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Anisotropic Diffusion Filtering for Correlated Multiple-Coil MRI

Gonzalo Vegas-Sánchez-Ferrero, Gabriel Ramos-Llordén, Rodrigo de Luis-García,
Antonio Tristán-Vega, Santiago Aja-Fernández

Abstract— **Recently, some methods have been proposed for filtering multi-coil MRI acquisitions with correlation between coils. Those methods are based on statistical models of noise to develop a Linear Minimum Mean Square Error (LMMSE) filter. The advantage of LMMSE-based filters stems from their simplicity and robustness. However, they exhibit some drawbacks: their performance strongly depends on the underlying statistical model and on the way the local moments are estimated. The first problem can be avoided when considering effective values provided by recent studies on the models of noise in multi-coil systems with correlation between coils. However, the local moments are estimated in square neighborhoods which can include different kinds of tissues. Thus, the local variance is biased towards upper values, which results in an inaccurate estimate in regions close to tissue boundaries. In this work we propose to overcome this problem by introducing an anisotropic diffusion step in the LMMSE estimate for correlated multi-coil systems which improves the estimation of the signal in regions where other LMMSE methods fail. Results demonstrate the better behavior in different noisy scenarios.**

Index Terms— **MRI, LMMSE, multi-coil, estimate, filtering.**

I. INTRODUCTION

Statistical models of noise in magnetic resonance (MR) depend on the scanner coil architecture. In the simplest case (a single-coil acquisition) the complex spatial MR data is typically assumed to be a complex Gaussian process, where real and imaginary parts of the original signal are corrupted with uncorrelated Gaussian noise with zero mean and equal variance. Thus, the magnitude signal calculated as the envelope of the complex signal is Rician distributed [1].

However, this simple case does not hold in modern architectures that include new acquisition technologies aiming to speed-up the acquisition of MRI. These methods are commonly referred to as Parallel Magnetic Resonance Imaging (pMRI), and are based on the simultaneous acquisition of different parts of the subsampled k-space by different receiving coils. As a consequence, in these more complex architectures, the Rician model is not valid any more.

Finding a proper statistical noise model in pMRI becomes important for anatomical evaluation and as a previous step for segmentation, registration or tensor estimation in diffusion tensor MRI (DT-MRI) [2]. When multiple-coil MR acquisition systems are considered, the noise in each receiving coil in the k-space is modeled as a complex stationary additive Gaussian noise process, with zero mean and equal variance [3]. Under the assumptions of noise components identically and independently distributed with no acceleration (the k-space is fully sampled); the composite magnitude signal (CMS) calculated as the sum of squares (SoS) of the received signal in each coil follows a non-central χ (nc-χ) distribution [4]. The noise variance is the same for all image points in both the k-space and x-space domains, i.e., the noise in the image may be considered spatial-stationary.

This statistical behavior does not hold when the coils in the scanner show a different variance of noise or under the presence of correlation between coils. In practical cases, correlation between coils exists, and therefore, the standard nc-χ model is not valid. In [5], the authors showed that if multiple coils and correlated noise are considered, the data do not strictly follow a nc-χ. However, for practical purposes, it can be modeled as such, but taking into account two effects: 1) Effective parameters must be considered. Thus, due to the correlation, the distribution is very similar to a nc-χ but considering a smaller number of coils and a greater variance of noise. 2) The effective parameters will also depend on the signal and hence on the position within the image. As a result, there will be different variance of noise in different areas of the image and the pattern of noise will be spatially variant, and the noise becomes non-stationary.

Due to these effects, an improper estimate of the effective parameters and a wrong supposition on the statistical model may cause an inaccurate estimate of the actual signal. Thus, in [6], a methodology to estimate the effective parameters in the presence of correlation between coils was proposed as well as the extension of the LMMSE filter to the case of correlated systems.

The main advantage of the LMMSE filters stems from their simplicity and robustness. However, their performance strongly depends on the underlying statistical model. An additional problem lies in the way the local moments are estimated. In those voxels corresponding to image edges, the square neighborhood comprises different kinds of tissues. Thus, the local variance increases and the LMMSE provides inaccurate estimates which results in a poor estimate of the actual value.

In this work, we propose to overcome this limitation by introducing an anisotropic diffusion step in the LMMSE estimate for correlated pMRI which improves the estimation of the signal in the regions where the LMMSE method proposed in [6] and conventional LMMSE methods fail. Results obtained from multiple-coil data evidence the better performance of the proposed filter with respect to other state-

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of-the-art methods for all the noisy scenarios considered.

II. Background

A. Noise Model

Noise in multiple coil systems, if the k-space is fully sampled and SoS is used to recover the CMS, is assumed to follow a nc-χ model [4], [7], [8] with parameters L (number of coils) and σ_n^2 (variance of noise in each coil) and with probability density function (PDF):

$$p_{M_L}(M_L|A_L,\sigma_n,L) = \frac{A_L^{1-L}}{\sigma_n^2} M_L^L e^{-\frac{M_L^2+A_L^2}{2\sigma_n^2}} I_{L-1}\left(\frac{A_L M_L}{\sigma_n^2}\right), \quad (1)$$

with $M_L(\mathbf{x}) > 0$, $A_L^2(\mathbf{x}) = \sum_{l=1}^{L} |A_l(\mathbf{x})|^2$ and $A_l(\mathbf{x})$ the original complex signal in each coil, $I_L(\cdot)$ the L-th order modified Bessel function of the first kind. In the background, this PDF simplifies to a central χ (c-χ).

When the variance of noise varies from coil to coil or there is some correlation between coils, the statistical distribution of data is affected. This deviation from the ideal case usually happens in multi-coil systems [9].

The general case can be formulated considering a covariance matrix, Σ, where the correlation terms are considered in the off-diagonal elements. For this case, the distribution does not follow a nc-χ. However, the nc-χ becomes a good approximation of the actual distribution when effective parameters of L and σ_n are considered [5]:

$$L_{\text{eff}}(\mathbf{x}) = \frac{A_T^2(\mathbf{x}) + (\text{tr}(\Sigma))^2}{\mathbf{A}^*(\mathbf{x})\Sigma\mathbf{A}(\mathbf{x}) + ||\Sigma||_F^2} \quad (2)$$

$$\sigma_{\text{eff}}^2(\mathbf{x}) = \frac{\text{tr}(\Sigma)}{L_{\text{eff}}(\mathbf{x})} \quad (3)$$

where $||\cdot||_F$ is the Frobenius norm and $\mathbf{A}(\mathbf{x}) = [A_1(\mathbf{x})\cdots A_L(\mathbf{x})]^T$.

From this equations, one should notice that the effective variance of noise increases due to the correlations between coils, whereas the effective number of coils is reduced. Additionally, both effective values will depend on the position, $\mathbf{x}$, so the distribution is non-stationary. However, the product, $L_{\text{eff}}(\mathbf{x})\sigma_{\text{eff}}^2(\mathbf{x}) = \text{tr}(\Sigma) = \sum_{l=1}^{L}\sigma_l^2 = L\sigma_n^2$, does not depend on the position.

B. Noise Filtering

In this section we present the filtering schemes that make use of the underlying statistical models of noise. The main advantage of using these methods is that they are based on an estimation philosophy. The simplest case is the so-called *conventional approach* (CA) which is an averaging of the squared signal with bias removal, assuming a Rician distribution of the data. Thus, the filtered signal is an estimate of the actual signal without any source of noise and, when there is no information to obtain a proper estimate of the signal, the noisy data is preserved. In the case of the nc-χ, the estimate is:

$$\widehat{A}(\mathbf{x}) = \sqrt{\max(\langle M_L^2(\mathbf{x})\rangle_{\mathbf{x}} - 2\sigma_n^2 L, 0)}. \quad (4)$$

Note that the sample estimation, $\langle\cdot\rangle_{\mathbf{x}}$, is used to estimate the second order moment:

$$\langle M_L^2(\mathbf{x})\rangle_{\mathbf{x}} = \frac{1}{|\eta(\mathbf{x})|}\sum_{\mathbf{y}\in\eta(\mathbf{x})} M_L^2(\mathbf{x}), \quad (5)$$

with $\eta(\mathbf{x})$ a neighborhood centered in $\mathbf{x}$, and $|\eta(\mathbf{x})|$ its cardinal.

Note also that this is a zero order estimate of the signal. Instead of this approach, a linear model can be adopted which takes into account not only the first moment of M_L^2 but also the second order moment in the local neighborhood. The linear model give rise to the LMMSE estimate. The LMMSE with Rician model is the simplest case when $L = 1$ and no correlation is considered. The importance of the formulation presented in Eqs. (2) and (3) is that the formulation of the filters do not need to be recalculated, since the effective values implicitly decorrelate the noise between coils and the nc-χ model can be used directly. The Rician LMMSE model was proposed in [2], whereas the nc-χ extension was presented in [10], [6]:

$$\widehat{A^2}(\mathbf{x}) = \langle M_L^2(\mathbf{x})\rangle_{\mathbf{x}} - 2L\sigma_n^2 + K_L(\mathbf{x})(M_L^2(\mathbf{x}) - \langle M_L^2(\mathbf{x})\rangle_{\mathbf{x}}), \quad (6)$$

where $K_L(\mathbf{x})$ is defined as:

$$K_L(\mathbf{x}) = 1 - \frac{4\sigma_n^2(\langle M_L^2(\mathbf{x})\rangle_{\mathbf{x}} - L\sigma_n^2)}{\langle M_L^4(\mathbf{x})\rangle_{\mathbf{x}} - \langle M_L^2(\mathbf{x})\rangle_{\mathbf{x}}^2}. \quad (7)$$

Note that, when there is some correlation between coils, the effective values shown in Eqs. (2) and (3) must be used instead of L and σ_n [6].

III. Anisotropic Diffusion LMMSE for Correlated Data

One of the problems of the LMMSE philosophy is that it is based on the computation of the mean and variance of the data being filtered according to an assumed noise model, which is usually accomplished by calculating local moments over square neighborhoods. In pixels corresponding to image contours, the local moments estimate results in an inaccurate calculation of the $K_L(\mathbf{x})$ term of Eq. (7), which bias the value towards 1, due to an increase of the local variance $\langle M_L^4(\mathbf{x})\rangle_{\mathbf{x}} - \langle M_L^2(\mathbf{x})\rangle_{\mathbf{x}}^2$. The effect is that image is not filtered. This is a desired property of the LMMSE philosophy, since it is preferable to preserve the noisy value rather than assume a poor estimate.

However, a proper estimate can be consider if the neighborhood is established in the tangent direction of the border instead of square neighborhoods. This way, the local moments can be calculated along the borders. For this purpose, we adopt the anisotropic scheme proposed in [11] for the Rician case and we adopt it for the correlated nc-χ. This way, the correlation of multiple coil acquisitions is considered and the behavior of the correlated nc-χ LMMSE is improved in the tissue contours.

We propose to adapt the correlated nc-χ LMMSE into a Partial Differential Equation scheme that models a diffusion process governed by the following equation:

$$\left\{ \begin{array}{lcl} u(\mathbf{x},0) & = & \langle M_L^2(\mathbf{x})\rangle_{\mathbf{x}} + K_L(\mathbf{x})(M_L^2(\mathbf{x}) - \langle M_L^2(\mathbf{x})\rangle_{\mathbf{x}}) \\ \dfrac{\partial u(\mathbf{x},t)}{\partial t} & = & \mathrm{div}(D\nabla u(\mathbf{x},t)) \end{array} \right. \tag{8}$$

where D is the diffusion tensor depending on the local statistics of the image and on the noise model. The matrix D can be expressed in a diagonal form with the eigenvectors $\mathbf{v}_i$, where i varies from 1 to the number of the dimensions of the image. In the 2D case it is defined as:

$$D = E \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix} E^T, \text{ with } \left\{ \begin{array}{lcl} \lambda_1 & = & K_L \\ \lambda_2 & = & 0 \end{array} \right. \tag{9}$$

with $E = (\mathbf{v}_1^t, \mathbf{v}_2^t)$. We choose the eigenbase obtained from the structure tensor calculated from $u(\mathbf{x},0)$ as follows:

$$T = G_\sigma * (\nabla_\sigma u(\mathbf{x},0) \cdot \nabla_\sigma u(\mathbf{x},0)^t) \tag{10}$$

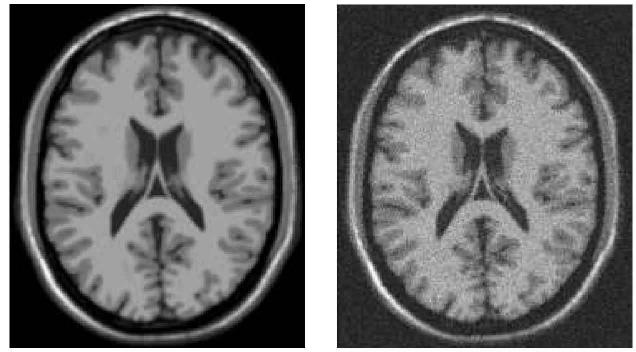
where G_σ is a Gaussian kernel with standard deviation σ, and $\nabla_\sigma u(\mathbf{x},0)$ is the gradient of the LMMSE estimate without bias correction filtered by a Gaussian kernel with standard deviation σ. The eigenbase E is obtained from the diagonalization of T.

The statistical information is included in the diffusion equation scheme by assigning $K_L(\mathbf{x})$ obtained from the nc-χ LMMSE as the eigenvalue associated to the eigenvector tangential to the contour. This definition takes the advantage of the coefficient K_L when it is close to 1, which means that there are more than one type of tissue in the neighborhood. In this case, filtering is performed along the detected contour.

Note that this formulation complements the correlated nc-χ LMMSE method, which makes use of isotropic neighborhoods, by adding an anisotropic diffusion in along the contours detected within the neighborhood of each voxel. Hence, the initial estimate is, at least, as good as the isotropic nc-χ LMMSE method and refines the estimate in the tissue contours in just a few iterations (we considered $t_{\text{end}} = 5$ iterations with a time step of 0.1). After the diffusion process, the bias is corrected as $\widehat{A}(\mathbf{x}) = \sqrt{\max(u(\mathbf{x},t_{\text{end}}) - 2L_{\text{eff}}(\mathbf{x})\sigma_{\text{eff}}^2(\mathbf{x}),0)}$. The 3D extension of this filter can be obtained easily considering a planar neighborhood obtained from the plane tangent to the contour of the image. A semi-implicit scheme was adopted for the numerical implementation of the diffusion equation.

IV. Results

To test the performance of the filters we considered a synthetic experiment. A phantom was built using different levels of noise for a noticeable level of correlation between coils ($\rho = 0.1$). The phantom is a two-dimensional synthetic slice (see Fig. 1) from a BrainWeb MR volume [12] with intensity values in $[0-255]$. An eight-coil system is simulated using an artificial sensitivity map coded for each coil so that $A_L^2(\mathbf{x}) = \sum_l |A_l(\mathbf{x})|^2$. The signal in each coil is corrupted with complex Gaussian noise with σ_n (in both the real and imaginary parts) ranging from 1 to 20. The CMS is reconstructed from the data in each coil using SoS. One hundred realizations were generated for each σ_n.



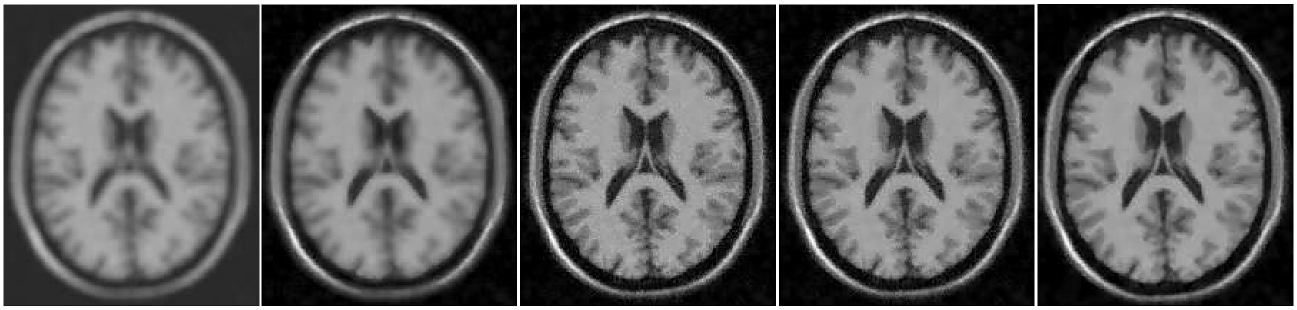
(a) Synthetic Image (b) Noisy Image

Fig. 1. Phantom obtained from a BrainWeb MR volume [12] and the simulated noisy image for an eight-coil system corrupted with $\sigma_n = 10$ in both the real and imaginary parts and correlation between coils $\rho = 0.1$.

The estimates of L_{eff} and σ_{eff} were obtained following the method proposed in [6]. All the isotropic local moments were calculated using 7×7 neighborhoods. Five filters were tested with the proposed phantom. 1) The conventional approach (CA) for the nc-χ statistical model described by Eq. (4). 2) The Rice LMMSE, where no correlations and $L = 1$ are assumed (Rice-LMMSE). 3) The nc-χ LMMSE without the assumption of correlation (nc-χ-LMMSE). 4) The correlated nc-χ LMMSE with the estimation assuming correlations between coils (c-nc-χ-LMMSE). 5) The proposed anisotropic version of the c-nc-χ-LMMSE method.

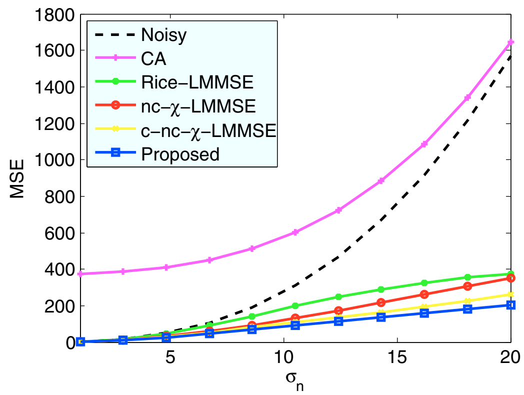
A visual comparison is depicted in Fig. 2 for $\sigma_n = 10$ and coils correlation $\rho = 0.1$. The performance of the c-nc-χ-LMMSE method results in a better estimate of the values in homogeneous regions compared to the nc-χ-LMMSE filter. This behavior owes the correction of the effective values. However, there still are some unfiltered regions in the boundaries of tissues because of the inaccurate calculation of the coefficient K_L. This effect is avoided with the proposed filter (Fig. 2.(e)) where the anisotropic filtering performed along the contour of tissues results in a better definition of edges. Both the CA and the Rice-LMMSE methods underestimate the value of K_L (in the case of the conventional approach is equal to zero) which provides an over-smoothed image due to a wrong statistical assumption (Rice instead of nc-χ) or an oversimplified estimate (zero order estimate in the case of CA).

The numerical comparison between methods was performed by using two quality indexes: the Structural Similarity (SSIM) index [13] and the Mean Squared Error (MSE). The SSIM provides a measure of the structural similarity between the ground truth and the estimated images. The closer to 1, the better the quality is. These quality measures were applied to those areas of the original image with intensities greater than zero in order to avoid any bias due to non relevant parts of the image. The results for an increasing σ_n are shown in Fig. 3. Note that the proposed method obtains the best performance compared to the rest of the LMMSE-based methods. This result is due to the better

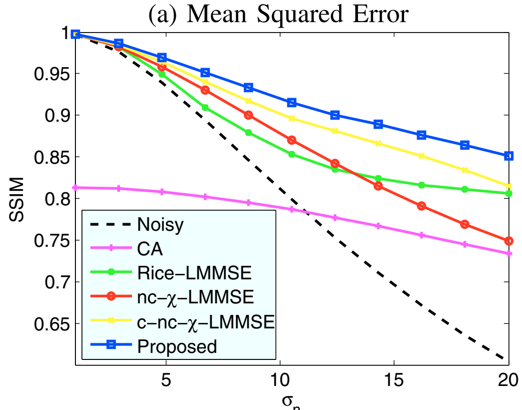


a) Conventional Approach (b) Rice-LMMSE (c) nc-χ-LMMSE (d) c-nc-χ-LMMSE (e) Proposed

Fig. 2. Filtered images from a phantom simulated in an eight-coil system for $\sigma_n = 10$ and correlation between coils $\rho = 0.1$.



(a) Mean Squared Error



(b) Structural Similarity Measure

Fig. 3. Quality measures for the LMMSE-based schemes for an increasing σ_n. These values were obtained as the mean value of 100 independent experiments of an eight-coil system and correlation between coils $\rho = 0.1$.

estimate obtained in the neighbors of the image contours, where different tissues cause an inaccurate estimate of K_L.

V. Conclusions

In this work we presented a methodology to avoid inaccurate estimates of the signal in neighborhoods close to tissue boundaries. This problem usually appears as noisy regions when applying LMMSE-based filters, which is due to an overestimation of the local variance. We propose to include an anisotropic diffusion step which acts in regions where the local variance is overestimated. Thus, the diffusion process performs an estimate in a tangential direction to the contours or tissues while the contours are preserved.

The proposed method takes advantage of the isotropic LMMSE for correlated multi-coil systems and performs anisotropic filtering in regions where the LMMSE cannot provide a suitable estimate of the variance due to the presence of multiple tissues in the neighborhood. Thus, the implementation of the proposed method provides an estimate of the signal at least as good as that one obtained with the isotropic LMMSE with the advantage of providing well defined contours of tissues in those regions where isotropic LMMSE methods fail. Results obtained with an increasing σ_n of noise and a noticeable correlation $\rho = 0.1$ exhibit the desired behavior of the proposed filter, which obtains better results for both the MSE and MSSIM in all noisy scenarios.

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