3D motion measurement system or electric goniometers can be effective for objective and quantitative evaluation in rehabilitation of motor function. However, these systems have some shortcomings in that the set-up of these systems are not easy, measurement condition is limited and costs of these systems are very high. Therefore, many studies have been performed in measurement of lower limb joint angles and stride length and so on with inertial sensors such as a gyroscope and an accelerometer, which are small, low cost and easy for settings [1-10].

In our previous studies, a method of measuring lower limb joint angles using wireless inertial sensors was developed to realize simplified wearable gait evaluation system for rehabilitation support, and the method was tested in measurement of gait of healthy subjects [11-12]. In the angle measurement method of our previous studies, Kalman filter was applied using the angle calculated from acceleration signals. Although the method was shown to have practical accuracy, measurement errors varied depending on movement speeds or subjects. Since the angle calculated from acceleration signals was influenced by acceleration caused by the impact and movement, those accelerations can be

considered as one of the causes of variation of measurement accuracy.

To reduce the influences of those accelerations, outputs of accelerometer were filtered with Butterworth low-pass filter with cut off frequency of 0.5Hz in our previous studies [11-12]. However, the low cut off frequency has a possibility of increasing measurement error since low cut off frequency causes delay in the low-pass filtering due to its large time constant.

In this paper, a method to reduce the influence of accelerations caused by the impact and movement in calculating angles using the Kalman filter was proposed, in which variable-gain Kalman filter was used with higher cut off frequency for the low-pass filtering. The proposed method was tested in measurement of lower limb angles of healthy subjects during treadmill walking.

## II. ANGLE MEASUREMENT METHOD BASED ON KALMAN FILTER

Figure 1 shows the block diagram of the angle measurement method used in our previous studies. The inclination angles of body segments are calculated by integrating outputs of gyroscopes. The integration error is corrected by the Kalman filter using the angles calculated from the outputs of the accelerometers. The joint angles are calculated from difference of inclination angles of the adjacent segments.

The state equation is represented by the error of the angle measured with gyroscopes  $\Delta\theta$  and bias offset of outputs of gyroscopes  $\Delta b$  as follows:

$$\begin{bmatrix} \Delta\theta_{k+1} \\ \Delta b_{k+1} \end{bmatrix} = \begin{bmatrix} 1 & \Delta t \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \Delta\theta_k \\ \Delta b_k \end{bmatrix} + \begin{bmatrix} \Delta t \\ 1 \end{bmatrix} w \quad (1)$$

Where  $w$  is error in measurement with gyroscopes,  $\Delta t$  is sampling period. Observation signal is the difference of angles

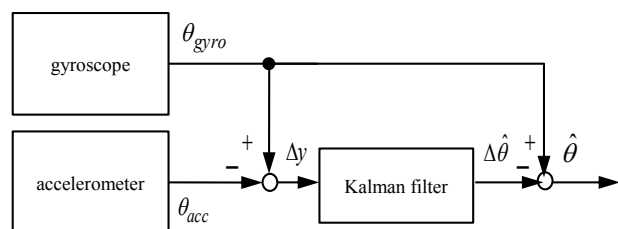


Fig. 1. Block diagram of the angle calculation method using Kalman filter in our previous studies.

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obtained from a gyroscope and an accelerometer  $\Delta y$ , which is given by:

$$\Delta y_k = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \Delta \theta_k \\ \Delta b_k \end{bmatrix} + v \quad (2)$$

Where  $v$  is error in measurement with accelerometer. On this state-space model, Kalman filter repeats corrections (Eq. (3)) and predictions (Eq. (4)):

$$\begin{bmatrix} \Delta \hat{\theta}_k \\ \Delta \hat{b}_k \end{bmatrix} = \begin{bmatrix} \Delta \hat{\theta}_k^- \\ \Delta \hat{b}_k^- \end{bmatrix} + \begin{bmatrix} K_1 \\ K_2 \end{bmatrix} (\Delta y_k - \Delta \hat{\theta}_k^-) \quad (3)$$

$$\begin{bmatrix} \Delta \hat{\theta}_{k+1}^- \\ \Delta \hat{b}_{k+1}^- \end{bmatrix} = \begin{bmatrix} 1 & \Delta t \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \Delta \hat{\theta}_k \\ \Delta \hat{b}_k \end{bmatrix} \quad (4)$$

Where  $K_1$  and  $K_2$  represent Kalman gain for  $\Delta \theta$  and  $\Delta b$ , respectively. Notations such as  $\Delta \hat{\theta}$  and  $\Delta \hat{\theta}^-$  represent estimated value and predicted value for  $\Delta \theta$ , respectively. For the initial condition,  $\Delta \hat{\theta}_0^-$  was set 0 and  $\Delta \hat{b}_0^-$  was set  $\Delta \hat{b}$  at the last measurement. The Kalman filter was applied repeatedly until its outputs converged.

A fixed value of noise ratio of the Kalman filter was used in our previous studies. Where, the noise ratio is the ratio of the covariance of observation noise and covariance of process noise. The Kalman gain increases as the noise ratio decreases, then the output become highly dependent on accelerometer. Oppositely, Kalman gain decreases as the noise ratio increases, then the output become highly dependent on gyroscope.

Figure 2 shows the proposed method in this paper, in which the noise ratio is adjusted based on the magnitude of the difference between the angle estimated by the Kalman filter and the angle calculated from acceleration signals. As shown in Eq. (3), the Kalman filter corrects the error of the angle measured with gyroscopes  $\Delta \theta$  and bias offset of outputs of gyroscopes  $\Delta b$  by using the difference of angles obtained from a gyroscope and an accelerometer  $\Delta y$ . Therefore, the proposed method changed the noise ratio to determine Kalman gain based on magnitude of influence of impact and motion acceleration. In this paper, the magnitude of influence of impact and motion acceleration was approximately determined by the difference between the angle estimated by the Kalman filter and the angle calculated from acceleration signals.

### III. EXPERIMENTAL METHOD

#### A. Parameter Values of Kalman Filter

The proposed method was tested in measurement of lower limb angles during treadmill walking comparing to the previous method using fixed parameter value. Method 1 is the previous method used in our research group, in which the noise ratio of Kalman filter is fixed and cut off frequency of the low pass filter for acceleration signals is very low. Method

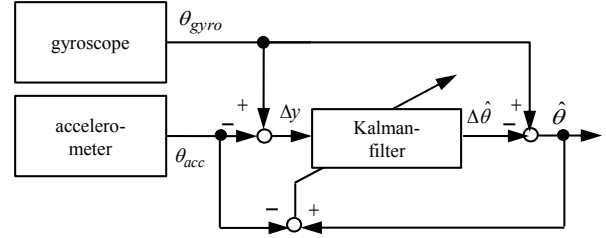


Fig. 2. Block diagram of the proposed method.

2 is the previous method with higher cut off frequency of the low pass filter. Method 3 is the proposed method, in which the noise ratio is adjusted based on Eqs. (7) ~ (10), and higher cut off frequency for the low pass filter is used. The 3 methods are summarized as follows:

(i) Method 1 (Previous method)

$$n = 10^6, f_c = 0.5 \text{ Hz} \quad (5)$$

(ii) Method 2 (Previous method with higher  $f_c$ )

$$n = 10^6, f_c = 10 \text{ Hz} \quad (6)$$

(iii) Method 3 (Proposed method)

$$n = 10^4, \text{ for } |\hat{\theta} - \theta_{acc}| \leq 1 \text{ deg} \quad (7)$$

$$n = 10^6, \text{ for } 1 \text{ deg} < |\hat{\theta} - \theta_{acc}| \leq 15 \text{ deg} \quad (8)$$

$$n = 10^8, \text{ for } 15 \text{ deg} < |\hat{\theta} - \theta_{acc}| \leq 60 \text{ deg} \quad (9)$$

$$n = 10^{13}, \text{ for } 60 \text{ deg} < |\hat{\theta} - \theta_{acc}| \quad (10)$$

$$f_c = 10 \text{ Hz} \quad (11)$$

Where  $n$  and  $f_c$  represent noise ratio of the Kalman filter and cut off frequency of Butterworth low-pass filter which was used when the outputs of accelerometer were filtered, respectively.  $\hat{\theta}$  and  $\theta_{acc}$  represent estimated angle by the Kalman filter and the angle calculated from acceleration signals, respectively.

#### B. Measurement Method of Angles

The inclination angles during treadmill walking were measured with 3 healthy subjects (male, 22-23 y.o.). The subjects walked on the treadmill for about 90sec at speeds of 1km/h (slow), 3km/h (normal), and 5km/h (fast). Five trials were performed for each walking speed. The wearable sensor system consists of seven wireless sensors (WAA006, Wireless Technologies) and a portable PC. The sensors attached on the feet, the shanks and the thighs of both legs, and lumbar region with stretchable bands. The sensors were put inside of pocket of the band. Acceleration and angular velocity signals of each sensor were measured with a sampling frequency of 100Hz, and were transmitted to PC via Bluetooth network.

The optical motion measurement system (OPTOTRAK, Nohrthern Digital Inc.) was used to measure reference data for

evaluating measured angles with the sensor system. The markers for reference data were attached on the left side of the body. The marker positions were measured with a sampling frequency of 100Hz.

There were offset difference between the sensor system and 3D motion analysis system because the markers for the reference signals could not be attached on the sensors. Therefore the difference was calculated as the mean value of the first 100 samples of the 1st measurement, and removed for evaluation. Then, root mean squared error (RMSE) between estimated angles and reference values were calculated.

#### IV. RESULTS

Figures 3, 4 and 5 show RMSE values of measured inclination angles of the foot, the shank, and the thigh, respectively. Average values of RMSE of the foot inclination angle (Fig.3) decreased with Method 3 comparing to the results of Method 1 and Method 2 for all walking speeds. For the shank inclination angle, slight improvement of RMSE values was shown for all walking speeds with Method 3 compared to the results of Method 1 and Method 2 (Fig.4). For the thigh inclination angle, as shown in Fig.5, the values of RMSE at the normal walking speed slightly reduced with Method 3 compared to Method 1 and Method 2. However, at the slow walking speeds and fast walking speeds, there was no improvement of RMSE with Method 3 comparing to the results of Method 1 or Method 2.

The values of RMSE with Method 2 decreased compared to the results of Method 1 in all parts at slow walking speed. However, at normal and fast walking speeds, the values of RMSE with Method 2 increased compared to the results of Method 1.

#### V. DISCUSSION

Measurement accuracy of the foot inclination angle was significantly improved with Method 3. Figure 6 shows the reference and measured foot inclination angles of one gait cycle at normal walking speed.  $\theta_{reference}$  and  $\theta_{acc}$  represents reference angle and angle calculated from acceleration signals, respectively. The x-axis shows the gait cycle. TO, HC, and FF represents the toe off, the heel contact, and the foot flat, respectively. As shown in Fig. 6, the estimated angles with Method 3 were almost equal with Method 1 between around the TO and the HC. The estimated angles improved with Method 3 between around the FF and the TO. It is considered that this is because the sensor attached on the foot was close to the stationary state at around the FF. At that time, the Kalman filter corrected the angle significantly, since error of the angles calculated from acceleration signals was very small. It is possible to decrease the angle measurement error between the FF and the TO by reducing noise ratio with Method 1. However, in that case, the error of estimated angle between around the TO and the HC is increased by the influence of acceleration caused by the movement and the impact. Method 3 reduced Kalman gain effectively at the time of around the TO and the HC, since the error of the angles calculated from acceleration signals increased significantly. Therefore, the Method 3 is considered to be effective in the angle measurement because the influence

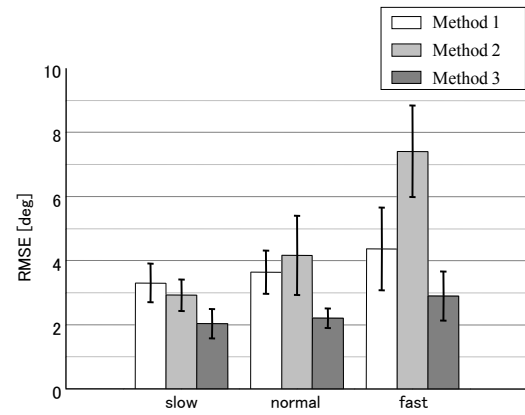


Fig. 3. Evaluation results of the foot inclination angles for each walking speeds. Average values obtained from the results of 5 trials of all subjects are shown.

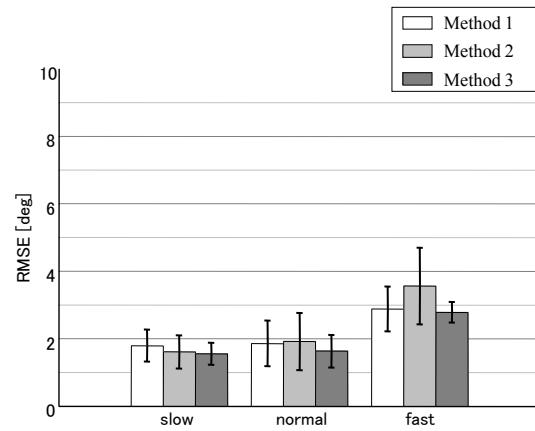


Fig. 4. Evaluation results of the shank inclination angles for each walking speeds. Average values obtained from the results of 5 trials of all subjects are shown.

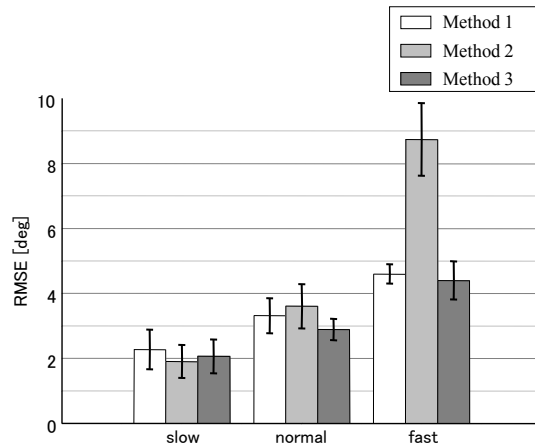


Fig. 5. Evaluation results of the thigh inclination angles for each walking speeds. Average values obtained from the results of 5 trials of all subjects are shown.

of acceleration caused by the movement and the impact could be decreased maintaining high accuracy between the HC and the TO.

Measurement accuracy of the shank and the thigh inclination angles were slightly improved with Method 3. In measurement of these angles, the noise ratios were not so greatly varied, since the magnitude of the error of the angle

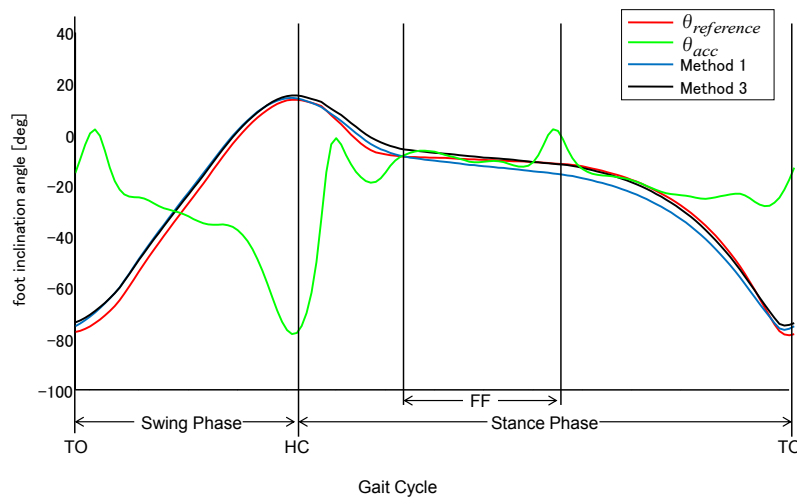


Fig. 6. Waveform of reference and measured foot inclination angles at normal walking speed.

calculated from acceleration signals did not fluctuate significantly during movements. This is considered to be one of the factors that improvement of measurement accuracy was scarcely seen with Method 3 in shank and thigh angles.

As shown in Figures 3, 4 and 5, the values of RMSE with Method 2 decreased at slow walking speed compared to the results of Method 1 in all parts. This suggests that the very low cut-off frequency for the low-pass filtering of the acceleration signal increased measurement error. The cause of increased error in the normal walking speed and the fast walking speed with Method 2 are considered to be influence of movement acceleration.

This paper focused only on the error of the angle calculated from acceleration signals in order to determine Kalman gain. It is considered that appropriate value of the noise ratio depends on magnitude of the offset drift of gyroscope. In addition, parameter values of Method 3, the noise ratio and the threshold to change the noise ratio, were determined by trial and error method. These are required to be studied in more detail for measurement of angles with the variable-gain Kalman filter.

## VI. CONCLUSION

In this paper, variable-gain Kalman filter based on the difference between the estimated angle and the angle calculated from acceleration signals was proposed and tested in measurement of lower limb inclination angles during treadmill walking with healthy subjects. Measurement accuracy of the foot inclination angle was significantly improved with the proposed method compared to the method of fixed parameter value. In addition, very low cut off frequency for the low pass filter was found to increase measurement error in some cases. Since the proposed method was found to be effective in angle measurement with inertial sensors, further studies on other methods to determine appropriate Kalman gain and parameter values to change the gain are expected.

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