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Directional Dual-Tree Complex Wavelet Packet Transform

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Abstract—**Doppler ultrasound systems, which are widely used in cardiovascular disorders detection, have quadrature format outputs. Various types of algorithms were described in literature to process quadrature Doppler signals (QDS), such as phasing filter technique (PFT), fast Fourier transform method, frequency domain Hilbert transform method and complex continuous wavelet transform. However for the discrete wavelet transform (DWT) case, which becomes a common method for processing QDSs, there was not a direct method to recover flow direction from quadrature signals. Traditionally, to process QDSs with DWT, firstly directional signals have to be extracted and later two DWTs must be applied. Although there exists a complex DWT algorithm called dual tree complex discrete wavelet transform (DTCWT), it does not provide directional signal decoding during analysis because of the unwanted energy leaks into its negative frequency bands. Modified DTCWT, which is a combination of PFT and DTCWT, has the capability of extracting directional information while decomposing QDSs into different frequency bands, but it uses an additional Hilbert transform filter and it increases the computational complexity of whole transform. Discrete wavelet packet transform (DWPT), which is a generalization of the ordinary DWT allowing subband analysis without the constraint of dyadic decomposition, can perform an adaptive decomposition of the frequency axis. In this study, a novel complex DWPT, which maps directional information while processing QDSs, is proposed. The success of proposed method will be measured by using simulated quadrature signals.**

I. Introduction

Quadrature signals, which are composed of in-phase and quadrature-phase parts, are used in many digital signal processing applications in many fields of science and engineering. Specifically, Doppler ultrasound is a non invasive diagnostic technique used to evaluate cardiovascular disorders in blood flow measurement systems, where signals are usually in quadrature format. In quadrature Doppler signals (QDSs), the blood flow direction information is encoded into the phase relationship between in-phase and quadrature-phase components [1]. Various methods in literature were described to obtain totally separated directional Doppler signals from the quadrature Doppler signals such as phasing filter technique (PFT), extended weaver receiver technique, complex fast Fourier transform method, frequency domain Hilbert transform method and spectral translocation method [2]. The PFT, which uses a Hilbert transform (HT) filter and a delay filter, is most widely used method in literature.

In [3], a complex continuous wavelet transform (CCWT) algorithm which maps the directional information in the scale domain, while doing the analysis, was proposed. However, for the conventional discrete wavelet transform case, an algorithm which works directly on quadrature signals and maps directional coefficients in the scale domain during analysis does not exist. Although there exists a complex DWT algorithm, called dual tree complex discrete wavelet transform (DTCWT) [4], which is an improved version of ordinary DWT, it does not provide directional signal decoding during analysis because of the unwanted energy leakages into its negative frequency bands. To overcome this drawback, a modified dual tree complex wavelet transform, which is a combination of PFT and DTCWT, was proposed in [5].

DWT represents discrete-time signals in dyadic subband decomposition. However, for most of the biomedical signals, the frequency decomposition provided by the DWT and also by the DTCWT might not be optimal. Generalization of the DWT in the discrete wavelet packet transform (DWPT) allows subband analysis without the constraint of dyadic decomposition. The DWPT performs an adaptive decomposition of the frequency axis. The specific decomposition may be selected according to an optimization criterion [6]. To find a more suitable decomposition, a “best” basis can be chosen from a structured dictionary of bases and, at the end of this procedure, analyzed signals can be represented by as few and large coefficients as possible [7]. This adaptive and sparse decomposition was used before in a wide class of problems like signal analysis, filtering or compression in literature.

Ordinary DWT lacks of being shift-invariant and it does not provide a geometrically oriented decomposition in multiple dimensions. These drawbacks are also valid for DWPT because it uses same high-pass/low-pass filter pair in the analysis and synthesis part of the transform. In the full DWPT, as well as both low-pass and high-pass outputs are iterated.

DTCWT was proposed as an alternative for the DWT that does have better shift-invariance property and provides better directional selectivity in M dimensions (M is equal to or greater than two). Shift-invariance property is very important in the process of quadrature Doppler signals because the

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direction information is encoded in the phase relationship between in-phase and quadrature-phase components.

The DTCWT employs two real DWTs; the first DWT can be thought as the real part of the transform while the second DWT can be thought as the imaginary part of the transform. The wavelet filter bank (FB) used in second tree was designed according to a specific rule in order to provide shift-invariance property. With respect to this rule, the second imaginary tree's wavelet FB is designed so that while it decomposes the input signal into different frequency bands (like the real tree), it additionally takes the HT of the input. Hence, the output of real tree and output of imaginary tree becomes a HT pair. Nevertheless, in DTCWT, this parity is not perfect because of the unwanted energy leakages into its negative frequency bands.

In [8], a dual-tree wavelet packet transform (DT-CWPT) was described which has the same shift-invariance and good directional selectivity properties of DTCWT, and it also has fewer energy leakages into its negative frequency bands.

In order to use DWPT in the analysis of quadrature Doppler signals, firstly directional signals (forward signal and reverse signal) must be obtained and only then DWPT can be applied to these two directional signals. In real-time applications, to process quadrature signals with DWPT, firstly a HT filter and a delay filter must be used in PFT to obtain directional signals and later two DWPTs can be used. This procedure increases the computational cost of the whole process. To decrease the computational cost of processing quadrature Doppler signals with ordinary DWPT, the HT property of the analysis and synthesis filters of DT-CWPT (proposed in [8]) can be used.

In this study, a novel method for processing quadrature Doppler signals with complex wavelet packet transform will be proposed, this method will be named as directional dual tree complex wavelet packet transform (DDT-CWPT). With DDT-CWPT, the HT filtering part in PFT will be eliminated. The required phase shift will be achieved by using the second wavelet FB in DT-CWPT.

II. METHOD

A. Quadrature Signals

To understand the proposed method, the nature of the quadrature signals must be examined in detail. A quadrature Doppler signal can be assumed as a complex signal, in which the real (*D(*n*)*) and imaginary (*Q(*n*)*) parts can be represented as the HT of each other.

In quadrature Doppler signals, the direction information is encoded in *D(*n*)* and *Q(*n*)* components. *D(*n*)* and *Q(*n*)* can also be represented in terms of the directional signals as

$$D(n) = s_f(n) + H[s_r(n)] \quad (1)$$

$$Q(n) = s_r(n) + H[s_f(n)] \quad (2)$$

where $s_f(n)$ and $s_r(n)$ is used for forward and reverse signals, respectively, and H[ ] is used for the HT.

In PFT, firstly the *D(*n*)* component's HT is taken and the resulting signal is added to and subtracted from delayed version of *Q(*n*)* component. As a result of these steps, forward and reverse direction signals are obtained. Extracting direction information using PFT can be described mathematically as below

$$H[D(n)] = H[s_f(n)] + H[H[s_r(n)]] = H[s_f(n)] - s_r(n) \quad (3)$$

When we add $H[D(n)]$ and $Q(n)$, we obtain

$$H[s_f(n)] - s_r(n) + s_r(n) + H[s_f(n)] = 2H[s_f(n)] \quad (4)$$

When we subtract $Q(n)$ from $H[D(n)]$, we obtain

$$H[s_f(n)] - s_r(n) - s_r(n) - H[s_f(n)] = -2s_f(n) \quad (5)$$

After those calculations as it can be seen above, the direction information is extracted.

In this study, to assess the proposed method, *D(*n*)* and *Q(*n*)* components will be simulated by using sines and cosines. The simulated signals can be seen below

$$D(n) = A\cos\left(2\times\pi\times n\times\frac{f_A}{f_s}\right) + B\sin\left(2\times\pi\times n\times\frac{f_B}{f_s}\right) \quad (6)$$

$$Q(n) = A\sin\left(2\times\pi\times n\times\frac{f_A}{f_s}\right) + B\cos\left(2\times\pi\times n\times\frac{f_B}{f_s}\right) \quad (7)$$

A and *B* are the amplitudes and they are equal. f_A and f_B are the frequencies of forward and reverse signals, and they are chosen as 400 Hz and 1200 Hz. f_s is the sampling frequency and chosen as 10 kHz. Assuming the signal is discrete, n is sample index.

B. Directional Dual-Tree Complex Wavelet Packet Transform

In order to extend the DTCWT into DT-CWPT, the first way can be iterating both its low-pass and high-pass perfect reconstruction FBs' outputs using the same filters set. This type of implementation was proposed previously in [9]. However, the resulting subbands are far from being analytic and there are significant energy leakages into negative frequency band. So, this approach does not fully have the desired perfect HT pair property.

To overcome this drawback of the straightforward construction, in [8], a new DT-CWPT, which has better HT pair property and has fewer energy leakages, was developed. This new DT-CWPT can also be named as approximately analytic DT-CWPT. The frequency responses of this transform and the other transforms can be seen in Figure 1.

It is possible to see from the figure that the energy leakages in analytic DT-CWPT are limited. This shows that the second tree is able to carry out the HT in a very effective way.

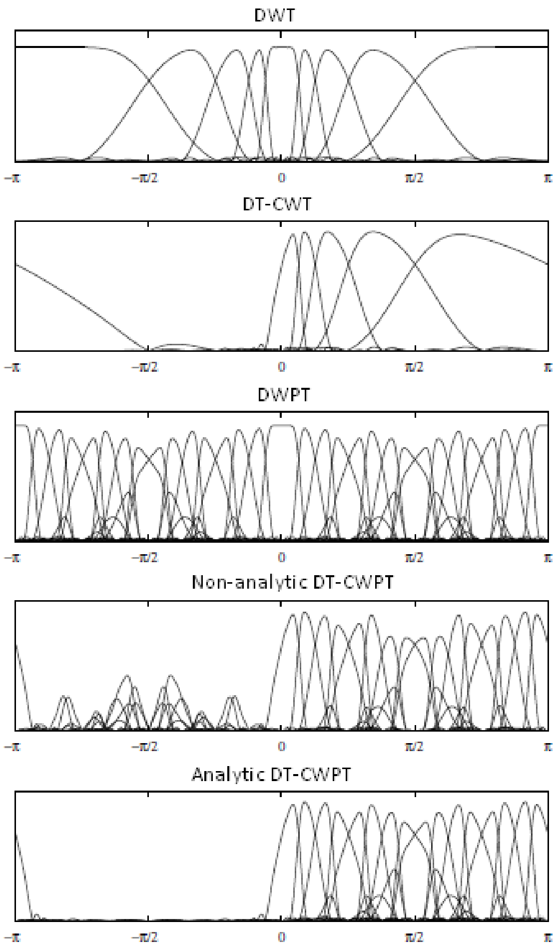

Fig. 1. Frequency responses of DWT, DTCWT, DWPT, non-analytic DT-CWPT, and analytic DT-CWPT. (This figure is taken from reference [8])

In PFT to extract directional information from quadrature signals, firstly a HT filter and delay filter must be used. In analytic DT-CWPT, the imaginary tree has the capability of taking the HT of input signal while decomposing it into different frequency bands. Hence, in order to obtain directional coefficients during analysis, two modifications can be done to analytic DT-CWPT. Firstly, instead of giving the same signal to the real and imaginary tree, $D(n)$ will be supplied to imaginary tree and $Q(n)$ will be supplied to real tree. By doing so, at the output of imaginary tree, Hilbert transformed coefficients of $D(n)$ will be obtained. Secondly, at the end of analysis part of the transform, the outputs of real and imaginary trees will be added and subtracted. These calculations will result the forward and inverse information coefficients. Later, these directional coefficients will be reconstructed by the synthesis filters resulting in directional signals. With these two modifications, the PFT part will be eliminated and this will decrease the computational complexity of whole transform. This new modified transform is named as directional DT-CWPT (DDT-CWPT) due to its property of obtaining directional coefficients during analysis. The structure of the proposed transform can be seen in Figure 2.

There are some non-analytic subbands (the lower and upper subbands) in this transform. However, in most cases QDSs are band limited signals. Therefore the directional signals are not affected by these non-analytic sub-bands.

III. RESULTS

A. *Performance Metric*

In order to measure performance of the proposed method, 4096 points simulated signals are created using the format mentioned before. These simulated signals are processed by using PFT and directional signals are obtained. Later, same signals are decomposed and reconstructed by using DDT-CWPT. As mentioned before, in analytic DT-CWPT the upper and lower subbands do not have analyticity property. Therefore there will be some differences between PFT output and the DDT-CWPT output. This difference is correlated with the width of these lower and upper subbands. Therefore different decomposition levels must be tested.

To measure the difference, an error metric given by (8) is defined,

$$Error = \frac{\sum_n \left| X_{PFT} - X_{DDT-CWPT} \right|}{\sum_n \left| X_{PFT} \right|} \times 100 \qquad (8)$$

X_{PFT} represents the directional signal obtained by the PFT and $X_{DDT\text{-}CWPT}$ represents the directional signal obtained by the DDT-CWPT. Error ratio is calculated for both forward and reverse signal, for various decomposition levels.

B. *Results*

In Figure 3, the directional outputs of the PFT and 6-levels DDT-CWPT can be seen. As expected, with DDT-CWPT, forward and reverse signals are obtained, and the results for both methods are very similar.

In Table 1, the error ratios for different decomposition levels can be seen. For low number of levels (3 and 4), the ratio is big as expected because in this situations the width of the lower and upper subbands are large. On the other hand, when the numbers of levels are increasing, the error ratio becomes smaller due to the narrower width of the lower and upper subbands.

Table 1. Error ratios for different levels

Level	Forward Error Ratio	Reverse Error Ratio
3	% 27.700	% 27.579
4	% 5.450	% 5.368
5	% 1.633	% 1.667
6	% 1.571	% 1.602
7	% 1.561	% 1.603
8	% 1.560	% 1.602

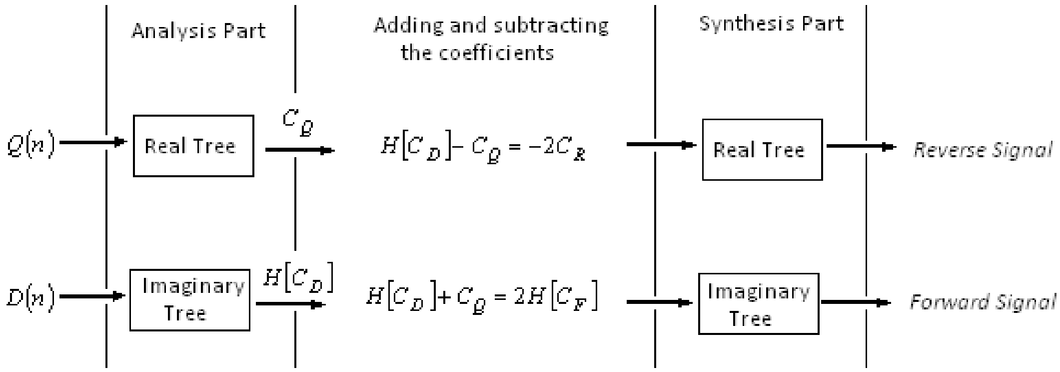

Fig.2. Structure of DDT-CWPT. C_Q represents coefficients of *Q*(n), C_D represents coefficients of *D*(n), C_F represents coefficients of forward signal, C_R represents coefficients of reverse signal. H[] stands for Hilbert Transform.

IV. CONCLUSION

In this study, a directional DT-CWPT, which is applied directly to QDSs and has the ability of extracting direction information during analysis, is proposed. With this proposed complex wavelet packet transform, the PFT step, which is used for extracting directional signals before wavelet analysis, is eliminated and the computational cost of the whole transform is reduced. To measure the performance of proposed method, a simulated quadrature signal is constructed and this signal is processed with PFT and DDT-CWPT. The outputs of both methods are compared with an error ratio metric. As it can be seen from Table 1, for levels higher than four, the difference is very small and in real-world applications most of the times decomposition level is chosen as five or higher.

It is known that in QDSs the information carrying parts, directional signals, have band-limited characteristic. Therefore, in real-world applications, the distorting affect of the energy leakages will be very limited and will not corrupt direction information.

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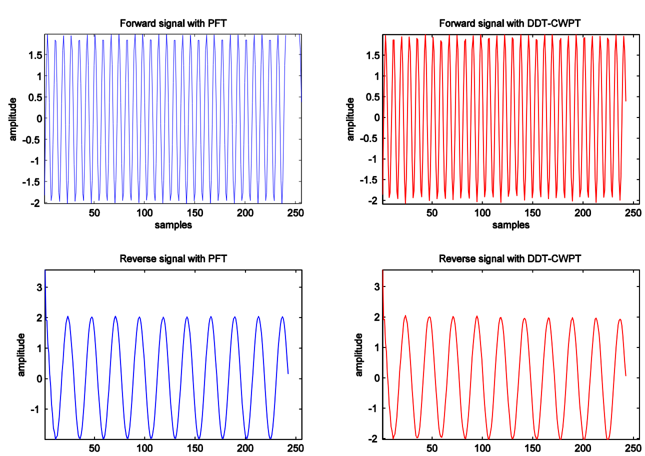

Fig.3. The first 256 points of obtained signals with PFT and DDT-CWPT.