35th Annual International Conference of the IEEE EMBS
Osaka, Japan, 3 - 7 July, 2013

Ultrasonic Guided Waves Dispersion Reversal for Long Bone Thickness Evaluation: a Simulation Study*

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Abstract — **It has been shown that ultrasonic guided waves have great potentials for long cortical bone evaluation. However, due to the multimodal dispersion, the received signals usually contain several mixed guided modes, which highly complicates the mode separation and signal processing. In the study, we showed that the use of dispersion reversal excitation allows the self-compensation of the dispersive modes in the long cortical bone. Two-dimension finite-difference time-domain (2D-FDTD) method was employed to simulate the propagation of two fundamental guided modes, symmetrical S0 and anti-symmetrical A0, in the long cortical bones. It was demonstrated that the pulse-like modes of S0 and A0 can be detected under the dispersion reversal excitations. The simulations also illustrated that the proposed dispersion reversal method can be used to evaluate the cortical thickness. Results are promising for the application of dispersion reversal method in ultrasonic assessment of the long cortical bone.**

I. Introduction

Due to the aging population, osteoporosis has been gained considerable attention. As an elastic wave, ultrasound is naturally sensitive to the material biomechanical properties and geometry with advantages of low-expense, portability and non-radiation [1]. Therefore, researchers have shifted the innovative emphasis towards ultrasonic bone evaluation and fracture monitoring [2-5]. Long cortical bone is a tubular structure and can support the propagation of different kinds of waves, for instance surface waves and guided waves [1, 5]. Current studies demonstrated that the ultrasonic guided waves is capable of assess the cortical thickness, bone fracture, and bone mineral density, which can open up promising applications [1]. However, in the field of the long bone evaluation, guided waves analysis is not as mature as the non-destructive evaluation in the industry. Great efforts have thus been taken into the signal processing for dispersion suppression and mode separation [6-8]. Time reversal method has been used to compensate the dispersive influence of guided waves and focus the plate defects [9]. Single mode tuning technique has also been proposed [10]. Although a number of studies have analyzed the applications of time reversal in the non-destructive evaluation, it has not been used for cortical bone evaluation. The main challenges could be the short diaphyseal length, unknown cortical thickness, multimodal overlap and also clinical limitation.

* This work was supported by the National Natural Science Foundation of China (11174060), the Ph.D. Programs Foundation of the Ministry of Education of China (20110071130004), the New Century Excellent Talents of the Ministry of Education of China (NCET-10-0349), and the China Postdoctoral Science Foundation (2012M520826).

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In this study, we applied the dispersion reversal method for long cortical thickness evaluation. The guided propagation function was firstly used to generate the theoretical single mode signals with wideband dispersion. The single mode waveforms (symmetrical S0 and anti-symmetrical A0) were then reversed and launched to excite the self-compensated pulses. To validate the proposed dispersion reversal method in the long cortical bone, two-dimension finite-difference time-domain (2D-FDTD) method was applied. Finally, we tested the use of dispersion reversal method for assessing long cortical bone thicknesses.

II. Methods

A. Theoretical Solutions of Guided Waves in Plates

Guided waves propagating in a solid plate (or layer) also named as Lamb waves, which are vibrations with traction-free boundary conditions. Lamb modes can be classified into symmetrical and anti-symmetrical systems. For an isotropic plate of thickness $2h$, the dispersion relation of these modes are governed by the Rayleigh-Lamb frequency equations [11]

$$(k^2-q^2)^2\tan(qh)+4k^2pq\tan(ph)=0 \text{ symmetrical modes.} \quad (1a)$$

$$(k^2-q^2)^2\tan(ph)+4k^2pq\tan(qh)=0 \text{ anti-symmetrical modes.} \quad (1b)$$

where k is the angular wave-number and numerically equal to ω / V_p, V_p is the phase velocity of the Lamb modes, and ω is the angular frequency. The coefficients p and q are given by

$$p^2=\frac{\omega^2}{V_L^2} \quad \text{and} \quad q^2=\frac{\omega^2}{V_T^2} \quad (2)$$

where V_L and V_T are the longitudinal and shear bulk wave velocities, respectively. After the root extraction of the Eq. (1), the dispersion curves can be expressed as wave-number k, phase velocity V_p, or group velocity V_g, versus frequency or frequency-thickness product.

B. Propagation of Dispersive Guided Waves

The physical mechanism of dispersion has been widely analyzed, but here we focus on the views of signal processing about the dispersive influence of the excitation. Under the assumption that a broadband incident pulse $f(t)$ is excited by a transmitter, the guided wave signals measured at distance x_0 can be defined by out-of-plane surface displacement $g(t)$ as [11]

$$g(t)=\int_{-\infty}^{+\infty}F(\omega)H(\omega)e^{-j\omega t}d\omega \quad \text{where } H(\omega)=e^{-jk(\omega)x_0} \quad (3)$$

where t is the time. The wave-number of the mode of interest $k(\omega)$ is a function of angular frequency ω, and $F(\omega)$

is the frequency spectrum of the excitation $f(t)$. The modal propagation, derived as Eq. (3), actually adds a phase adjustment term $e^{-jk(\omega)x_0}$ to the excitation $F(\omega)$, and from the time shift property of Fourier transform, it is known that the factor $e^{-jk(\omega)x_0}$ will change the time delay of harmonics of $F(\omega)$. This phenomenon is the so-called dispersion.

C. Dispersion Reversal of the Single Mode

Assuming a single mode signal $g_S(t)$ is obtained by the Eq. (3), the spectrum of the $g_S(t)$ is $G_S(\omega) = F(\omega)e^{-jk_S(\omega)x_0}$. The dispersion reversal signal is $g_S(\tau_0 - t)$, where τ_0 is a time constant. As automatic dispersion compensation, the dispersion reversal single mode forward propagating in the waveguide can gradually regress backward to the original pulse.

III. Simulations

A. Dispersive Signal Generation

According to the Eq. (3), single modal signals of S0 and A0 can be obtained. Material parameters of the cortical bone are about density of ρ = 1.5 g/cm^3, longitudinal wave velocity of V_l = 4063 m/s and shear wave velocity of V_t = 1846 m/s [12]. The synthesized S0 and A0 signals plate are given in Fig. 1, where the cortical thickness is 3 mm, the propagation distance is 100 mm.

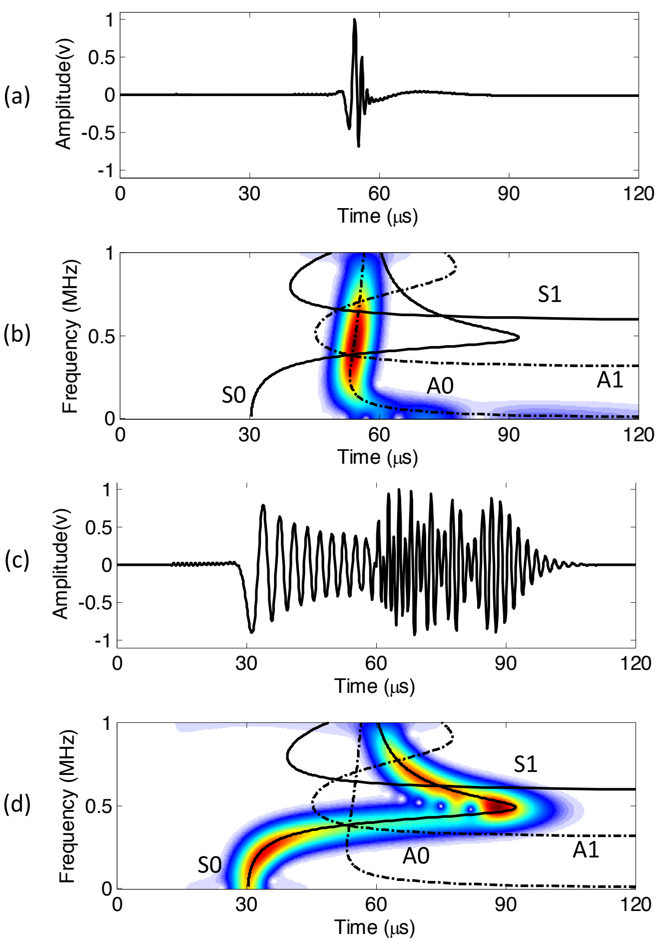

Fig. 1. Synthesized wideband dispersive signals of 3 mm thick bone plate, (a) A0 waveforms, (b) A0 time frequency representation, (c) and (d) are the waveform and time frequency representation of S0. The center frequency is 0.5 MHz, and -3 dB bandwidth is 0.5 MHz.

As shown in Fig.1 (a) and (c), the single modal signals of S0 and A0 can be theoretically synthesized. The time frequency representation of S0 and A0 are calculated by the short time Fourier transform in Fig. 1 (b) and (d). The dash and dot lines in Fig. 1 (b) and (d) are the theoretical dispersive curves of S0, A0, S1 and A1. The time frequency representations of the synthesized A0 and S0 are in good accordance to the theoretical dispersion curves. To avoid the excitation of high-order modes, such as S1 and A1, in the simulation analysis, we only discussed in the low frequency range (<0.3 MHz).

B. Dispersion Reversal Simulation

2D-FDTD was employed for the dispersion reversal simulation. Fig. 2 shows the simulation model of a 300 mm long cortical layer. For consistency of comparison, the distance between the receiver and transmitter was fixed at 100 mm to imitate the flat diaphyseal region of the long cortical bone. The dispersion reversal single modes of S0 and A0 were used as the excitations to testify the self-compensation effects. Absorbing boundaries were used on the two sides of the cortical bone plate to attenuate the wave reflection. The top and bottom boundaries were satisfied by the traction-free condition.

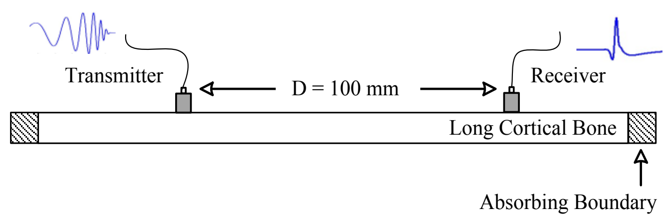

Fig. 2. The simulation model.

To obtain the symmetrical vibration of S0 Mode, a transducer pair was arranged on the two sides of the cortical plate with the identical excitation signals. With regard to the anti-symmetrical mode A0, only a contact transducer was used.

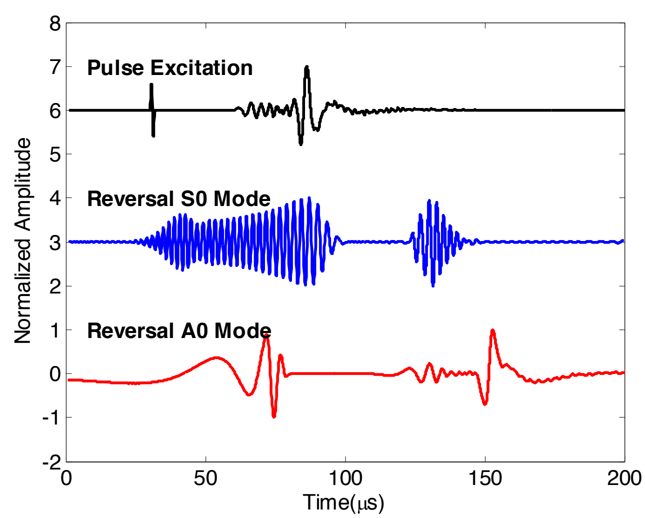

Fig. 3. Comparison of the pulse excitation and dispersion reversal excitation of the S0 and A0, the cortical thickness is 3 mm.

Figure 3 shows the simulated signals under the traditional pulse excitation and dispersion reversal excitations of the modes S0 and A0. The duration of the pulse excitation is 5 μs. The center frequency of the A0 and S0 are 0.1 MHz and 0.3 MHz, respectively.

The dispersion reversal results of S0 and A0 are plotted by blue and red lines, respectively. It can be found that the original dispersive excitation, *i.e.* reversal S0 and A0 modes can be self-compensated to the pulse-like signals. Relatively weak S0 component can also be detected under the A0 excitation. As can be found in the reversal A0 excited signals, the highly dispersive A0 was focused as a one cycle pulse, and two separated S0 and A0 can be identified without modal overlap.

IV. Results

According to the former simulations, the accurate dispersion reversal signals of the certain bone plate can achieve a full compensation. The accurate wideband dispersion reversal signals can be calculated from the dispersion function in Eq. (3) with the correct bone parameters, including density, velocity, and cortical thickness. In this study, we only analyzed the thickness determination and suppose the density and velocity are known.

A series of dispersion reversal signals were then generated with the cortical thickness parameter varying from 1 mm to 6 mm. As can be seen in Fig. 4, 6 different dispersion reversal signals of A0 were obtained. The energy of these 6 signals is the same. The center frequency of the signals is 0.1 MHz and the bandwidth is of 0.3 MHz.

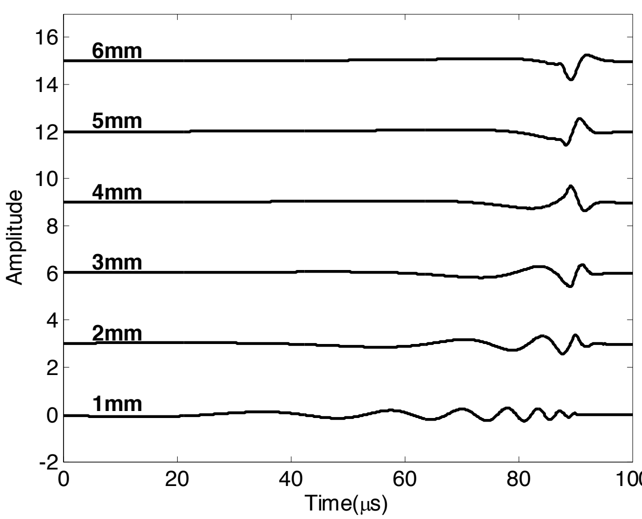

Fig. 4. The different dispersion reversal signals, the cortical thicknesses are from 1 mm to 6 mm.

The dispersion reversal signals were further used to excite the A0 mode in the same bone plate with a constant cortical thickness of 3 mm. Fig. 5 shows the simulated signals of the 3 mm bone plate under different dispersion reversal excitations. From the top to the bottom, there are the simulated signals with cortical thickness parameter varied from 6 mm to 1 mm. Although both the S0 and A0 were excited by the singular contact transducer, under the excitation of the 3 mm A0 dispersion reversal signal, the fully-compensated A0 pulse was successfully obtained with the strongest pulse energy and the shortest duration.

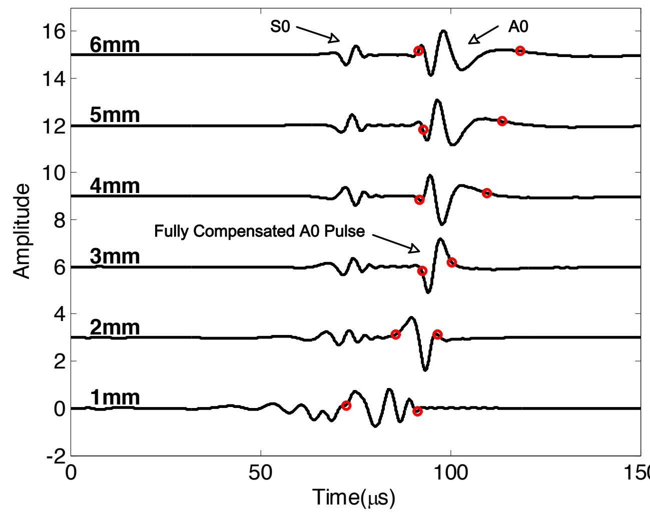

Fig. 5. Simulated signals of the 3 mm bone plate. The thickness parameter of dispersion reversal signals is varied from 1 mm to 6 mm.

As can be seen in Fig. 5, the A0 pulses were extracted by 15% threshold of the corresponding signal amplitudes. The duration and amplitude of the self-compensated pulse were then calculated to describe the signals in Fig. 5. Quantitative analysis of the self-compensated pulses with the thickness parameter variation are presented in Fig. 6, where the Fig. 6(a) and (b) are the durations and amplitudes of the A0 pulses in Fig. 5.

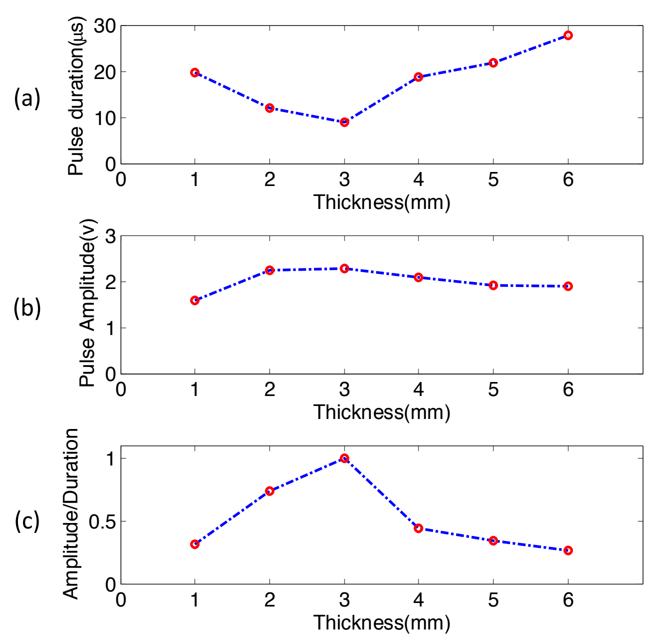

Fig. 6. Analysis of the self-compensated A0 pulses with the thickness parameter variation. (a) Duration, (b) Amplitude, c) Normalized ratio of amplitude and duration.

It can be found that the pulse durations start from 19.79 μs to 9.0 μs and end at 27.8 μs. The minimum duration is obtained at the 3 mm. Similarly, the amplitudes of the A0 pulses increase from 1.59 to 2.28, and decrease to 1.90. The amplitude maximum is at 3 mm. However, the difference of the amplitudes is not as significant as the durations. The general trend should be that under the correct self-compensation, the amplitude reaches the maximum and the pulse duration achieves the minimum. Consequently, a combined parameter, amplitude/duration, should be more sensitive than the amplitude and duration. Fig. 6(c) shows the normalized curve of the ratio of the amplitude and duration, the curve peak locates at the 3 mm, which illustrates that the cortical thickness can be determined by the self-compensation effects of the dispersion reversal signals.

V. Discussion

The mathematical models of the dispersion generation, reversal and self-compensation were introduced. The dispersion function can be used to synthesize the wideband single mode signals. These signals can be reversed as excitations to stimulate the specific modal pulses in the bone plates. Since the strongest pulse energy and shortest pulse duration come out by using the correct synthetic parameters, the cortical thickness evaluation can be achieved.

Limited by the clinical application, the traditional contact transducers are usually adopted in the long cortical bone evaluation. Under the circumstance, the anti-symmetrical modes are primarily excited and received. Consequently, we mainly investigated the A0 mode excitation and dispersion. The simulations of A0 mode suggest that dispersion reversal method could be used for long cortical bone thickness evaluation.

However, as can be seen in Fig. 3, the use of contact transducer pair allows to only excite the symmetrical mode S0, and thus the proposed dispersion reversal method is also suitable for the self-compensation of S0.

As for the long bone evaluation, the soft tissue has high influence of the modal excitation, attenuation and velocity, which was not considered in the study. The simulations are preliminary, and should be more accurate for clinical application.

VI. Conclusion

It is demonstrated in the paper that the synthesized dispersion reversal signals can be used to stimulate the pulse-like mode components. The 2D-FDTD bone plate simulation illustrated that based on the excitation method of the single dispersion reversal mode, the S0 and A0 pulse can be obtained. The pulse amplitude and duration may be sensitive to the cortical thickness. The reported results are encouraging to step toward the application of the dispersion reversal method in the ultrasonic assessment of long cortical bone. Future work will focus on the analysis of the soft tissue influence and *in vivo* experiments.

Acknowledgment

The authors acknowledge Lida Xiao for his creative contributions and discussions within his undergraduate study.

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