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Noise Estimation in Magnetic Resonance SENSE Reconstructed Data

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***Abstract*— Parallel imaging methods allow to increase the acquisition rate via subsampled acquisitions of the k-space. SENSE is one of the most popular reconstruction methods proposed in order to suppress the artifacts created by this subsampling. However, the SENSE reconstruction process yields to a variance of noise value which is dependent on the position within the image. Hence, the traditional noise estimation methods based on a single noise level for the whole image fail. Accordingly, we propose a novel method to recover the complete spatial pattern of the variance of noise in SENSE reconstructed images up from the sensitivity maps of each receiver coil. Our method fits applications in statistical image processing tasks such as image denoising.**

***Index Terms*— Noise estimation, parallel imaging, SENSE, Magnetic Resonance**

I. INTRODUCTION

Magnetic Resonance (MR) data is known to be affected by several sources of quality deterioration, due to limitations in the hardware, scanning times, movement of patients, or even the motion of molecules in the scanning subject. One source of degradation that affects most of the acquisitions is thermal noise. The presence of noise over the acquired MR signal is a problem that affects not only the visual quality of the images, but also may interfere with further processing techniques such as registration or tensor estimation in Diffusion Tensor MRI. Emerging techniques that demand large amounts of data, such as High Angular Resolution Diffusion Imaging (HARDI), in order to reduce the acquisition time, also reduce the temporal averaging; as a consequence, the noise power is increased proportionally to the square root of the speedup.

One of the most direct approaches to cope with acquisition noise in MRI (but not the only one) is signal estimation via noise removal. Traditionally, noise filtering techniques in different fields have been based on a well-defined prior statistical model of data, usually a Gaussian model. Noise models in MRI have allowed the natural extension of many well known techniques to cope with features specific of MRI. Many examples can be found in the literature, such as the Conventional Approach (CA) [1], Maximum Likelihood (ML), linear estimators [2], or adapted non-local mean (NLM) schemes [3], [4]. All of these methods explicitly need an estimation of the variance of noise. They usually assume a single coil configuration in which noise is modeled as a complex Gaussian process and therefore the magnitude signal is the Rician distributed envelope. In any case, the noise parameters are usually considered constant through the image, i.e., the noise is stationary. The CA method and the NLM, for instance, estimate the signal by simply subtracting the bias term from the expected value of the square of the magnitude $M(\mathbf{x})$:

$$\widehat{I(\mathbf{x})} = \sqrt{E\{M^2(\mathbf{x})\} - 2\sigma_n^2}, \qquad (1)$$

where σ_n^2 is the variance of noise that must be known or estimated.

Although the stationary Rician model is widely used in literature, the fact is that nowadays, due to time restrictions, most acquisitions are usually accelerated by using Parallel MRI (pMRI) reconstruction techniques, which allow to increase the acquisition rate via subsampled acquisitions of the **k**-space. Many reconstruction methods have been proposed in order to suppress the aliasing and underlying artifacts created by this subsampling, being SENSE [5] dominant among them. From a statistical point of view, such a reconstruction will affect the stationarity of the noise in the reconstructed data, i.e. the spatial distribution of the noise across the image. As a result, if SENSE is used, the magnitude signal may be considered Rician [6], [7] but the value of the statistical parameters, and in particular the variance of noise σ_n^2, will vary for different image locations, i.e. it becomes **x**-dependent.

Noise estimators proposed in literature (see for instance [2], [8]) are based on the assumption of a single σ_n^2 value for all the pixels in the image. Accordingly, those methods do not apply when dealing with pMRI. Noise estimators must therefore be reformulated in order to cope with these new image modalities.

In this paper we propose a method to estimate the spatially distributed variance of noise σ_n^2 from the magnitude signal when SENSE is used as pMRI technique. The method is based on the study of the distribution of noise in SENSE.

II. STATISTICAL NOISE MODEL IN SENSE RECONSTRUCTED IMAGES

Prior to the definition of the estimators, the statistical noise model in SENSE must be properly defined. Many studies have been made about this topic from a SNR or a g-factor (noise amplification) point of view [5], [7], [9]. Since this paper is focused on the σ_n^2 value estimation rather than a SNR level, an equivalent reformulation must be done, more coherent with the signal and noise analysis usually assumed for noise estimation.

In multiple coil scanners, the image received in coil l-th, $S_l(x, y)$, can be seen as an *original image* $S_0(x, y)$ weighted by the sensitivity of that coil:

$$S_l(x, y) = C_l(x, y)S_0(x, y), \quad l = 1, \cdots, L \qquad (2)$$

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An accelerated pMRI acquisition with a factor r will reduce the matrix size of the image at every coil. The signal in one pixel at location (x, y) of l-th coil can be now written as [10]:

$$S_l(x,y) = C_l(x,y_1)S_0(x,y_1) + \cdots + C_l(x,y_r)S_0(x,y_r) \quad (3)$$

In what follows, let us call $S_l^{\mathcal{S}}(x,y)$ to the subsampled signal at coil l-th and $S^{\mathcal{R}}(x,y)$ to the final reconstructed image. Note that the latter can be seen as an estimator of the original image $S^{\mathcal{R}}(x,y) = \widehat{S_0}(x,y)$ that can be obtained from eq. (3). For instance, for $r = 2$ for pixel (x,y), $S^{\mathcal{R}}(x,y)$ is obtained as

$$\begin{bmatrix} S_1^{\mathcal{R}} \\ S_2^{\mathcal{R}} \end{bmatrix} = \begin{bmatrix} \mathbf{W}_1 & \mathbf{W}_2 \end{bmatrix} \times \begin{bmatrix} S_1^{\mathcal{S}} & \cdots & S_L^{\mathcal{S}} \end{bmatrix}. \quad (4)$$

In matrix form for each pixel and arbitrary r

$$S_i^{\mathcal{R}} = \mathbf{W}_i \times \mathbf{S}^{\mathcal{S}} \quad i = 1, \cdots, r. \quad (5)$$

with $\mathbf{W} = [\mathbf{W}_1, \cdots \mathbf{W}_r]$ a reconstruction matrix created from the sensitivity maps at each coil. These maps, $\mathbf{C} = [\mathbf{C}_1, \cdots, \mathbf{C}_L]$ are estimated through calibration right before each acquisition session. Once they are known, the matrix $\mathbf{W}$ reduces to a least-squares solver for the overdetermined problem $\mathbf{C}(x,y) \times \mathbf{S}^{\mathcal{R}}(x,y) \simeq \mathbf{S}^{\mathcal{S}}(x,y)$ [5], [10]:

$$\mathbf{W}(x,y) = (\mathbf{C}^*(x,y)\mathbf{C}(x,y))^{-1}\mathbf{C}^*(x,y). \quad (6)$$

The correlation between coils may be incorporated in the reconstruction as a pre-whitening matrix for the measurements, and $\mathbf{W}(x,y)$ becomes then a weighted least squares solver with correlation matrix $\mathbf{\Sigma}$:

$$\mathbf{W}(x,y) = (\mathbf{C}^*(x,y)\mathbf{\Sigma}^{-1}\mathbf{C}(x,y))^{-1}\mathbf{C}^*(x,y)\mathbf{\Sigma}^{-1}.$$

The SNR of the fully sampled image and the image reconstructed with SENSE are related by the so-called g-factor, g [9], [10]:

$$\text{SNR}_{\text{SENSE}} = \frac{\text{SNR}_{\text{full}}}{\sqrt{r} \cdot g} \quad (7)$$

However, in our problem we are more interested on the actual noise model underlying the SENSE reconstruction and on the final variance of noise. The final signal $S_i^{\mathcal{R}}$ is obtained as a linear combination of $S_l^{\mathcal{S}}$, where the noise is Gaussian distributed. Thus, the resulting signal is also Gaussian, with variance:

$$\sigma_i^2 = \mathbf{W}_i^* \mathbf{\Sigma} \mathbf{W}_i. \quad (8)$$

Since $\mathbf{W}_i$ is position dependent, i.e. $\mathbf{W}_i = \mathbf{W}_i(x,y)$, so will be the variance of noise, $\sigma_i^2(x,y)$. For further reference, when the whole image is taken into account, let us denote the variance of noise for each pixel in the reconstructed data by $\sigma_{\mathcal{R}}^2(\mathbf{x})$.

Note now that all the lines $S_i^{\mathcal{R}}$ reconstructed from the same data $S_l^{\mathcal{S}}$ will be strongly correlated, since they are basically different linear combinations of the same Gaussian variables.

All in all, noise in the final reconstructed signal $S^{\mathcal{R}}(x,y)$ will follow a complex Gaussian distribution. If the magnitude is considered, i.e. $M(x,y) = |S^{\mathcal{R}}(x,y)|$, the final magnitude image will follow a Rician distribution [7], just like single-coil systems.

To sum up: (1) Subsampled multi coil MR data reconstructed with Cartesian SENSE follow a Rician distribution at each point of the image; (2) The resulting distribution is non-stationary. This means that the variance of noise will vary from point to point across the image; (3) The final value of the variance of noise at each point will only depend on the covariance matrix of the original data and on the sensitivity map, and not on the data themselves; (4) Each pixel in the final image will be strongly correlated with all those pixels reconstructed from the same original data. Each pixel is correlated with $r - 1$ other pixels.

III. Noise Estimation in SENSE

In the background of a SENSE MR image, where the SNR is zero due to the lack of water-proton density in the air, the Rician PDF simplifies to a (non-stationary) Rayleigh distribution, whose second order moment is defined as

$$\text{E}\{M^2(\mathbf{x})\} = 2 \cdot \sigma_R^2(\mathbf{x}). \quad (9)$$

Since $\sigma_R^2(\mathbf{x})$ is $\mathbf{x}$-dependent, $\text{E}\{M^2(\mathbf{x})\}$ will also show a different value for each $\mathbf{x}$ position. Let us assume that each coil in the $\mathbf{x}$-space is initially corrupted with uncorrelated Gaussian noise with the same variance σ_n^2 and there is a correlation between coils ρ^2 so that matrix $\mathbf{\Sigma}$ becomes

$$\mathbf{\Sigma} = \sigma_n^2 \begin{pmatrix} 1 & \rho^2 & \cdots & \rho^2 \\ \rho^2 & 1 & \cdots & \rho^2 \\ \vdots & \vdots & \ddots & \vdots \\ \rho^2 & \rho^2 & \cdots & 1 \end{pmatrix} = \sigma_n^2 \left(\mathbf{I} + \rho^2[\mathbf{1} - \mathbf{I}]\right).$$

with $\mathbf{I}$ the $L \times L$ identity matrix and $\mathbf{1}$ a $L \times L$ matrix of 1's. For each $\mathbf{x}$ value, we define the global map

$$\mathcal{G}_{W_i} = \mathbf{W}_i^* \left(\mathbf{I} + \rho^2[\mathbf{1} - \mathbf{I}]\right) \mathbf{W}_i, \quad i = 1, \cdots, r$$

Global map $\mathcal{G}_W(\mathbf{x})$ can be easily infered from the $\mathcal{G}_{W_i}$ values. Note that $\mathcal{G}_W(\mathbf{x})$ is strongly related to the g-factor [9]. Eq. (9) then becomes

$$\text{E}\{M^2(\mathbf{x})\} = 2\ \sigma_n^2\ \mathcal{G}_W(\mathbf{x}) \quad (10)$$

and

$$\sigma_n^2 = \frac{\text{E}\{M^2(\mathbf{x})\}}{2\ \mathcal{G}_W(\mathbf{x})} \quad (11)$$

By using this regularization, we can assure a single σ_n^2 value for all the points in the image. Following the noise estimation philosophy in [2], [8], we can now define a noise estimator based on the local sample estimation of the second order moment:

$$\langle M^2(\mathbf{x})\rangle_{\mathbf{x}} = \frac{1}{|\eta(\mathbf{x})|} \sum_{\mathbf{p} \in \eta(\mathbf{x})} M^2(\mathbf{p}),$$

with $\eta(\mathbf{x})$ a neighborhood centered in $\mathbf{x}$. $\langle M^2(\mathbf{x})\rangle_{\mathbf{x}}$ is known to follow a Gamma distribution [8] whose mode is $2\sigma_n^2(|\eta(\mathbf{x})| - 1)/|\eta(\mathbf{x})|$. Then

$$\text{mode}\left\{\frac{\langle M_L^2\rangle_{\mathbf{x}}}{\mathcal{G}_W(\mathbf{x})}\right\} = 2\sigma_n^2 \frac{|\eta(\mathbf{x})| - 1}{|\eta(\mathbf{x})|} \approx 2\sigma_n^2$$

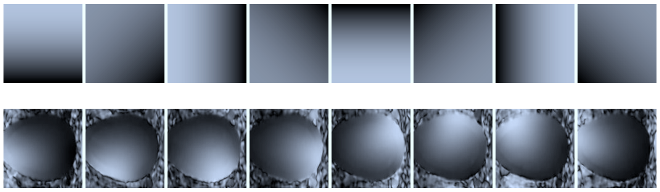


Fig. 1. Sensitivity Maps used for the experiments. Top: synthetic sensitivity map. Bottom: Map estimated from real acquisition.

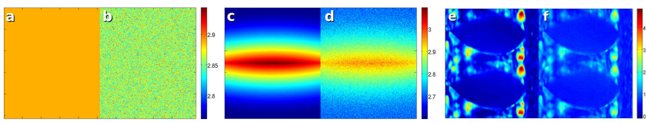


Fig. 2. Maps of $\sigma^2_{\mathcal{R}}(\mathbf{x})$ in the final image: (a-c-e): Theoretical values. (b-d-f): Estimated from samples. (a-b) Synthetic Sensitivity Map with no correlation. (c-d) Synthetic Sensitivity Map with correlation between coils. (e-f) Real sensitivity map with correlation between coils (log scale).

when $|\eta(\mathbf{x})| >> 1$. The estimator is then defined as

$$\widehat{\sigma^2_n} = \frac{1}{2}\text{mode}\left\{\frac{\langle M_L^2(\mathbf{x})\rangle_{\mathbf{x}}}{\mathcal{G}_W(\mathbf{x})}\right\} \tag{12}$$

and consequently the noise in each pixel is estimated as

$$\widehat{\sigma^2_{\mathcal{R}}}(\mathbf{x}) = \frac{1}{2}\text{mode}\left\{\frac{\langle M_L^2(\mathbf{x})\rangle_{\mathbf{x}}}{\mathcal{G}_W(\mathbf{x})}\right\}\mathcal{G}_W(\mathbf{x}) \tag{13}$$

This estimator is only valid over the background pixels. However, as shown in [2], [8], no segmentation of these pixels is needed: the use of the mode operator allow us to work with the whole image. On the other hand, to carry out the estimation, the sensitivity map of each coil and the correlation between coils must be known beforehand. These parameters are needed for the SENSE encoding, and thus, they can be easily obtained.

IV. Experiments and Results

We will **first** test the variation of parameter $\sigma^2_{\mathcal{R}}(\mathbf{x})$ across the image in SENSE. To that end, we work with two sensitivity maps belonging to 8-coil systems as shown in Fig. 1: one synthetic sensitivity map (top) and a real map (bottom), estimated from a T1 acquisition done in a GE Signa 1.5T EXCITE, FSE pulse sequence, 8 coils, TR=500msec, TE=13.8msec, 256×256 and FOV: 20cm×20cm. For the sake of simplicity we assume a normalized variance at each coil $\sigma_l^2 = 1$ since it will not affect the experiment. We will simulate two different configurations, first, assuming that there is no initial correlation between coils, and second, assuming a correlation coefficient of $\rho^2 = 0.1$. From the data, and using the theoretical expression in eq. (8) we calculate the variance of noise for each pixel in the final image. In order to test the theoretical distributions, 5000 samples of 8 complex 256×256 Gaussian images with zero mean and covariance matrix $\mathbf{\Sigma}$ are generated. The $\mathbf{k}$-space of the data is subsampled by a 2x factor and reconstructed using SENSE and the synthetic sensitivity field. We estimate the variance of noise in each point using the second order moment of the Rayleigh distribution [8]:

$$\sigma^2_{\mathcal{R}}(\mathbf{x}) = \frac{1}{2}E\{M^2(\mathbf{x})\}.$$

We estimate the $E\{M^2(\mathbf{x})\}$ along the 5000 samples.

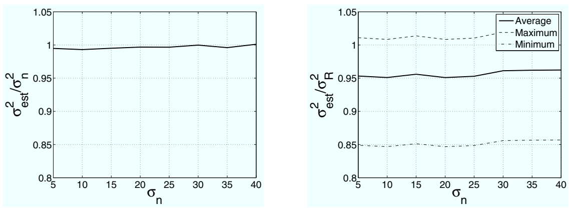
Visual results are depicted in Fig 2. For the synthetic maps, when no correlations are considered, the final variance of noise will not depend on the position $\mathbf{x}$. Therefore, in this particular case $\sigma^2_{\mathcal{R}}(\mathbf{x}) = \sigma^2_{\mathcal{R}}$. The estimated values in Fig 2-(b) show a noise pattern that slightly varies around the real value (note the small range of variation). In this very particular case, the noise can be considered to be spatially stationary, and the final image (leaving the correlation between pixels aside) is equivalent to one obtained from a single-coil scanner.

When correlations are taken into account, even using the same synthetic sensitivity map, results differ. In Fig. 2-(c), the theoretical value shows that the standard deviation of noise of the reconstructed data is not the same for every pixel, i.e., the noise is no longer spatial-stationary. The center of the image shows a larger value that decreases going north and south. So, in this more realistic case, the $\sigma^2_{\mathcal{R}}(\mathbf{x})$ will depend on $\mathbf{x}$, which can have serious implications for future processing, such as model based filtering techniques. The estimated value in Fig. 2-(d) shows exactly the same non-homogeneous pattern across the image. In the last experiment, Fig. 2-(e) and Fig. 2-(f), a real sensitivity map is used, and correlation between coils is also assumed. Again, the noise is non-stationary. To increase the dynamic range of the images, the logarithm has been used to show the data.

Secondly, we will validate the noise estimation capability of the proposed method by carrying out an experiment with a 2D synthetic slice from a BrainWeb MR volume [11], with intensity values in $[0-255]$.The average intensity value for the White Matter is 158, for the Gray Matter is 105, for the cerebrospinal fluid 36 and 0 for the background. An 8-coil system is simulated using the artificial sensitivity in Fig. 1. Image in each coil is corrupted with Gaussian noise with std σ_n ranging in $[5-40]$ and $\rho^2 = 0.1$ between all coils. The $\mathbf{k}$-space is uniformly subsampled by a factor of 2 and reconstructed using SENSE. Note that the variance of noise of the subsampled images in each coil is amplified by a factor r [5]: $(\sigma_n^2)_{\text{sub}} = r \times \sigma_n^2$.

Results for the experiment are shown in Fig 3-(a): the average of the 100 experiments divided by the actual value of σ_n^2 is depicted. Accordingly, the closer to 1, the better the estimation. From the figure it can be seen that the estimation is very accurate for all the considered values of σ_n. The estimation is similar to the one carried out for single coil data in [2]. However, the goodness of the estimation lies in the fact that the sensitivity maps are available. We repeat the estimation assuming that the maps are not available, and considering a single $\sigma^2_{\mathcal{R}}$ value for the whole image:

$$\widehat{\sigma^2_{\mathcal{R}}} = \frac{1}{2}\text{mode}\left\{\langle M_L^2(\mathbf{x})\rangle_{\mathbf{x}}\right\} \tag{14}$$



(a) With known $\mathcal{G}_W(\mathbf{x})$ (b) Blind estimation

Fig. 3. Estimation of the variance of noise from SENSE. The average of 100 experiments is considered.

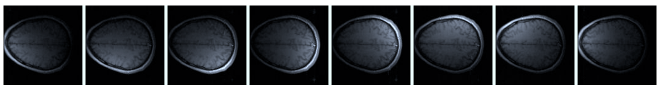


Fig. 4. Slice from a brain T1 acquisition done in a GE Signa 1.5T EXCITE with 8 coils.

We define the ratio $\widehat{\sigma^2_{\mathcal{R}}}/\sigma^2_{\mathcal{R}}(\mathbf{x})$ and we calculate the average, the minimum and maximum values across the image, and the average along 100 samples. Results are depicted in Fig 3-(b). The estimated value presents a constant bias of around 5% for all values. The estimated value will be in a range from 85% to 100% of the original value. Hence, if $\mathcal{G}_W(\mathbf{x})$ is unknown, estimating an individual value of σ_n^2 will only be acceptable for certain applications, whenever they are robust enough to cope with a bit deal of bias and a higher deal of uncertainty in this parameter.

Finally, an experiment is carried out with data from a real acquisition, see Fig. 4, with sensitivity map in Fig. 1-bottom. First, as a golden standard, parameter σ_n is estimated from the Gaussian complex data:

Real component	$\widehat{\sigma_n} = 4.1709$
Imag. component	$\widehat{\sigma_n} = 4.0845$

Then a subsampled acquisition is simulated and reconstructed with SENSE. σ_n is first estimated using eq. (12) and then, assuming the map $\mathcal{G}_W(\mathbf{x})$ is unknown, using eq. (14). Results are as follows:

Magnitude ($\mathcal{G}_W(\mathbf{x})$ known)	$\widehat{\sigma_n}/\sqrt{r} = 4.1728$
Magnitude ($\mathcal{G}_W(\mathbf{x})$ unknown)	$\widehat{\sigma_n}/\sqrt{r} = 4.8404$

Note that the value estimated using the proposed method is totally consistent with the estimation done over the original complex Gaussian data. The blind estimation method, on the other hand, overestimates the noise level, but it can still be within an acceptable error rate for some applications. However, note that this time, the method overestimates the value, unlike the previous experiment, in which it underestimates it. The using of this simplification will go along with an uncertainty on the direction of the bias.

V. Conclusions

The proper modeling of the statistics of thermal noise in MRI is crucial for many image processing and computer aided diagnosis tasks. While the stationary Rician model has been the keystone of statistical signal processing in MR for years, the stationarity assumption is no longer valid when parallel imaging and SENSE reconstruction are considered. The main assumption for single coil acquisitions is that the noise is stationary, and therefore a single value of σ_n^2 characterizes the whole data set. However, when pMRI and SENSE are considered, due to the reconstruction process, the variance of noise becomes $\mathbf{x}$-dependent, with a different value for each pixel.

To overcome the problems of non-stationarity we have proposed a novel noise estimation technique to be used with SENSE reconstructed data. The estimation of the spatially variant $\sigma^2_{\mathcal{R}}(\mathbf{x})$ is of paramount importance, since the knowledge of this parameter will allow us to re-use many of the methods in literature proposed for single-coil Rician models. In most cases it will suffice with changing an scalar σ_n^2 value by the spatially dependent $\sigma^2_{\mathcal{R}}(\mathbf{x})$.

The estimation method has shown to be accurate, robust and easy to use. However, it also shows some limitations. First, correlation between coils must be known beforehand, as well as the sensitivity map from each coil. Finally, some post processing software in the scanner may add a mask to data, which eliminates part of the background, drastically reducing the number of points available for noise estimation.

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