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# A sliding window approach to detrended fluctuation analysis of heart rate variability

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*Abstract*— **The analysis of heart rate variability (HRV) aids in the diagnosis of various diseases related to the malfunction of the autonomic nervous system. Traditional approaches for analysis of HRV require the signal to be reasonably stationary during the period of observation. This is not possible when analyzing long duration signals. Detrended fluctuation analysis (DFA) is robust to this issue, as it removes external interferences ("trends") and considers only intrinsic characteristics which are present throughout the signal. DFA is typically performed by segmenting the signal into shorter windows. This has two undesirable effects: (i) if the signal length is not a multiple of the window length, then at least one window will have fewer samples than the others; and (ii) discontinuities are observed on the detrended signal at the edges of each window. Both issues may be addressed using a sliding window. We propose and evaluate this idea, comparing its results with those obtained using the traditional approach. Experiments using different kinds of random and real HRV signals are presented. Statistically significant differences were observed with the proposed approach, especially with respect to $\alpha_2$ values. The proposed method also presented a great reduction in $\alpha_1$ error for white noise, which is a good model for congestive heart failure, with respect to $\alpha_1$ correlations.**

## I. Introduction

The analysis of heart rate variability (HRV) is a non-invasive approach for studying pathologies related to the sympathetic/parasympathetic balance of the autonomic control over the cardiovascular system [1]. Traditional methods for analysis of HRV include time-domain analysis, frequency-domain analysis, and geometrical techniques. Such approaches generally require the signal to be stationary, i.e., the characteristics of the HRV signal must not change considerably within the observation period. This is most problematic when studying long duration signals, as these are more susceptible to external influences.

Detrended fluctuation analysis (DFA) is a tool especially useful for analysis of non-stationary HRV signals [2], and is able to distinguish signals with pathological characteristics from normal signals [2,3]. The goal of DFA is to calculate two coefficients, $\alpha_1$ and $\alpha_2$, which reflect the short-term and long-term correlation, respectively, of a "detrended" signal.

One important step of DFA involves segmenting the signal into shorter windows. This has two undesirable effects. First, if the signal length is not a multiple of the window length, and no samples are to be discarded, then at least one window will have fewer samples than the others. If a window is considerably shorter than the others, than the energy of the detrended signal within that window will be much lower than that of the rest of the signal. Second, discontinuities are observed on the detrended signal at the edges of each window. These two effects are observed especially when calculating the $\alpha_2$ coefficient, which requires using large windows.

Both issues may be addressed using a sliding window approach. We propose and evaluate this idea, comparing its results with those obtained using the traditional, non-overlapping window, approach. Experiments using different kinds of random and real HRV signals are presented.

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## II. Materials and Methods

### A. Traditional DFA calculation

RR intervals present complex variability patterns, and long-term correlations [2]. DFA quantifies these correlations, and is able to distinguish the intrinsic features of the HRV signal from the non-stationary external trends [2].

The calculation of DFA is based on finding two coefficients, $\alpha_1$ and $\alpha_2$, that characterize the correlation properties of HRV signals. The traditional approach for calculating these two coefficients for a signal $RR(n)$ of length $N$ can be described as follows [2,3]:

1. Obtain $y(n) = \sum_{\eta=0}^{n}[RR(\eta) - \overline{RR}(\eta)]$, where $\overline{RR}(\eta)$ is the mean value the HRV signal $RR(n)$.

2. The "integrated signal", $y(n)$, is segmented into multiple windows of length $l_k$. For each window, a first-order least-square fitted polynomial is calculated. The "trend signal", $y_k(n)$, is a piecewise-linear approximation of $y(n)$, which is obtained by replacing the samples of $y(n)$ with the values obtained by linear fitting within each window.

3. Calculate the approximation error, $e_k(n)$, i.e. the "detrended signal", as: $e_k(n) = y(n) - y_k(n)$.

4. Calculate the root mean square value of the approximation error, i.e.: $E_k = \sqrt{\frac{1}{N}\sum_{n=0}^{N-1} e_k^2(n)}$.

5. Repeat steps 2 through 4 for multiple integer values of $l_k$.

6. Obtain the curve $f(x)$, where $x = \log_{10}(l_k)$, as $f(x) = \log_{10}(E_k)$. $E_k$ has an exponential relationship with $l_k$, thus $f(x)$ will have a linear relationship with $x$.

7. Therefore, a first-order least-square fitted polynomial is calculated for $f(x)$, for values of $x$ such that $4 \leq l_k \leq 16$. The angular coefficient of this first-order polynomial will be called $\alpha_1$.

8. Step 7 is repeated for values of $x$ such that $16 \leq l_k \leq N$. The angular coefficient of this polynomial will be called $\alpha_2$.

*B. A sliding window approach to DFA*

In the algorithm proposed above, if $N / l_k \notin \mathbf{Z}$, then the length of the last window of the signal would be less than $l_k$ (Fig. 1a). Alternatively, the samples from the last window could be discarded. With the first approach, the accuracy of $E_k$ calculation would be affected, because the energy of the approximation error $e_k(n)$ within the shorter window would be lower than on the rest of the signal. This happens because $y_k(n)$ becomes more similar to $y(n)$ as the window length gets smaller, resulting in reduced approximation error. The second approach could also provide reduced accuracy, because the signal would not be analyzed in its entirety.

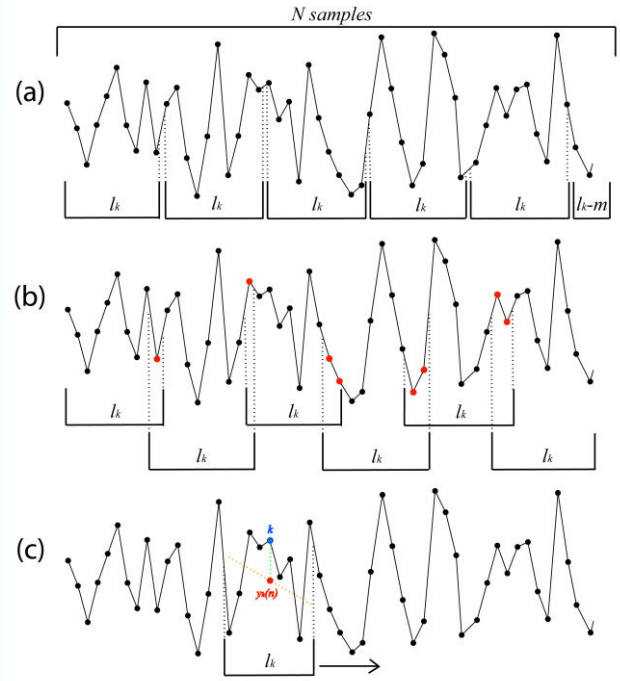


Figure 1. Different approaches for segmenting a $N$-sample signal into multiple segments (windows) of length $l_k$: (a) traditional non-overlapping window approach; (b) overlapping window approach; and (c) proposed sliding window approach.

We wish that all windows are exactly $l_k$-sample long, and that no samples are discarded. This can be achieved using overlapping windows (Fig. 1b). However, this would not eliminate the discontinuities observed in the detrended signal, at the edges of each window. These are due to the piecewise-linear characteristic of the trend signal (Fig. 2a).

In order to obtain a smooth trend signal, and thus eliminate the discontinuities in the detrended signal, we go one step further: we propose a sliding window approach. Instead of simply using overlaps, we proposed to replace step 2 of the above algorithm with the two following:

> *"For each of the **N** samples of the integrated signal, **y(n)**, i.e. for **η = 1, 2, …, N**, the value of **$y_k$(η)** is obtained by: (i) taking a segment of **y(n)**, with **$l_k$** samples, centered around the **η**–th sample of **y(n)** (precise centering is not possible for **η < $l_k$/2** or **η > N − $l_k$/2**); (ii) least-square fitting a first-order polynomial, **y'(n)**, to said segment; (iii) evaluating this polynomial at **n = η**, and making **$y_k$(η) = y'(η)**."*

This process is illustrated in Fig. 1c. An example of the smooth trend signal obtained with this approach is shown in Fig. 2b. Note the detrended signal with no discontinuities.

*C. Quantitative evaluation*

In order to compare the proposed sliding-window approach with the traditional non-overlapping window approach, we used different kinds of both random and real HRV signals, as follows.

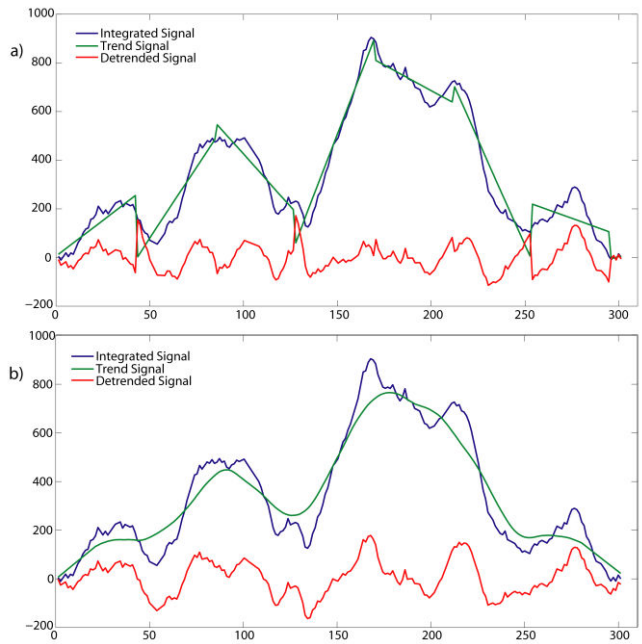


Figure 2. Integrated signal, trend signal and detrended signal obtained with: (a) the traditional non-overlapping window approach; and (b) the proposed sliding window approach.

The random signals were created with different power-law correlation characteristics: white noise, pink noise ($1/f$) and Brownian noise ($1/f^2$) [2,3]. The expected $\alpha$ coefficient for each of these kinds of noise are 0.5, 1.0 and 1.5, respectively [2]. The mean error (from these expected values) and standard deviation associated with each approach were calculated for groups of 250 random signals of each kind. Each random signal was created with 4096 samples.

The real HRV signals were obtained from the Physiobank database (http://www.physionet.org/physiobank/) [4, 5, 6]. The selected signals are from eleven normal subjects, 8 women and 3 men, during sleeping hours [4]; nine elite athletes, 3 women and 6 men, during sleeping hours [4]; eight Chi meditators, 5 women and 3 men, during meditation periods and during rest periods [4]; twelve subjects, 11 men e 1 woman, with sleep apnea, whose data were obtained while they were sleeping [5]; and seven subjects, 5 women and 2 men, with mild epileptic seizures [6]. One outlier was removed from the two groups of signals from meditators. Ectopic beats, and false positives and negatives were identified, removed and corrected (by cubic spline interpolation), using the ECGLab software [7]. Each signal has about 20,000 samples.

The results obtained with the two approaches were compared using statistical tests. Student's t-test was used for the random signals, as all groups showed a positive result to the normality test (Shapiro-Wilk test with significance $\alpha$ = 0,05). For the real HRV signals, the nonparametric Friedman ANOVA test was used, because not all groups tested positive for normality. For both random and real signals, the relation between the results from the two approaches was also evaluated, based on the correlation and regression coefficients.

## III. Results

Table 1 presents the mean error (from the expected value, $\alpha$) and standard deviation for the coefficients $\alpha_1$ and $\alpha_2$, calculated using the traditional (non-overlapping window) and proposed (sliding window) approaches. The expected value was defined as 0.5, 1.0, and 1.5, for white, pink and Brownian noise, respectively [2]. For white noise signals, the proposed approach presented better accuracy ($\|(e_1,e_2)\|$ = 0.072) than the traditional approach ($\|(e_1,e_2)\|$ = 0.095). While the $\alpha_2$ error is larger with the proposed method than with the original approach, this is compensated by a great reduction in $\alpha_1$ error, which is much more predominant in this case. This is an important result, as white noise is a good model for congestive heart failure with respect to $\alpha_1$ correlations [2]. For pink and Brownian noise, the mean error is larger for the proposed method ($\|(e_1,e_2)\| \approx 0.058$) than for the original approach ($\|(e_1,e_2)\|$ = 0.051), but only slightly. The precision of the two approaches appears to be similar (approximately the same standard deviation) for all three kinds of noise, and for both $\alpha_1$ and $\alpha_2$.

TABLE 1: Mean error (from the expected value, $\alpha$) and standard deviation for the coefficients $\alpha_1$ and $\alpha_2$, calculated using the traditional (non-overlapping window) and proposed (sliding window) approaches. The expected value was defined as 0.5, 1.0, and 1.5, for white, pink and Brownian noise, respectively [2].

| | | mean ($\times 10^{-3}$) | | std. deviation ($\times 10^{-3}$) | |
|---|---|---|---|---|---|
| | | trad. | prop. | trad. | prop. |
| $e_1$ † | white | 90 | 64 | 12 | 11 |
| | pink | 29 | 38 | 16 | 13 |
| | Brownian | 2 | 25 | 16 | 12 |
| $e_2$ ‡ | white | −8 | −16 | 29 | 30 |
| | pink | −19 | −25 | 42 | 42 |
| | Brownian | −23 | −25 | 54 | 52 |
| $\|(e_1,e_2)\|$ ♯ | white | 95 | 72 | 13 | 14 |
| | pink | 51 | 58 | 25 | 25 |
| | Brownian | 51 | 57 | 33 | 31 |

† where $e_1 = \alpha_1 - \alpha$.
‡ where $e_2 = \alpha_2 - \alpha$.
♯ Euclidian norm of the error vector, $(e_1,e_2)$.

Student's t-test showed that, for the random signals, the $\alpha_1$ and $\alpha_2$ coefficients calculated using the sliding window approach are statistically different ($p < 0,05$) from those calculated with the non-overlapping window approach. For the real HRV signals, however, statistically different results ($p < 0,05$) were observed for all groups of subjects only for $\alpha_2$ (Table 2). This can be explained by the fact that $\alpha_1$ calculation uses only very short windows ($4 \leq l_k \leq 16$), for which the issue of having a single window shorter than the others won't be significant, given the total length of the signal. Nevertheless, two of the six groups presented significantly different results also for $\alpha_1$: meditators, during meditation; and apneic patients.

TABLE 2: Results ($p$ values) from the ANOVA tests, comparing the traditional (non-overlapping window) and proposed approaches.

| | $\alpha_1$ | $\alpha_2$ |
|---|---|---|
| normals | 0.37 | 0.04* |
| athletes | 0.74 | 0.10 |
| meditators (rest) | 0.26 | 0.01* |
| meditators (meditating) | 0.01* | 0.01* |
| apneics | 0.00* | 0.02* |
| epileptics | 0.71 | 0.01* |

* statistically significant difference ($p < 0,05$).

Table 3 presents the results of an evaluation of the relation between the results from the traditional and proposed approaches, based on the correlation ($r$) and regression ($m$) coefficients between the two. For $\alpha_2$, the two approaches are extremely correlated ($r \approx 1$) and present very similar results ($m \approx 1$). For the real HRV signals, the same is true for $\alpha_1$. For the noise signals, the correlation between $\alpha_1$ values measured with the two approaches are somewhat correlated ($0.75 < r < 0.90$), but not as similar: the dynamic range of the $\alpha_1$ values calculated with the proposed approach seem to decrease (relative to that of the traditional approach) as the true value of $\alpha_1$ increases ($m$ decreases from 0.84 for white noise to 0.6 for Brownian noise), which suggests higher precision.

TABLE 3: Evaluation of the relation between the results from the traditional and proposed approaches.

| | $\alpha_1$ | | $\alpha_2$ | |
|---|---|---|---|---|
| | correlation coefficient | regression coefficient | correlation coefficient | regression coefficient |
| white noise | 0.87 | 0.84 | 0.98 | 1.03 |
| pink noise | 0.83 | 0.68 | 0.98 | 0.98 |
| Brownian noise | 0.78 | 0.60 | 0.97 | 0.95 |
| real HRV signals | 1.00 | 1.06 | 1.00 | 1.01 |

Figure 3 presents the values of $\alpha_1$ and $\alpha_2$ calculated for real HRV signals from different groups of subjects, using the traditional and proposed approaches. While statistically significant differences were observed between the two approaches, this visual analysis shows that they provided very similar results for these subjects. This corroborates the results from Table 3. This visual analysis also suggests that both approaches are able to differentiate between most of the studied groups, with the exception of normal and athletes. A statistical evaluation of this statement will be presented in a future work.

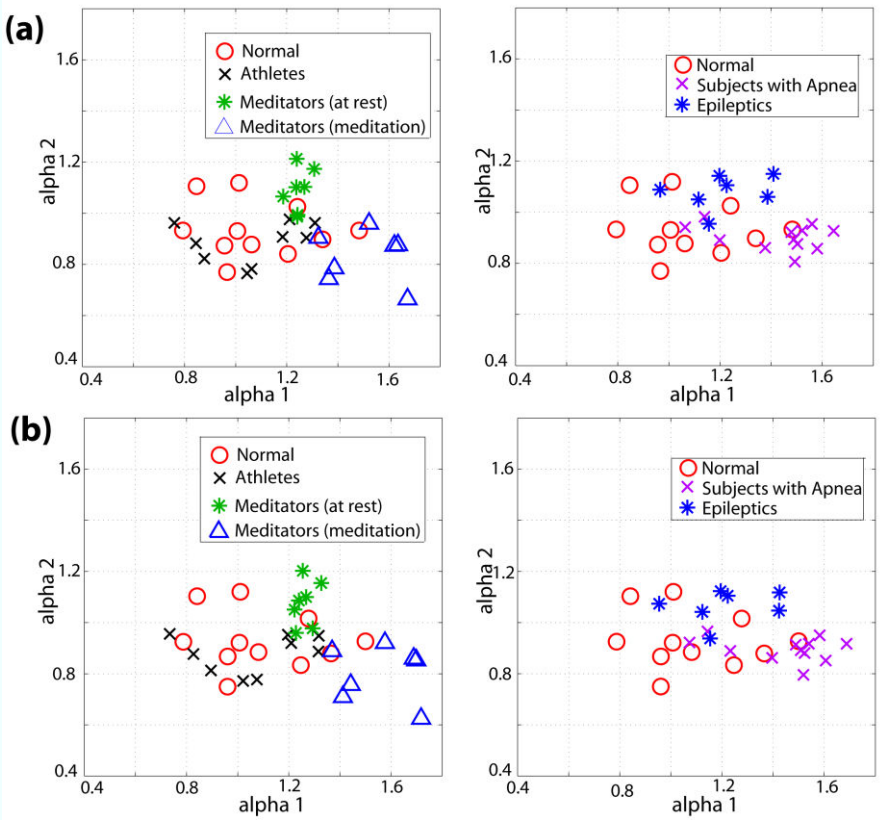


Figure 3. Coefficients $\alpha_1$ and $\alpha_2$ calculated for real HRV signals from different groups of subjects using: (a) the traditional non-overlapping window approach; and (b) the proposed sliding window approach.

## IV. Conclusion

This paper proposed a sliding window approach to detrended fluctuation analysis of heart rate variability, which provides a smooth detrended signal, instead of a piecewise-linear signal. The proposed approach aims to correct two issues with the traditional non-overlapping window approach, regarding uneven window lengths and discontinuities in the detrended signal.

The values of $\alpha_1$ and $\alpha_2$ coefficients calculated using the sliding window approach are statistically different from those calculated with the non-overlapping window approach. While the results from the two approaches are quantitatively similar, extremely correlated, and with similar precision for both $\alpha_1$ and $\alpha_2$, the proposed approach presented some advantages for specific cases. For white noise signals, for example, the proposed approach presented better accuracy than the traditional approach, with a great reduction in $\alpha_1$ error. This is an important result, as white noise is a good model for congestive heart failure with respect to $\alpha_1$ correlations. Both approaches seem able to visually differentiate between most of the studied groups. A statistical evaluation of DFA's ability to differentiate between groups of subjects will be presented in a future work.

The proposed approach has been incorporated into the DFA software designed by Leite *et al.* [3]. This software tool implements DFA in a friendly graphical interface, and is available at http://pgea.unb.br/~joaoluiz/. Since it has been proposed for analysis of HRV, DFA has shown potential for separating between different pathologies and autonomic conditions. The proposed sliding window approach, combined with this friendly tool, can be a significant contribution towards making DFA more well-understood and accepted by the scientific community.

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