

Optimization of surgical planning of total hip arthroplasty based on computational anatomy*

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Abstract— This paper describes a method for automated optimization of total hip arthroplasty (THA) planning incorporating joint functionalities. The optimal planning is formulated as maximum a posterior (MAP) estimation, which ensures the best-balance of joint functionalities and bone-implant spatial relations based on their statistical models derived from the training datasets prepared by an experienced surgeon. According to the performance evaluation, four of the six functionalities of the automatically optimized plans were almost equivalent to those of surgeon's plans, and two of them were improved. We consider these results showed a potential usefulness of the proposed method.

I. INTRODUCTION

In the field of computer aided surgery, surgical computer-aided design (CAD) / computer-aided manufacturing (CAM) paradigm have been recently proposed [1]. The surgical CAD/CAM have realized more accurate surgery, as CAD systems enable surgeons to perform 3-dimensional (3D) pre-operative planning and CAM systems enable intra-operative navigation and robotization [2-7]. Especially in total hip arthroplasty, the CAM systems have been intensively studied in the past decade, on the other hand, less attention has been paid to the CAD systems. On the CAD systems, surgeons determined the size (one parameter) and 3D positions (three parameters), and 3D angles (three parameters) for each implants with considering various evaluation criteria which indicates the implant compatibility and hip joint functionalities [4-6]. As some of the criteria have trade-off relationships, finding the best-balanced solution often become time-consuming tasks for surgeons. Therefore automation of surgical CAD is highly desirable.

In this paper, we describe a method for automated optimization of total hip arthroplasty (THA) planning using computational anatomy and hip joint functionalities. As far as we know, there are no reports of automated optimization methods of pre-operative THA planning based on a maximum a posterior (MAP) estimation approach except ours. In our previous works, we proposed the automated optimization methods for single implant [8][9], however, they are not able to deal with combined implants. Then, we formulate the optimal planning as a MAP estimation, which ensures the

best-balance of bone-implant spatial relations and hip joint functionalities based on their statistical models derived from the training datasets. To evaluate the proposed method, we compared the implant size and hip joint functionalities between the plans of the proposed methods and surgeon's.

II. METHODS

The artificial hip joint is consisted of four components: the implant placed in pelvis is called cup, the implant placed in femur is called stem, joint component of pelvic side is called insert, and joint component of femoral side is called head. We assume that the pelvis and femur shapes reconstructed from patient 3D CT data are given as input datasets. The coordinate systems of pelvis, femur, and hip joint were assumed to be determined automatically by the bony landmarks of SSM [10].

Figure 1 shows the system overview of automated optimization of pre-operative planning for combined pelvic and femoral implants. Firstly, we describe the MAP formulation of our previous method for single-implant planning (pelvic cup). Secondly, we propose the MAP formulation of automated optimization method for combined implant planning based on computational anatomy and hip joint functionalities.

MAP formulation of our previous method for single-implant planning (pelvic cup)

In our previous work [8], the statistical shape model (SSM) of combined shapes of the pelvis and implanted acetabular cup was constructed from the training datasets of the cup plan, which is called as pelvis-cup merged statistical shape model (PC-SSM). Let X and Y be pelvis and cup shape parameters represented in SSM, respectively. And let Θ be the pre-operative planning parameters. As the cup size (r), position, and orientation (described as 4×4 matrix T) are implicitly embedded in the cup shape, we describe cup shape parameters as $Y(\Theta)$. The SSM defines prior probability $p(X, Y)$ modeled by Gaussian distribution whose covariance matrix is obtained by principal component analysis. Then, the problem is regarded as

$$\Theta^* = \arg \max_{\Theta} p(X, Y(\Theta)|D) \quad (1)$$

$$\Theta = \{T, r\} \quad (2)$$

Based on the Bayes' rule, the posterior probability $p(X, Y|D)$ could be described below.

$$p(X, Y(\Theta)|D) \propto p(X, Y(\Theta))p(D|X, Y(\Theta)) \quad (3)$$

As D depends only on X , $p(X, Y)p(D|X)$ amounts to $p(X, Y)p(D|X, Y)$. Therefore, Eq. (1) is rewritten by

$$\Theta^* = \arg \max_{\Theta} p(X, Y(\Theta))p(D|X) \quad (4)$$

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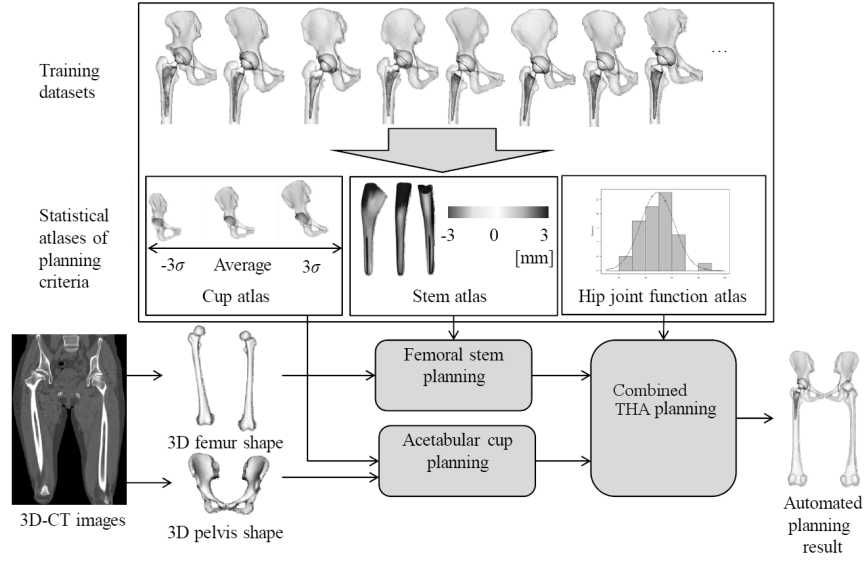


Figure 1. Schematic overview of automated combined pre-operative planning system.

where $p(D|X)$ is a likelihood, and $p(X)$ is a prior probability of X in the PC-SSM. In optimization of combined pre-operative planning, we obtain the optimal planning parameters Θ^* with maximizing Eq.(4). Now we define the likelihood $p(D|X)$ and prior probability $p(X)$. The likelihood becomes higher value when the pelvis shape generated by X become closer to the patient anatomy. Then we define the likelihood function with pelvis shape generated by X and shape differences with patient anatomy. When we let $s_j(X)$ be the j -th vertex of the polygonal model X , we define the shortest distance between $s_j(X)$ and patient anatomy D . If we assume that L_j obey gaussian distribution with mean value of 0 and variance of α , the likelihood $p(D|X)$ is defined as

$$p(D|X) = \left(\prod_{j=1}^g \frac{1}{\sqrt{2\pi\alpha}} \exp\left(-\frac{L_j^2}{2\alpha}\right) \right)^{\frac{1}{g}} \quad (5)$$

$$L_j = l(s_j(X), D) \quad (6)$$

where $l(s_j(X), D)$ indicates the shortest distance from $s_j(X)$ to D . Next, we define the prior probability $p(X, Y(\Theta))$. The prior probability amounts to the probability distribution function of $X, Y(\Theta)$ in the PC-SSM. Now we let the number of elements of $X, Y(\Theta)$ be h and assume that each element b_k ($k=1, \dots, h$) of $X, Y(\Theta)$ obey gaussian distribution with mean value of 0 and variance of λ_k . Then $p(X, Y(\Theta))$ is written below.

$$p(X, Y(\Theta)) = \frac{1}{(\sqrt{2\pi})^h \sqrt{|P|}} \exp\left(-\frac{1}{2} \sum_{k=1}^h \frac{b_k^2}{\lambda_k}\right) \quad (7)$$

$$P = \begin{bmatrix} \lambda_1 & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & \lambda_h \end{bmatrix} \quad (8)$$

where P is the covariance matrix of shape parameters $X, Y(\Theta)$. Determining b_k is equivalent to determining Θ , as finding b_k means finding $X, Y(\Theta)$. When we take the logarithm of Eq.(4), Θ^* is given by

$$\Theta^* = \arg \max_{\Theta} (\ln(p(X, Y(\Theta))) + \ln(p(D|X))) \quad (9)$$

By substituting Eq.(5) and Eq.(7) for Eq.(9), Eq.(10) is obtained.

$$\Theta^* = \arg \min_x \left(\frac{1}{g} \sum_{j=1}^g \frac{L_j^2}{\alpha} + \sum_{k=1}^h \frac{b_k^2}{\lambda_k} \right) \quad (10)$$

Therefore, the system find a cup pre-operative plan which minimizes Eq.(10) in the automated optimization of cup shape parameters.

Extending the MAP formulation to incorporate joint functionalities

As it is required to adjust not only the fitness of implants but also hip joint functionalities in pre-operative planning of combined pelvic and femoral implants, we extend the MAP formulation in order to optimize all of them. The joint functionalities are related to both the cup and stem. In this study, we assume that the stem plan is determined only using the femur-stem statistical model described in our previous work [9] because automated stem planning is sufficiently stable by itself.

The combined pre-operative planning parameters Θ' is consisted of cup size r' , 4×4 matrix T' describing position and pose of cup, and head offset o of femoral side component. Let Z be joint functionality parameters and $p(Z)$ be their prior probability distributions. As the combined planning parameters are implicitly embedded in the cup planning parameters and normalized joint functionality parameters, we describe them as $Y(\Theta')$ and $Z(\Theta')$, respectively. The problem is regarded as

$$\Theta'^* = \arg \max_{\Theta'} p(X, Y(\Theta'), Z(\Theta')|D) \quad (11)$$

$$\Theta' = \{T', r', o\} \quad (12)$$

Based on the Bayes' rule, $p(X, Y(\Theta'), Z(\Theta')|D)$ could be described below.

$$p(X, Y(\Theta'), Z(\Theta')|D) \propto \frac{p(X, Y(\Theta'), Z(\Theta')) p(D|X, Y(\Theta'), Z(\Theta'))}{p(X, Y(\Theta'), Z(\Theta'))} \quad (13)$$

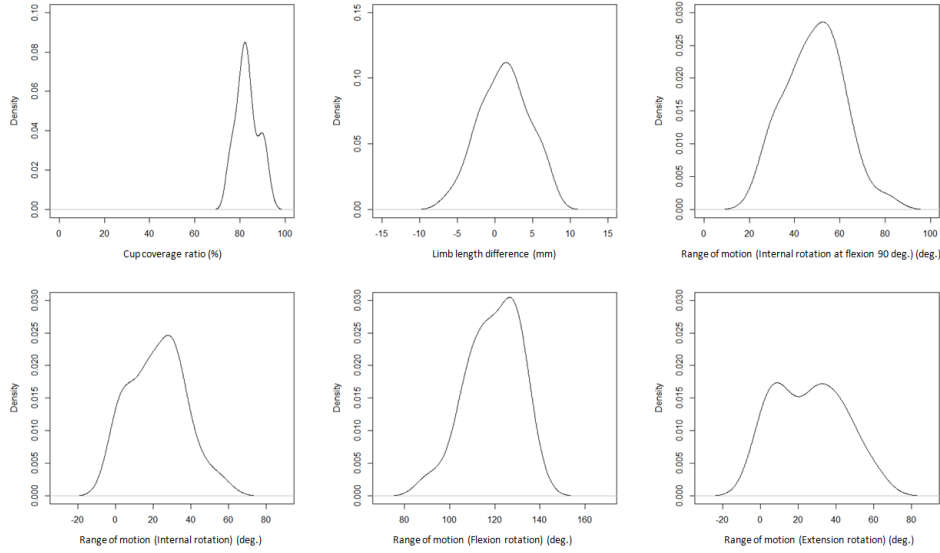


Figure 2. Statistical model for selecting solution from candidates.

$p(X, Y(\theta'), Z(\theta')|D)$ is able to be described as $p(D|X)$, as patient anatomy D depends only on X . When we assume that X , $Y(\theta')$ and $Z(\theta')$ are independent, $p(X, Y(\theta'), Z(\theta'))$ is able to be described as $p(X, Y(\theta'))p(Z(\theta'))$. Then, Eq.(11) is re-written as

$$\theta'^* = \arg \max_{\theta'} p(X, Y(\theta'))p(Z(\theta'))p(D|X) \quad (14)$$

Thus, in the optimization of combined pre-operative planning parameters, the CAD system finds the cup planning parameters $Y(\theta')$ and joint functionality parameters $Z(\theta')$ which maximize $p(X, Y(\theta'))p(Z(\theta'))p(D|X)$.

Now, the problem is how to model $p(Z(\theta'))$ for each functionality parameters from the training datasets. High probability values in $p(Z(\theta'))$ should mean high functionality as well as frequent occurrence. In order to derive a suitable model of $p(Z(\theta'))$, we assume that experienced surgeons aim at recovery of the highest functionalities in the best-balanced manner. This means that higher functionality should have occurred more frequently in the training datasets. In order to represent the above assumption, we model $p(Z(\theta'))$ with kernel density estimation method. The distribution (histogram) of each raw functionality parameter (e.g. ROM) obtained from the training datasets is converted to density data and the probability density function is estimated with this method. Now, let m be the number of elements of $Z(\theta')$, $p(Z)$ is defined as

$$p(Z) = \frac{1}{(\sqrt{2\pi})^m} \exp\left(-\frac{1}{2} \sum_{i=1}^m z_i^2\right) \quad (15)$$

When we take the logarithm of Eq.(11), θ'^* is given by

$$\theta'^* = \arg \max_{\theta'} (\ln(p(X, Y(\theta')))) + \ln(p(Z(\theta'))) + \ln p(D|X) \quad (16)$$

By substituting Eq.(5), Eq.(7), and Eq.(15) for Eq.(16), Eq.(17) is obtained.

$$\theta'^* = \arg \min_{\theta'} \left(\frac{1}{g} \sum_{j=1}^g \frac{L_j^2}{\alpha} + \sum_{i=1}^m z_i^2 + \sum_{k=1}^h \frac{b_k^2}{\lambda_k} \right) \quad (17)$$

Therefore, the system find an optimal combined pre-operative THA plan which minimizes Eq.(10) with one-by-one search in the automated optimization of cup shape parameters and hip joint functionality parameters.

III. RESULTS

As retrospective study, we used 37 datasets of past THA plans as the training datasets, and adopted 25 cases, where none of them were included in the training datasets, for the automated optimization of combined pre-operative planning. As the input datasets, we used manually segmented pelvis and femur shapes from CT data. We incorporated the cup coverage ratio as well as range of motion (ROM) and limb length difference (LLD) as joint functionality parameter. Four pattern of ROM (internal rotation at 90-degree flexion, internal rotation, flexion, extension) were considered. Figure 3 shows a typical case of automatically optimized THA plans. In this case, cup coverage ratio and LLD of the plan were equivalent to those of the surgeon's plan and ROM was largely improved.



Cup coverage [%]	80.7	78.5
Limb length difference [mm]	3.6	3.4
Range of motion [degree] (Internal rotation at 90-degree flexion)	50.6	30.6
Range of motion [degree] (Internal rotation)	10.6	9.4
Range of motion [degree] (Flexion rotation)	130.6	119.4
Range of motion [degree] (Extension rotation)	39.4	29.4

(a) Optimal combined plan (b) Surgeon's plan

Figure 3. Typical case of total planning.

TABLE I. COMPARISON OF EVALUATION VALUES OF HIP

	Optimal combined plan	Surgeon's plan
Cup coverage ratio [%]	82.3±8.1	81.9±8.6
Leg length difference (ABS) [mm]	2.4±2.0	3.0±2.9
	$p < 0.05$	
Range of motion [deg.] (Ant. at Flex.90)	46.5±13.5	38.3±18.8
Range of motion [deg.] (Internal rotation)	30.0±10.3	27.5±13.0
	$p < 0.05$	
Range of motion [deg.] (Flexion rotation)	120.0±11.0	113.7±12.6
Range of motion [deg.] (Extension rotation)	35.2±9.1	31.8±13.6

Table 1 shows the average values of the joint functionalities of the optimal combined plans and surgeon's plans. Statistical significance between the optimal plans and surgeon's plans was confirmed in ROM (internal rotation at 90-degree flexion) and ROM (flexion rotation) ($p < 0.05$), while no significance between the optimal plans and surgeon's plans in all the other functionalities. The average size differences between the optimal plans and the surgeon's plan were 0.80 size in stem and 0.64 size in cup. In addition, the average computation time of the proposed optimization method was 7.6 minutes.

IV. DISCUSSION

As the results of the experiments, two of the six functionalities were significantly improved and four of them were equivalent to those of the surgeon's plan. And the percentage of the cases whose size of stem and cup were within 1 size difference, when compared with the surgeon's plan, were 88 percent. These showed the usefulness of the proposed optimization method which incorporated the statistical model of the computational anatomy and joint functionalities. In three of 25 cases, the LLD differences of the optimal plans were large. Those cases were thought to be statistically irregular cases which had severe deformation of host bone (pelvis) for optimizing positions of implants with the statistical approach.

In three of 25 cases, some of the ROM of surgeon's plan were extremely small. It was considered that this was caused by the impingement of fragments of femoral neck, which should be removed, with pelvis. Therefore, we will revise the bone removal algorithm for femoral neck, which is implemented in the optimization method for combined THA plan.

V. CONCLUSION

We have described an optimization method for surgical planning of THA based on computational anatomy and joint functionality evaluation. The method fully utilizes the training datasets of past 3D THA plans to construct statistical models of the bone-implant spatial relations and joint functionalities. The objective function, that is, the posterior probability in the MAP formulation, was automatically

generated from the training datasets. As the results of optimized values of the functionalities were close to those of the surgeon's plan, we considered the method worked properly.

In this paper, we used manual segmentation as input bone 3D shape datasets. However, we have already developed automated CT segmentation software and showed that clinically acceptable accuracy was attainable [10]. Thus, we will start experiments using automatically segmented 3D shape datasets in prospective study.

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