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Accelerating k-t sparse using k-space aliasing for dynamic MRI imaging

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Abstract— **Dynamic imaging is challenging in MRI and acceleration techniques are usually needed to acquire dynamic scene. K-t sparse is an acceleration technique based on compressed sensing, it acquires fewer amounts of data in k-t space by pseudo random ordering of phase encodes and reconstructs dynamic scene by exploiting sparsity of k-t space in transform domain. Another recently introduced technique accelerates dynamic MRI scans by acquiring k-space data in aliased form. K-space aliasing technique uses multiple RF excitation pulses to deliberately acquire aliased k-space data. During reconstruction a simple Fourier transformation along time frames can unaliase the acquired aliased data. This paper presents a novel method to combine k-t sparse and k-space aliasing to achieve higher acceleration than each of the individual technique alone. In this particular combination, a very critical factor of compressed sensing, the ratio of the number of acquired phase encodes to the number of total phase encode (n/N) increases therefore compressed sensing component of reconstruction performs exceptionally well. Comparison of k-t sparse and the proposed technique for acceleration factors of 4, 6 and 8 is demonstrated in simulation on cardiac data.**

I. INTRODUCTION

For the required spatial and temporal resolution in the magnetic resonance imaging (MRI), all k-t space data needs to be acquired for faithful reconstruction of dynamic scene. However the most pronounced limitation of MRI is its slow data acquisition speed that limits its use for most of the dynamic imaging scenarios. Therefore, acceleration techniques are needed to alleviate the imaging speed limitation of dynamic imaging for cardiac function, fMRI, time resolved angiography, and perfusion studies. All the acceleration techniques in MRI aims at acquiring less amount of the data than the required and approximate the missing data by using additional information about the object or imaging process. The accelerated imaging techniques can be broadly classified into two categories: Parallel Imaging and Compressed Sensing. Parallel imaging methods exploits spatial coil sensitivity information to estimate the missing data while compressed sensing exploits sparsity of the image to find the missing data. These techniques reduces data acquisition time by exploiting spatial, temporal or spatiotemporal redundancy. Techniques such as Keyhole imaging, BRISK, TRICKS, CURE and VIPR [1-5] exploits temporal redundancy to reduce scan time. Techniques such as TSENSE, TGRAPPA and HYPR [6-8] exploit temporal redundancy followed by spatial redundancy. Techniques such as UNFOLD, k-t Blast, k-t SENSE, k-t FOCUS and k-t SPARSE [9-12] exploits spatiotemporal redundancy to accelerate dynamic MRI scans.

Recently introduced technique of k-space aliasing [13] overlaps different portions of k-space and acquires the overlapped k-space data. The acquired data is then resolved by taking a simple Fourier transformation along time. Separately k-t sparse [12] is another technique that exploits sparsity in k-t space and reconstructs imaging sequence by acquiring less than required amount of data. In this work we propose a novel method to combine k-t sparse and k-space aliasing to achieve higher acceleration than each of the individual techniques alone.

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II. BACKGROUND

A. *k-space aliasing*

K-space aliasing technique uses tailored signal excitation module consisting of multiple RF excitation pulses and gradient waveforms to deliberately overlap distinct phase (PE) encodes and acquire them simultaneously. In k-space aliasing, excitation pulse in any conventional pulse sequence is replaced by tailored signal excitation module called aliasing module. Fig. 1 shows a typical aliasing module for acceleration factor of 3. When aliasing module designed for k-space aliasing factor of 3 is substituted instead of excitation pulse in any conventional pulse sequence then it will make the sequence to acquire three overlapped PE at a time. In Fig.1 the spins excited by RF_1 belong to the PE determined by (G_{pe1}+G_{pe2}+G_{pe3}), spins excited by RF_2 belongs to the PE determined by (G_{pe2}+G_{pe3}) and spins excited by RF_3 belongs to the PE determined by G_{pe3}. The phase of the RF excitation pulse in the module can be used to tag the overlapped PE with distinct phase information. This phase information is then used in reconstruction to unaliase the overlapped PE.

Fig. 2 graphically describes the k-space aliasing acquisition method. Fig. 2(a) shows the desired dynamic scene to acquire, Fig. 2(b) shows how the k-space data would be acquired using conventional imaging sequence where A(t), B(t) and C(t) are top, middle and bottom portions of k-space respectively. Fig. 2(c) shows the overlapped data acquired by using k-space aliasing module. The k-space aliasing module in this case is designed in such a way that top, middle and bottom portions of k-space overlap on each other. During acquisition the phases of RF_1, RF_2 and RF_3 are set to ϕ_1t, 0 and ϕ_2t respectively, where t is an integer which is equal

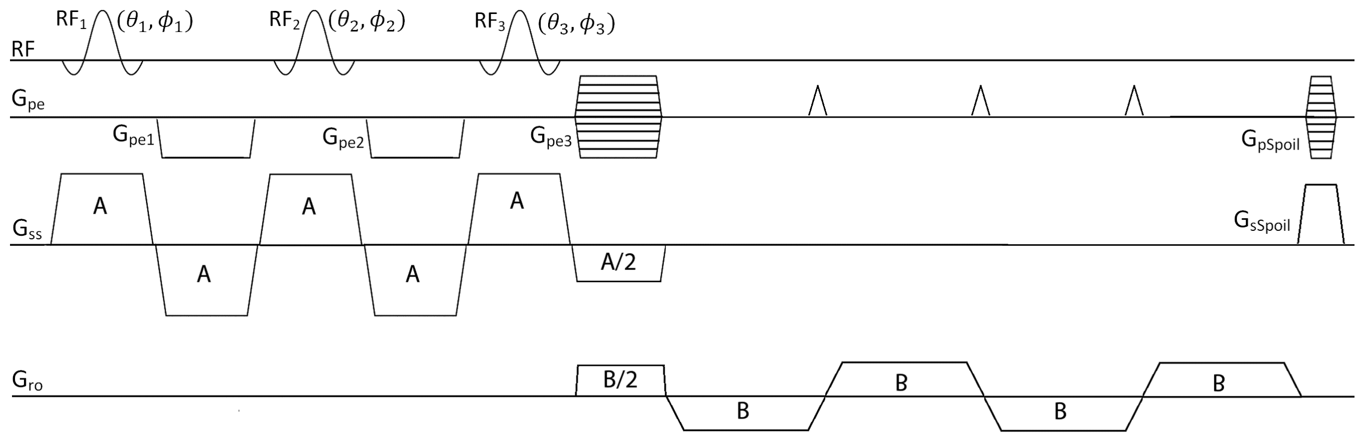

Fig. 1: Aliasing module designed for k-space aliasing factor of 3 inserted into interleaved gradient echo (IGEPI) sequence, here θ is the flip angle of RF excitation pulse, ϕ is the phase of the RF excitation pulse, A is the area under slice select gradient, G_{pe} is phase encoding axis, G_{ss} is slice select gradient axis and G_{ro} is readout gradient axis

to the time frame number. Therefore the acquired aliased k-space data is given by

$$K_{alias}(t) = [A(t)e^{j\phi_1 t} + B(t) + C(t)e^{j\phi_2 t}] \tag{1}$$

In reconstruction, a Fourier transformation is taken along time direction that separates the overlapped portion of the k-space and places the spectrum of A(t) at ϕ_1, spectrum of B(t) at 0 and spectrum of C(t) at ϕ_2 on the temporal frequency axis. If $k_1(t)$, $k_2(t)$ and $k_3(t)$ are three k-space points from A(t), B(t) and C(t) respectively that were overlapped during acquisition due to k-space aliasing, then after taking Fourier transformation along time these overlapped points would be separated as shown in Fig. 3. Each of these $K_1(f)$, $K_2(f)$ and $K_3(f)$ can be extracted by using a simple Fermi filters and rearranged appropriately followed by inverse Fourier transformation along time to generate full k-space data set.

B. *k-t sparse*

K-t sparse is a technique that exploits spatiotemporal sparsity to accelerate dynamic MRI scans. Equidistant under-sampling in k-t space results in aliasing artifact in spatial – temporal frequency (x-f) space. However random under-sampling of k-t space as shown in Fig. 4 results in incoherent aliasing artifact in x-f space. The aliasing artifact is also incoherent in sparse transform domain (Wavelet transform in space and Fourier transform in time). This incoherent aliasing artifact in transform domain can be removed by minimizing the sparse representation of k-t space subjected to data fidelity constraint by solving the following non-linear minimization program

$$\underset{x}{\text{Min}} \quad \|\psi x\|_{l_1} + \lambda \|Fx - y\|_{l_2} \tag{2}$$

Where x is the dynamic scene and ψ represent the sparsifying transform operator, F is under-sampled Fourier transform operator, y is acquired data after random order under-sampling of phase encodes and λ is a regularization parameter that enforces data consistency and determines the allowed noise level in the reconstructed image.

III. PROPOSED TECHNIQUE

In k-t sparse Fourier transform is used along time and wavelet tranform is used along phase encode direction to sparsify the imaging sequence. Fig. 5(a) shows k-space data acquired without using k-space aliasing module and Fig. 5(c) shows sparse representation of the imaging scene. Similarly Fig. 5(d) shows the data acquired using k-space aliasing module designed for factor of 3; Fig. 5(e) shows the images reconstructed using 2D Fourier transform of aliased k-space data and Fig. 5(f) shows their sparse representation. It is evident from Fig. 5(d) that the aliased k-space data is also sparse in the same transform domain however the level of sparsity has changed and it has become less sparse compared to Fig. 5(c).

If aliasing module is used in any pulse sequence then effectively we are acquiring data from an aliased or overlapped k-space. Hereafter, we term the phase encodes acquired using aliasing module as aliased phase encodes (**APE**). While using k-space aliasing module the sequence developer has the flexibility to choose which APEs to acquire. Therefore in order to combine k-space aliasing with k-t sparse, we propose to randomly under-sample the APEs that would result in incoherent aliasing artifact in x-f space. Here we represent X_u to be 2D Fourier transform of K_{alias} ($K_{alias} \overset{FFT}{\longleftrightarrow} X_u$). Similarly, imaging sequence X to be 2D Fourier transform of full k-space data set K ($K \overset{FFT}{\longleftrightarrow} X$).

1) First the acquired data (aliased and under-sampled) is subject to a minimization program that approximate the overlapped k-t space instead of full k-t space.

$$\underset{x}{\text{Min}} \quad \|\psi.X_u\|_{l_1} + \lambda \|F.X_u - y\|_{l_2} \tag{3}$$

where λ is the regularization parameter for solving minimization program and F is Fourier transformation

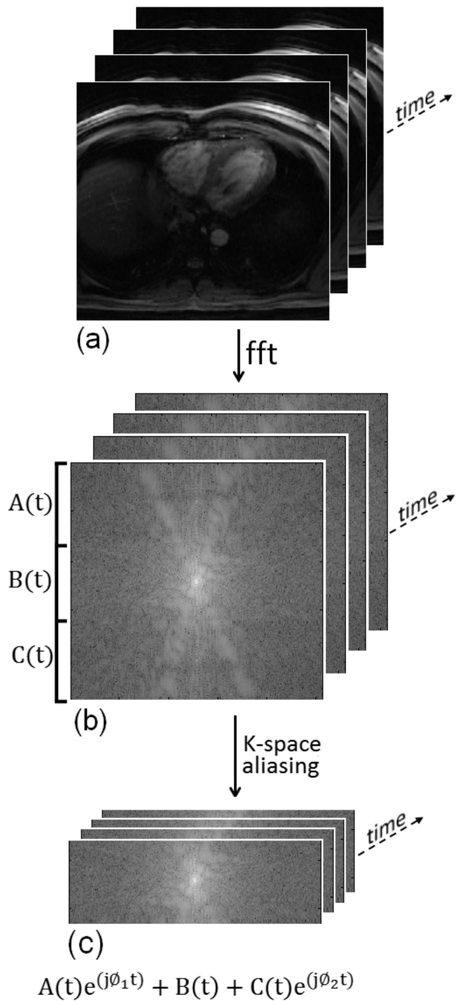

Fig. 2: **(a)** Sequence of desired images; **(b)** k-space needs to be acquired to form the desired images, here A(t), B(t) and C(t) are top middle and bottom protin of the k-space **(c)** Overlapped k-space consist of sum of top, middle and bottom portion of k-space that would be acquired by using k-space aliasing module

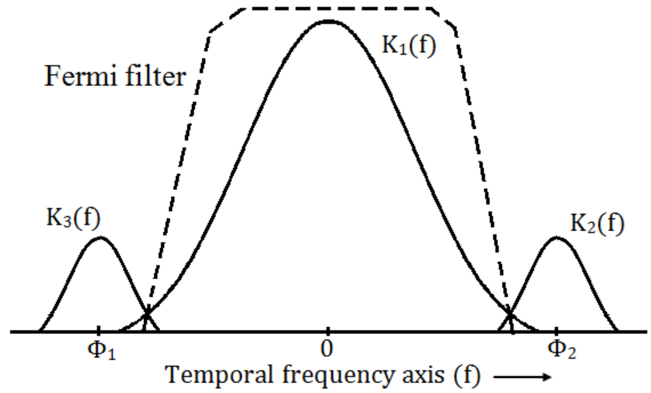

Fig. 3: Temporal spectrum of one point in overlapped k-space data after taking Fourier transformation along time. $K_1(f)$ comes from the middle portion of k-space and has larger bandwidth and magnitude than the other two $K_2(f)$ and $K_3(f)$ that has come from the top and bottom portion of k-space. The dotted line shows a Fermi filter designed to extract $K_1(f)$.

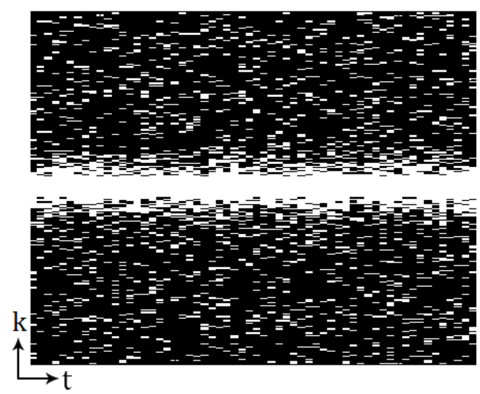

Fig. 4: Each time frame when randomly under-sampled in phase encode direction with different sampling pattern would result in 2D random under-sampling pattern in k-t space as shown in this figure.

operator.

2) Once the aliased k-space data K_{alias} is recovered through step1, then applying a simple Fourier transformation along time and use of fermi filter can recover the desired k-space data (X) as discussed in k-space aliasing section.

IV. SIMULATION RESULTS

The feasibility of the proposed technique was validated on cardiac triggered dataset acquired using Siemens Skyra 3T human scanner. Informed consent was taken from the volunteer in accordance with institute policy. Aliaising module designed for acceleration factor of 3 was inserted in place of RF excitation pulse in an interleaved gradient echo EPI sequence to acquire overlapped k-space data set with echo train length (ETL) = 4, FOV = 300×300 mm^2, TR = 12 ms, cardiac phases = 48, segment size = 4, number of segment = 16, APEs = 80, number of heart beats = 16 and acquisition window = 700 ms (acquisition time after ECG trigger). Echo train length of 4, results in acquisition of 4 APEs for each cardiac phase giving temporal resolution of 12 ms. Echo shifting [14] was implemented in the IGEPI sequence to suppress ghosting due to FAT. The images were reconstructed with k-space unaliasing reconstruction resulting in 48 time frames of size 240x240 pixels. These images were used as reference image for all the simulation presented here.

For simulating k-t sparse, each time frame was under-sampled in phase encode direction with different sampling pattern that would result in 2D random under-sampling pattern in k-t space (Fig. 4). The minimization program given in eq.(2) was solved using nonlinear conjugate gradient algorithm to recover the imaging sequence. Result for the same is shown in Fig. 6(b), (c) and (d).

For simulating k-t sparse and k-space aliasing combina-

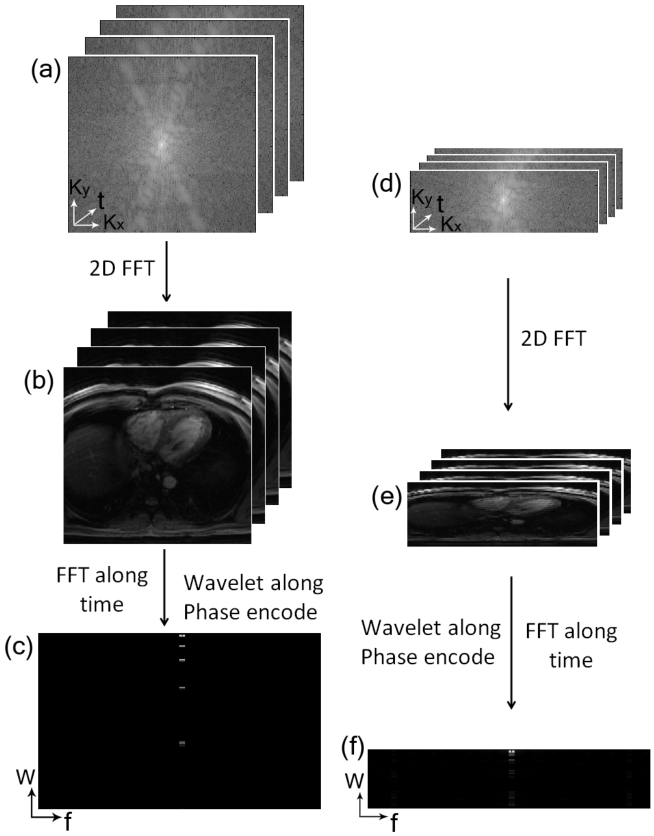

Fig. 5: **(a)** k-space time frames; **(b)** imaging sequence; **(c)** sparse representation of k-t space; **(d)** overlapped k-space data acquired using k-space aliasing module; **(e)** images obtained by 2D FFT of acquired overlapped data; **(f)** sparse representation of overlapped k-t space.

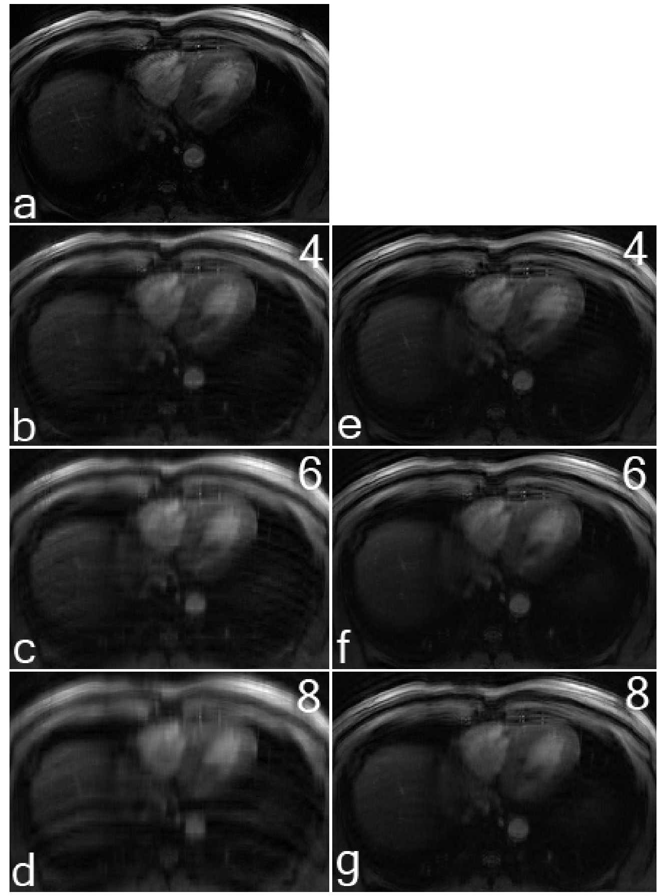

Fig. 6: **(a)** One time frame of reference image; **(b), (c)** and **(d)** are reconstructed image by k-t sparse for acceleration factors of 4, 6 and 8 respectively; **(e), (f)** and **(g)** are reconstructed image by the proposed method for total acceleration factor (k-space aliasing acceleration factor $\times$ k-t sparse acceleration factor) of 4 (3×1.33), 6 (3×2) and 8 (3×2.66) respectively.

tion, first the k-space data was overlapped as shown in Fig. 2(c) to form aliased k-space; then random under-sampling of aliased phase encode (APE) was done to generate a 2D random under-sampling in k-t space. In reconstruction, first eq.(3) was solved to recover aliased k-space data that was then subject to k-space unaliasing reconstruction method to recover full k-space data set. The result for the same is shown in Fig. 6(e), (f), and (g). In case of the proposed combination of k-space aliasing and k-t sparse the total acceleration factor would be multiplication of acceleration factor due to k-space aliasing and k-t sparse. It is evident from the simulation results that the proposed method preserves resolution of image and removes incoherent aliasing artifact better than k-t sparse.

Table.1 shows that mean square error (MSE) at different acceleration factors for the k-t sparse alone and the proposed 'combination of k-space aliasing and k-t sparse'. The MSE for the proposed combination was always less than the k-t sparse alone. Therefore, the proposed combination performs better than the k-t sparse at all acceleration factors. Moreover as the acceleration factor increases, the increase in MSE for the k-t sparse was more than that of increase in MSE for the proposed combination. Therefore, the proposed combination becomes even more superior to the k-t sparse at higher acceleration factors.

TABLE I

MEAN SQUARE ERROR AT DIFFERENT ACCELERATION FACTORS

Acceleration factor	k-t sparse	Proposed method
4	5.5890×10^{-5}	4.9813×10^{-5}
6	5.5977×10^{-5}	4.9817×10^{-5}
8	5.6100×10^{-5}	4.9846×10^{-5}

V. DISCUSSION

K-space aliasing exploits redundancy in temporal domain by allocating targeted temporal bandwidth to different region of k-space, while k-t sparse exploits sparsity of k-t space in transform domain. Therefore each of the technique exploits different redundancy of the imaging process hence their combination provides acceleration that would not be achievable by each of the individual technique alone.

A critical factor in k-t sparse reconstruction is the ratio n/N, where n is the number of acquired PEs and N is the number of targeted PEs to be reconstructed after l_1 minimization. This ratio will be very low when k-t sparse alone is used as compared to the k-t sparse and k-space aliasing combination. For instance, if k-t sparse alone was used and the total number of targeted PEs were 300 and the acquired PEs were only 50, then the ratio n/N would be 0.1667 . However, for the 'k-t sparse and k-space aliasing combination' the target N will be aliased k-space consisting of only 100 APEs, and the acquired APEs will still be 50. This increases the ratio n/N to 0.5, thereby dramatically increasing the performance of l_1 minimization component of the proposed combination. Also this technique applies the l_1 minimization on aliased k-space (reduced dimension) data, thereby significantly reducing the required computation time to solve non-linear program.

K-space aliasing has been combined with GRAPPA and compressed sensing (CS) in [15]. When combined with GRAPPA, the achievable acceleration factor would be limited by g-factor of the coil geometry. However when combined with CS, the achievable acceleration factor would be limited by sparsity of image and incoherence due to sampling pattern. This scheme differs from the previously presented scheme [15], as it exploits spatiotemporal sparsity rather than only spatial sparsity and exploits 2D incoherence rather than 1D incoherence during reconstruction.

VI. CONCLUSIONS

A novel sampling method to rapidly acquire k-t space is proposed. The proposed method reconstructs high frame rate MR images by exploiting both sparsity and temporal redundancy. This method has potential to be used in clinical applications requiring high spatial and temporal resolution.

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