

# Comparison of a Discrete Wavelet Transform Method and a Modified Undecimated Discrete Wavelet Transform Method for Denoising of Mammograms\*

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**Abstract**— The purpose of this study was to evaluate the performance of a conventional discrete wavelet transform (DWT) method and a modified undecimated discrete wavelet transform (M-UDWT) method applied to mammographic image denoising. Mutual information, mean square error, and signal to noise ratio were used as image quality measures of images processed by the two methods. We examined the performance of the two methods with visual perceptual evaluation. A two-tailed *F* test was used to measure statistical significance. The difference between the M-UDWT processed images and the conventional DWT-method processed images was statistically significant ( $P < 0.01$ ). The authors confirmed the superiority and effectiveness of the M-UDWT method. The results of this study suggest the M-UDWT method may provide better image quality as compared to the conventional DWT.

## I. INTRODUCTION

Breast cancer has caused a significant number of deaths in woman population and continues to be a significant public health problem in the world [1]. Mammography is one of the most effective methods of early breast cancer detection [2, 3]. However, it is not always perfect and adequate because of the fuzzy nature of the mammograms and the low contrast between the breast cancer and its surroundings [4]. Image processing has been suggested as a way to improve performance of mammography [5]. Image processing techniques applied for mammography could be used to smooth noise, equalize systematic variations in density or gray level, and enhance local contrast and sharpness of calcifications.

Image denoising plays a vital role in the field of digital mammography. Because of its importance, there has been an enormous amount of research dedicated to the subject of denoising and many methods have been reported in the literature [6-8]. Several approaches have been proposed with the use of discrete wavelet transform (DWT) [9, 10]. The DWT is very efficient from the computational point of view, but it is shift variant. Therefore its denoising performance can change drastically if the starting position of the signal is shifted. In order to achieve the shift-invariance and get more complete characteristic of the analyzed signal, the

undecimated discrete wavelet transform (UDWT) method has been proposed [11-13]. The reported methods were robust and effective. But, the methods were not advantageous from computational aspects. Recently, Matsuyama *et al* proposed a modified UDWT (M-UDWT) approach to mammographic denoising both for improving image quality and for decreasing image processing time-consuming [14]. The main features of the proposed method include the incorporation of the use of hierarchical correlation of the coefficients of the UDWT and iterative use of undecimated multi-directional wavelet transforms at two consecutive levels.

The purpose of this study was to compare images obtained by applying the conventional DWT method with images obtained by the M-UDWT method. In this work, a simulation study was undertaken for selection of an optimal wavelet basis function to perform wavelet analysis. In this simulation study, mutual information (MI), mean square error (MSE), and signal to noise ratio (SNR) were used as evaluation measures for the selection. After determination of an optimal wavelet basis function, we applied both the DWT and the M-UDWT methods to 30 clinical mammograms for image denoising. The performances of the two methods were compared by visual evaluation. The experiments demonstrated that the M-UDWT method is superior to the conventional DWT method in terms of image quality.

## II. MATERIALS AND METHODS

### A. Conventional Discrete Wavelet Transform Method

The 2-dimensional (2D) discrete wavelet transform (DWT) corresponds to multi-resolution approximation expressions. In practice, the DWT is carried out using 4 channel filter banks (for each level of decomposition) composed of a low-pass and a high-pass filter and each filter bank is then 1/2 down sampling of the previous frequency [15]. Thus the original image can be decomposed to 4 sub-images, i.e., approximation and its horizontal, vertical and diagonal wavelet coefficients. By repeating this procedure, it is possible to obtain wavelet transform of any level. The basic steps of the 2-D DWT method for denoising are decomposition, wavelet noise thresholding, and reconstruction. The detailed algorithm of the 2-D DWT can be found in the literature [16, 17].

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## B. Modified Undecimated Discrete Wavelet Transform Method

The UDWT has been discovered for various purposes and is also known stationary wavelet transform or redundant wavelet transform [18, 19]. The key point is that it is redundant, shift invariant, linear, and it gives a better approximation to the continuous wavelet transform than the approximation provided by the orthonormal DWT. Unlike the DWT, the UDWT does not incorporate the down sampling operations. Thus, the approximation coefficients (low-frequency coefficients) and detailed coefficients (high-frequency coefficients) at each level are the same length as the original signal.

The basic algorithm of the conventional UDWT is that it applies the transform at each point of the image and saves the detailed coefficients and uses the approximation coefficients for the next level. The size of the coefficients array does not diminish from level to level [20]. This decomposition is further iterated up to level 4. After computing the UDWT of the image, thresholding of the detailed coefficients at all levels is performed by applying the universal threshold. The wavelet coefficients are subjected to soft thresholding.

The main steps of the M-UDWT method are outlined below [14]:

- 1) Apply undecimated discrete wavelet transform to the noisy image up to level 2 to produce the noisy wavelet coefficients.
- 2) Compute the hierarchical correlations of the detailed coefficients between level 1 and level 2 for three different (horizontal, vertical and diagonal) directions.
- 3) Select appropriate threshold values based on the obtained hierarchical correlation values.
- 4) Apply the selected threshold values to the coefficients of level 1 to remove the noise, and obtain the modified detailed coefficients for level 1.
- 5) Apply inverse wavelet transform to the modified wavelet coefficients to obtain a denoised image.
- 6) Repeat steps 1-5 again, leading to obtain a final, denoised image.

The major differences between the M-UDWT method and the conventional UDWT method are as follows. First, the conventional UDWT decomposed the original image up to resolution level 4. In contrast, the proposed UDWT method only needs to perform the computation up to resolution level 2 and repeat the computation one time. Second, the conventional UDWT thresholded the detailed coefficients at all 4 levels with the same thresholding value, while the M-UDWT method utilized the hierarchical correlation of the coefficients between the level 1 and level 2 of the three detailed coefficients for thresholding. That is, the thresholding value were various and dependent on the nature of the noise.

## C. Selection of Wavelet Basis Functions

We evaluated 5 different wavelet basis functions, namely, discrete FIR approximation of Meyer wavelet (dmey), Daubechies order 2 (db2), Symlets order 7 (sym7), Coiflets order 1 (coif1), and Coiflets order 5 (coif5), as candidates for selection as the most suitable basis function for the M-UDWT. In this work, we employed MI [21], the MSE and SNR as measures of image quality for selecting the optimal wavelet basis function to be used in denoising mammographic images. Computer simulated images were designed and used for the selection phase. The simulated images consisted of 8 strips with different width and various contrast. The simulated images were regarded as different thickness of fibers, which realistically depicts one of the major signs of breast cancer in a mammogram.

## D. Image Dataset

Mammograms were obtained from the data base of the Japanese Society of Medical Imaging Technology [22]. The original screen-film mammograms were collected from several medical institutions and they were digitized using a film digitizer with a pixel size of  $100 \times 100 \mu\text{m}$  and 10-bit gray-level resolution. The size of each image was  $2510 \times 2000$  pixels. A region of interest with a fixed size of  $200 \times 200$  pixels was manually selected. A total of 30 mammograms (14 normal cases and 16 abnormal cases) obtained from the database were used for investigation of the performance of the conventional DWT and the M-UDWT methods.

## E. Visual Perceptual Evaluation

The obtained 30 mammograms were processed using both the M-UDWT and the conventional DWT methods. Thus, a total of 90 images including the original images were used for image quality valuation. In this study, Scheffe's method of paired comparison was employed for visual performance analysis [23, 24]. The visual evaluation was conducted by seven experienced radiological technologists (ranging from 15 to 25 years of experience). All images were evaluated on a pair of popular medical 3M monochrome liquid-crystal display (LCD) monitors ( $2048 \times 1536$  matrix, 700:1 contrast ratio, Mediotto, Nagano, Japan). Each observer reviewed the images independently. The reading time was limited to less than 20 seconds for each reading. The 7 observers independently evaluated one pair of images, which were shown on the monitors at a time, using a 5-point grading scale (-2 points to +2 points). If the image shown on the left is much better than that shown on the right in terms of overall image quality, the left image is given +2 points; the left image is given +1 point when it is slightly better than the right one; the left image is given 0 point, when both images show the same image quality. In contrast, if the image shown on the left is much poorer than that shown on the right in terms of overall image quality, the left image is given -2 points; the left image is given -1 point when it is slightly poorer than the right one. Comparisons were made by use of three possible combinations, that is, original/DWT, original/M-UDWT, and DWT/M-UDWT combinations. Each pair of images was determined randomly. Also, the two separate images (left side vs. right side) were arranged on a random basis.

### III. RESULTS AND DISCUSSION

Results for the simulated noisy images processed by the 5 wavelet basis functions are presented in Table I. It is obvious from the table that the wavelet-processed image with db2 basis function gave the best result among the 5 basis functions in all three quality metrics. Thus, we selected db2 basis function for the M-UDWT and DWT methods.

TABLE I. COMPARISON OF THREE IMAGE QUALITY MEASUREMENTS OF FIVE DIFFERENT WAVELET BASIS FUNCTIONS FOR SIMULATED NOISY IMAGES

Image quality measures	Wavelet basis function				
	dmey	db2	sym7	coif1	coif5
MI (bit)	0.68	0.81	0.72	0.79	0.69
MSE	58.43	50.20	55.87	51.01	57.1
SNR(dB)	27.93	29.10	28.21	28.93	28.04

MI: mutual information, MSE: mean square error, SNR: signal to noise ratio.

An example of the results of applying the M-UDWT method is shown in Fig. 1. Figs. 1(a) and 1(b) are the original and the M-UDWT processed images, respectively. Perceptually, the processed image is less noisy. Fig. 1(c) is the vertical wavelet coefficient of the subband at level 1, and Fig. 1(d) is the profile of the coefficient distribution traced from the line indicated on the image (Fig. 1(c)). Fig. 1(e) shows the new coefficients of the subband at level 1 after performing the second iteration of the processing of the M-UDWT method, and Fig. 1(f) illustrates the profile of the coefficient distribution traced from the line indicated on the image (Fig. 1(e)). In comparison of the coefficient distributions as shown in Figs. 1(d) and 1(f), it is found that the noise has been significantly reduced. This demonstrates the effectiveness of the M-UDWT method.

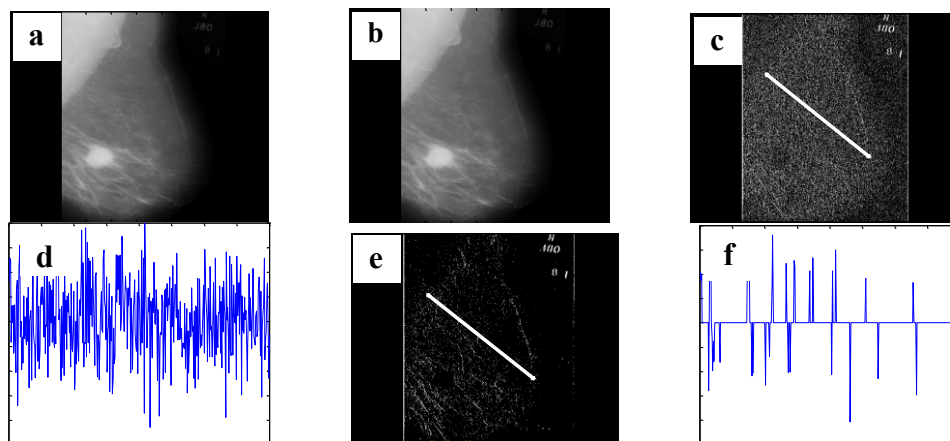


Figure 1. An example showing images and plots of the detailed (vertical) coefficients. **a** Original image, **b** M-UDWT-processed image, **c** vertical wavelet coefficient of sub-band at level 1, **d** profile of the coefficient distribution traced from the line indicated in **c**, **e** new coefficients of sub-bands at level 1, and **f** profile of the coefficient distribution traced from the line indicated in **e**.

The results of scoring for the three combinations by the seven observers are listed in Table II. From the preference scores shown on the right-most column of Table II, the images processed by the M-UDWT method had the best quality. Fig. 2 illustrates visual evaluation results using Scheffe's method of paired comparisons. The results are depicted by a preference ranking map for the three image groups, i.e., original, the conventional DWT-processed, and the M-UDWT-processed image groups. The figures shown on the horizontal line of the map are average preference degrees of the three groups. The average preference degrees were obtained from the average main effects by use of the data shown in Table II. The images processed by the M-UDWT-method shows the highest ranking, followed by the conventional DWT-method processed and the original images. A 2-tailed *F*-test was used to measure statistical significance. The differences between the M-UDWT-method processed images and the conventional DWT-method processed images were statistically significant ( $P < 0.01$ ). The differences between the conventional DWT-method processed images and the original images were statistically significant ( $P < 0.05$ ).

### IV. CONCLUSION

In this study, we compared and evaluated the performance of a conventional DWT method and an M-UDWT method applied to mammographic image denoising. The results of visual assessment indicated that the images processed with the M-UDWT method showed statistically significant superior image quality over those processed with the conventional DWT method. Our research results demonstrated the superiority and effectiveness of the M-UDWT approach. We used mutual information as an evaluation measure for selection of wavelet basis function. The assessment results were consistent with those measured with MSE and SNR. Future work will focus on the combination of the M-UDWT method with contrast enhancement method for further improvement in image quality of mammograms.

TABLE II. RESULTS OF SCORING FOR THE THREE COMBINATIONS BY THE SEVEN OBSERVERS.

Combination	Observer							Sum
	A	B	C	D	E	F	G	
Original / DWT	-2.0	-1.5	-1.0	-1.3	-1.1	-0.8	-1.0	-8.7
Original / M-UDWT	-2.0	-1.0	1.0	-0.2	-1.0	-1.0	-0.2	-4.4
DWT /M- UDWT	-2.0	-1.8	-2.0	-1.0	-1.0	-1.0	-1.3	-10.1

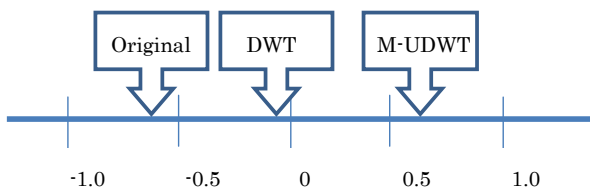


Figure 2. Preference ranking map for the three image groups: original, conventional DWT-processed, proposed UDWT-processed mammograms.

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