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Theory of Dynamic Pulsatile Spectroscopy for Photoplethysmographic Signals Analysis

V O Rybynok, *Member, IEEE*, P A Kyriacou, *Senior Member, IEEE*

***Abstract*—Photoplethysmography (PPG) is a technique that uses light to non-invasively obtain a volumetric measurement of an organ with each cardiac cycle. Pulse Oximetry (PO) is an empirical technique which allows the arterial blood oxygen saturation (SpO_2) evaluation from the PPG signals. There have been many reports in the literature suggesting that other arterial blood chemical components can be evaluated from the PPG signals. Most attempts to perform such evaluation on empirical bases have failed, especially for components concentrations. This paper introduces a non-empirical rational theory called Dynamic Pulsatile Spectroscopy (DPS) which can be used to analytically investigate the phenomena of PPG. The DPS theory provides the mathematically rigid method of how PPG signals can be used for arterial blood analysis to evaluate its chemical component concentrations and molar fractions spectroscopically and transcutaneously. It also highlights what other signals might be required for such evaluation. DPS opens the possibility of extending PPG application for blood analysis beyond conventional PO. The DPS basic principles are introduced in this paper.**

***Index Terms*—PPG, Photoplethysmography, Pulse Oximetry**

I. Introduction

Photoplethysmography (PPG) is a measurement technique that uses light to noninvasively obtain a volumetric measurement of an organ with each cardiac cycle [1].

Pulse Oximetry (PO) is a well established empirical technique which allows the arterial blood oxygen saturation (SpO_2) evaluation from PPG signals [1]. The unique property of PO is its ability to look directly into the arterial blood stream through the skin bypassing other parts of the vascular tissue (e.g. bones or muscles). There have been many reports in the literature suggesting that other arterial blood chemical component molar fractions and concentrations can be evaluated from PPG signals obtained at multiple wavelengths [2].

The theory of Dynamic Pulsatile Spectroscopy (DPS) presented in this paper began as an attempt to establish a technique which could be used to non-invasively evaluate arterial blood glucose concentration using PPG signals similarly as they are used in PO to evaluate SpO_2. Despite the great success of PO, the physical principles behind the PPG signals are not well understood. Consequently, all previous attempts to estimate arterial blood glucose concentration using PPG signals were based on empirical studies. Methods which are utilized to reconstruct empirical relations can be made to be advanced and sophisticated however, they will fail even if a single statistically significant parameter is not included into the analysis. For more than 30 years all empirical attempts to non-invasively evaluate arterial blood glucose concentration were unsuccessful [3].

V. O. Rybynok and P. A. Kyriacou are with the School of Engineering and Mathematical Sciences, City University London, EC1V 0HB, UK (e-mail: v.rybynok@gmail.com; p.kyriacou@city.ac.uk).

Therefore, in this work the decision was made to start from scratch and attempt to rationally explain the relevant physical principles behind the PPG phenomena before committing to any further developments. This explanation had to be done by means of a rational model that would be based on the fundamental laws of physics and would allow mathematical analysis of the PPG signals and arterial blood constituents.

A similar rational model is available for SpO_2 in PO [4]. That model utilizes the conventional Beer-Lambert law, neglects scattering within vascular tissue, and assumes that arterial blood chromophores consist of only oxygenated and deoxygenated hemoglobins. Despite those oversimplifying assumptions, the equations produced from that model allow evaluation of SpO_2 values from PPG signals quite accurately in respect to the empirical relations established with invasive blood oxygen saturation measurements. Although the analytical solution obtained from the rational model is not as accurate as the relations obtained empirically, it theoretically proves that such non-invasive SpO_2 evaluation is generally possible. The analytical solution equations also reveal which parameters should be used for empirical calibration (i.e. R value).

The DPS theory generalizes PO rational model by taking vascular tissue scattering into consideration and including other arterial blood analytes into the analysis. The PO rational model equations can be derived from the DPS equations. The research questions which DPS theory answers are: what assumptions should be made about vascular tissue to construct its rational pulsatile spectroscopy model; how to include an arbitrary arterial blood component into the model equations; and what parameters should be measured and computed in order to evaluate analytes concentrations and molar fractions.

From a practical perspective, DPS equations allow PPG researchers and engineers to theoretically prove if a certain arterial blood component can be accessed non-invasively using PPG signals. DPS equations may also reveal whether and how it is possible to evaluate molar fractions of those analytes with a given PPG monitoring system, how many and what PPG light wavelengths are required for such evaluation, and what other signals apart from PPG should be monitored to enable evaluation of the blood analytes concentrations.

In order to combine light scattering with conventional absorption spectroscopy, DPS theory is developed using a method named Beer-Lambert law along Non-Linear mean Light Pathways (BLNLP).

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II. Method

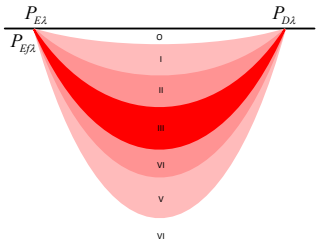
In this paper only the main ideas of BLNLP are introduced. It is sufficient to provide the basics required for the following DPS derivations, for a more detailed discussion on BLNLP refer to [5].

In BLNLP, light is accepted as a flux of elementary particles – photons. Speed of photons is equal to the speed of light. This speed is defined by the electromagnetic properties of the media where light propagates. A light photon can be emitted absorbed or scattered by an optical electron of an atom or molecule, collectively called matter particle. Photon energy is proportional to the light wavelengths and the proportionality coefficient is the Plank constant. Additional postulates used in BLNLP were deduced by analysis of the following physical models: Beer-Lambert law, Monte Carlo light scattering modeling and light energy transport integral equation [6].

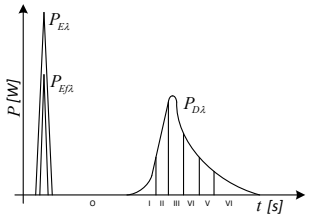
Although the individual photons motion is determined statistically by the photon scattering and scattering phase distribution functions, the average light energy propagation pathways within the vascular tissues are deterministic and not random. Fig. 1 demonstrates this physical concept. It shows the light flux envelope between emitter and detector in a scattering media such, which is known as "banana".

This banana shaped envelope represents a space area along which most photons travel from the light emitter to the light detector. Such envelopes can be obtained by light scattering modeling on the homogeneous, plain, and semi-infinite media models utilizing the Monte Carlo method to solve the light radiation transfer equation. It might also be obtained by measuring the light radiant intensities at various points within the matter sample volume in-vitro [6].

Photons motion can be explained from two points of view: light pulse propagation delay experiments Fig. 1b (i.e. dynamic), and light transfer equation solution with Monte Carlo analysis Fig. 1a (i.e. static). Time interval "I" during which the first part of the light pulse energy arrives to the light detector corresponds to the shortest distance in the banana envelope. Time interval "II" corresponds to the longer distance, and so on up to the end of the light pulse propagation time interval

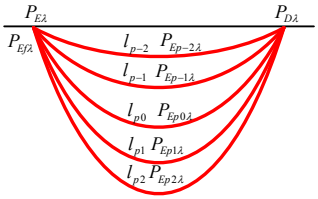


(a) Light flux envelope between emitter and detector in a scattering media such as vascular tissue, known as "banana".

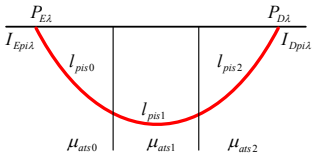


(b) Concept of the light pulse propagation delay experiment in a scattering media with absorption [6].

Fig. 1: Roman numbers (I–VI) on both diagrams correspond to parts of the light power which propagates from emitter to detector at different time intervals.



(a) Mean light pathways concept within scattering media.



(b) Mean light pathway example within scattering non-homogeneous composite media.

Fig. 2: "Banana" shaped light propagation envelope is split into smaller "canoe" shaped envelopes by the fraction of light going through them (Fig. 1a). Each smaller envelope can be approximated by the mean light pathway l_{pi} going through the smaller envelopes medians.

"VI" (Fig. 1).

Light energy propagation envelope can be split into a series of smaller "canoe" shaped envelops within which certain fraction of light energy propagates through the matter Fig. 1a. Each of those envelopes can be approximated by the mean light pathway going through the envelop median Fig. 2a. In this approximation, fractions of the light energy corresponding to each envelope are propagating along the mean light pathways Fig. 2a. Each mean light pathway corresponds to the certain light pulse propagation delay time interval Fig. 1b.

Thus, after introducing the concept of the light propagation pathway in the scattering media, light intensity can also be introduced for the resulting scattered light. Light intensity can be computed for each point of the mean light pathway curve along its tangent and at the emitter–detector direction. Thus light intensity degrades along the mean pathways according to the Beer-Lambert law, apart from the fact that the Beer-Lambert integral is taken along the mean light pathways rather than a conventional straight line:

$$A_{p_i\,\lambda} = -\log \frac{I_{\mathrm{D}\,p_i\,\lambda}}{I_{\mathrm{E}\,p_i\,\lambda}} = \mu_{\mathrm{at}\,\lambda} \cdot l_{p_i\,\lambda} \tag{1}$$

Let's suppose that there are m_λ matter segments along the mean light pathway, each with different atenuation coefficients $\mu_{\mathrm{at}\,\lambda}$ (three segments example is shown in Fig. 2b), then the following equation can be stated:

$$A_{p_i\,\lambda} = \sum_{j=0}^{m_\lambda} \mu_{\mathrm{at}\,s_j\,\lambda} \cdot l_{p_i\,s_j\,\lambda} \tag{2}$$

Similarly as in the conventional Beer-Lambert law, attenuation coefficient $\mu_{\mathrm{at}\,s_j\,\lambda}$ is computed as following:

$$\mu_{\mathrm{at}\,s_j\,\lambda} = \sum_{k=1}^{n_{s_j}} a_{k\,\lambda} \cdot c_{s_j\,k} \tag{3}$$

Absorptivity coefficient $a_{k\,\lambda}$ has the same meaning as in absorption spectroscopy—a function of wavelength λ, unique for each chromophore k present in the segment j of the sample. $c_{s_j\,k}$ is a chromophore k concentration within the segment j (Fig. 2b). Equations (1)–(3) form the basis for the DPS theory.

III. Results

The diagram in Fig. 3a pictures a single mean pathway (as in Fig. 2a) when light propagates through a vascular tissue. $I_{\mathrm{E}\,\lambda}$ is a light intensity at the emitter side and $I_{\mathrm{D}\,\lambda}$ is a light intensity at the detector side. When arterial blood refills arteries and capillaries within the vascular tissue during diastole, the mean light pathway expands causing more light to get absorbed by the incoming layer of blood. When blood leaves the capillaries during systole, the mean light pathway contracts causing less light to get absorbed (1). Thus, $I_{\mathrm{D}\,\lambda}$ changes with blood inflow and outflow. If $I_{\mathrm{E}\,\lambda}$ does not change between time t_0 and t, then the following equation can be written for the absorbance (1) variation:

$$\Delta A_\lambda(t_0,t) = A_\lambda(t) - A_\lambda(t_0) = -\log\frac{I_{\mathrm{D}\,\lambda}(t)}{I_{\mathrm{D}\,\lambda}(t_0)} \tag{4}$$

Let $\mu_{\mathrm{at\,AB}\,\lambda}(t)$ be the absorption coefficient for the arterial blood as in (3); $l_{\mathrm{AB}\,\lambda}(t)$ is a total mean light pathway within all arterial blood segments as in Fig. 2b; and $A_{\mathrm{T}\,\lambda}(t)$ is a total absorbance within non-arterial blood segments as in (2). Then:

$$A_\lambda(t) = \mu_{\mathrm{at\,AB}\,\lambda}(t)\cdot l_{\mathrm{AB}\,\lambda}(t) + A_{\mathrm{T}\,\lambda}(t) \tag{5}$$

Two assumptions can be made about (5): $\mu_{\mathrm{at\,AB}\,\lambda}(t)$ does not change significantly between t_0 and t (i.e. $\mu_{\mathrm{at\,AB}\,\lambda}(t) = \mu_{\mathrm{at\,AB}\,\lambda}(t_0)$); and $A_{\mathrm{T}\,\lambda}(t)$ is not altered much by the blood pulsation (i.e. $A_{\mathrm{T}\,\lambda}(t) = A_{\mathrm{T}\,\lambda}(t_0)$). From these assumptions and (4), (5), the following equation can be written:

$$\Delta A_\lambda(t_0,t) = \mu_{\mathrm{at\,AB}\,\lambda}(t)\cdot\Delta l_{\mathrm{AB}\,\lambda}(t_0,t) \tag{6}$$

where $\Delta l_{\mathrm{AB}\,\lambda}(t_0,t)$ is a Mean Light Pathway Variation (MLPV), which represents a thin layer of arterial blood responsible for the light absorption change between systole and diastole heart cycles:

$$\Delta l_{\mathrm{AB}\,\lambda}(t_0,t) = l_{\mathrm{AB}\,\lambda}(t) - l_{\mathrm{AB}\,\lambda}(t_0) \tag{7}$$

The light intensity (I_{D}, Fig. 3a) used in BLNLP and the light power (P_{D}, Fig. 3) sensed via photodetector are two different light energy characteristics. By definition, detected light power can be computed by integrating light intensity over photodetector's surface and over all mean light pathways leading to the photodetector (Fig. 2a). In order to be able to use light power instead of light intensity in (4) as it is done in PO [4], one more assumption has to be made: $\Delta l_{\mathrm{AB}\,\lambda}(t_0,t)$ is not significantly different from one light pathway to another. With this assumption, (4) can be transformed as following:

$$\Delta A_\lambda(t_0,t) = -\log\frac{P_{\mathrm{D}\,\lambda}(t)}{P_{\mathrm{D}\,\lambda}(t_0)} \tag{8}$$

Combining (6) with (3) produces set of linear equations, which in DPS theory is named DPS equations:

$$\left.\begin{aligned}\frac{\Delta A_{\lambda_j}(t_0,t)}{\Delta l_{\lambda_j}(t_0,t)} &= \tilde{\mu}_{\mathrm{at\,AB}\,\lambda_j}(t) && \text{(9a)}\\ \tilde{\mu}_{\mathrm{at\,AB}\,\lambda_j}(t) &= \sum_{i=1}^{n_{\mathrm{AB}}} a_{\mathrm{AB}\,c_i\,\lambda_j}\cdot c_{\mathrm{AB}\,c_i}(t) && \text{(9b)}\end{aligned}\right\}$$

$$j = [1\ldots n_{\mathrm{AB}}] \tag{9c}$$

Since in this set MLPV $\Delta l_{\lambda_j}(t_0,t)$ is introduced as a physical quantity with the meaning illustrated in Fig. 1, rather than phenomenologically, it opens the possibility to solve those equations in respect to the blood constituents concentrations $c_{\mathrm{AB}\,c_i}(t)$. Indeed, absorbance variation $\Delta A_{\lambda_j}(t_0,t)$ can be computed directly from PPG signals using (8). Absorptivities $a_{\mathrm{AB}\,c_i\,\lambda_j}$ can be obtained individually for each blood component using conventional spectroscopy. Direct measurement of the MLPV is still a research question. However, there is a clear relationship between light pulse propagation delay experiment and MLPV. The peak of the wave $P_{\mathrm{D}\,\lambda}(t)$ in Fig. 1b should move left and right on the time scale with each heartbeat. The time variation between $P_{\mathrm{E}\,\lambda}(t)$ spike and $P_{\mathrm{D}\,\lambda}(t)$ peak should be proportional to the MLPV with a proportionality coefficient equal to the speed of light in arterial blood. Therefore, theoretically it is possible to measure MLPV with at least the light pulse propagation delay experiment.

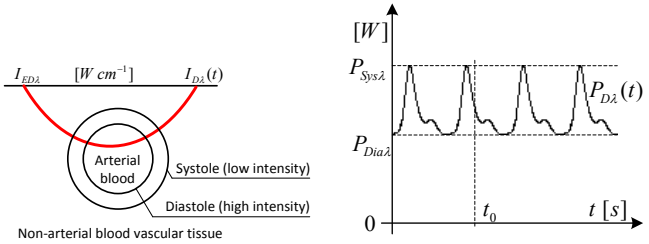
Indexes j in (9c) identify the blood components taken into account. The smaller the concentration of a given component and the smaller its absorptivity, the higher PPG/MLPV system resolution will be required to evaluate its concentration. The component with the smallest absorption coefficient (3) defines the total number of components which have to be included into equations n_{AB}, as concentration variations of the components with the higher absorption coefficients will affect the evaluation accuracy. Component absorptivities have to be different by at least one wavelength to avoid equations linear dependency. The number of wavelengths at which PPGs should be monitored must be equal to or greater than the number of blood components, whose concentrations are taken into DPS equations.

Similarly as in PO [4], another assumption can be made on MLPV equality between different light wavelengths:

$$l_{\mathrm{AB}\,\lambda_i} = l_{\mathrm{AB}\,\lambda_j} = l_{\mathrm{AB}} \tag{10}$$

From (9a) and (9c), the following ratio can be introduced:

$$R_{\lambda_j} = \frac{\Delta A_{\lambda_j}(t_0,t)}{\Delta A_{\lambda_{n_{\mathrm{AB}}}}(t_0,t)} \tag{11}$$



(a) Light-tissue interaction diagram during cardiac cycles–induced vascular tissue volume changes.

(b) The PPG signal obtained from a tissue represented by the light–tissue interaction diagram on Fig. 3a.

Fig. 3: This model is built according to (2) and scattered light power distribution along the mean light pathways (Fig. 2a).

The proportion taken by a blood component in respect to all the components present in blood is defined as a molar fraction:

$$MF_{\text{AB}\,c_i}(t) = \frac{c_{\text{AB}\,c_i}(t)}{\sum_{k=1}^{n_{\text{AB}}} c_{\text{AB}\,c_k}(t)} \tag{12}$$

Substituting (10) and (12) into (9a) and rearranging it produces the following equation:

$$\Delta A_{\lambda_j}(t_0,t) = \Delta l_{\text{AB}}(t_0,t) \cdot \left(\sum_{k=1}^{n_{\text{AB}}} c_{\text{AB}\,c_k}(t) \right) \cdot \cdot \sum_{i=1}^{n_{\text{AB}}} a_{\text{AB}\,c_i\,\lambda_j} \cdot MF_{\text{AB}\,c_i}(t) \tag{13}$$

Substituting (13) into (11) results in the following:

$$R_{\lambda_j} = \frac{\sum_{i=1}^{n_{\text{AB}}} a_{\text{AB}\,c_i\,\lambda_j} \cdot MF_{\text{AB}\,c_i}(t)}{\sum_{i=1}^{n_{\text{AB}}} a_{\text{AB}\,c_i\,\lambda_{n_{\text{AB}}}} \cdot MF_{\text{AB}\,c_i}(t)} \tag{14}$$

Since the sum of all molar frations $MF_{\text{AB}\,c_i}$ as defined in (12) is equal to one, then the following equation can be stated:

$$MF_{\text{AB}\,c_{n_{\text{AB}}}} = 1 - \sum_{i=1}^{n_{\text{AB}}-1} MF_{\text{AB}\,c_i}(t) \tag{15}$$

Combining (15) and (14) produces the following set of equations:

$$\left.\begin{aligned} \nu_{\text{AB-1}\,\lambda_j}(t) &= \sum_{i=1}^{n_{\text{AB}}-1} a_{\text{AB}\,c_i\,\lambda_j} \cdot MF_{\text{AB}\,c_i}(t) && \text{(16a)} \\ \nu_{c_{n_{\text{AB}}}\,\lambda_j}(t) &= a_{\text{AB}\,c_{n_{\text{AB}}}\,\lambda_j} \cdot \left(1 - \sum_{i=1}^{n_{\text{AB}}-1} MF_{\text{AB}\,c_i}(t)\right) && \text{(16b)} \\ R_{\lambda_j} &= \frac{\nu_{c_{n_{\text{AB}}}\,\lambda_j}(t) + \nu_{\text{AB-1}\,\lambda_j}(t)}{\nu_{c_{n_{\text{AB}}}\,\lambda_{n\text{AB}}}(t) + \nu_{\text{AB-1}\,\lambda_{n\text{AB}}}(t)} && \text{(16c)} \end{aligned}\right\}$$

$$j = [1 \ldots (n_{\text{AB}} - 1)] \tag{16d}$$

This set of equations is a part of DPS theory and it is named Static Pulsatile Spectroscopy (SPS) equations. The number of unknown molar fractions $MF_{\text{AB}\,c_i}(t)$ in these equations is equal to the number of equations and they are solvable. Unlike DPS, SPS equations do not require extra parameters apart from the PPG signals.

Logically, SPS is an expansion of the PO technique, while DPS is an attempt to identify what additional parameters (i.e. MLPV) should be measured is order to extend this technique for the evaluation of arterial blood analytes concentrations. It is easy to see that SpO_2 is the molar fraction as defined in (12) and the theoretical relation of SpO_2 and R values in OP is a special case of the more general SPS equations. To prove this, assumption should be made that blood analytes consist of oxygenated and deoxygenated hemoglobins. Thus, with two components two light wavelengths are required. Absorptivity coefficients for both types of hemoglobin can be obtained using conventional spectroscopy. Substituting those absorptivities into (16) and solving it in respect to $MF(t)$ produces equation which matches exactly the theoretical SpO_2 equation in PO [4].

IV. Discussion and Conclusion

DPS theory introduced in this paper is based on a number of basic postulates and simplifying assumptions borrowed from a few well established sciences and technologies. PPG analysis within DPS theory leads to DPS and SPS equations. According to our knowledge and conventional literature, terms "Static Pulsatile Spectroscopy" and "Dynamic Pulsatile Spectroscopy" have been introduced in this work for the first time. Based on those equations corresponding techniques can be established:

- SPS is a technique which involves building a set of equations at a number of different light wavelengths using non-invasively obtained PPG signals and neglecting MLPV differences between them. Real-time solution of SPS equations allows non-invasive evaluation of arterial blood analytes *molar fractions*.
- DPS technique supplements PPG signal with the MLPV, which in theory could be evaluated non-invasively by dynamically measuring light propagation delay within vascular tissue and assuming certain value for the speed of light in arterial blood. Real-time solution of DPS equations should allow non-invasive evaluation of arterial blood analytes *concentrations*.

These techniques open the possibility of extending PPG application for non-invasive blood analysis beyond conventional PO. However, to take advantage of these possibilities a number of technical problems must be addressed: a generic high resolution and multiwavelengths PPG monitoring system must be developed; MLPV monitoring principles should be established and the corresponding technology developed; the whole blood spectrochemical decomposition should be performed, i.e. representing experimentally obtained blood absorption coefficient as a sum of its chemical components absorption coefficients (as in (3)).

In our current and future work we have already developed a high resolution modular PPG system, and working on the MLPV monitoring system.

We hope that this paper will stimulate further research in multiwavelength PPG and in whole blood spectroscopy which may lead to the invention of the new generation non-invasive optical physiological monitoring devices.

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