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A Fast Pulse Design for Parallel Excitation with Gridding Conjugate Gradient

Shuo Feng[1] and Jim Ji[2]

Abstract— **Parallel excitation (pTx) is recognized as a crucial technique in high field MRI to address the transmit field inhomogeneity problem. However, it can be time consuming to design pTx pulses which is not desirable. In this work, we propose a pulse design with gridding conjugate gradient (CG) based on the small-tip-angle approximation. The two major time consuming matrix-vector multiplications are substituted by two operators which involves with FFT and gridding only. Simulation results have shown that the proposed method is 3 times faster than conventional method and the memory cost is reduced by 1000 times.**

I. INTRODUCTION

Spatially tailored RF (TRF) pulses Magnetic Resonance Imaging (MRI) in can excite arbitrary valued spatial patterns. Parallel excitation (pTx) [1-3] techniques exploit the additional degree of freedom provided by the multiple transmit channels to shorten the RF pulse duration and reduce the specific absorption rate (SAR) [4, 5]. The combination of TRF and pTx is regarded as the promising method to address challenges in the high field MRI, such as field inhomogeneity and high SAR [6].

One widely used method under the small-tip-angle approximation [7] is the spatial domain method [3]. In this method, a specified target pattern and a k-space trajectory are specified and a set of linear system equations is built. The pulses can be designed by solving the linear system using various numerical methods such as conjugate gradient (CG). One major problem of such a pulse design is the high computation cost since each iteration will require two matrix-vector multiplications. And. Generally, it can take $2-5$ minutes [8] to design a moderate 3-D pulse, which can prevent the parallel excitation technique from being used in real-time applications. Meanwhile, the large system matrix has to be clearly specified before design which will require memory allocations on the level of several gigabyte. The entire design will require memory size several times of that.

Currently, some methods have been reported to accelerate the spatial domain pulse design method. For example, by employing the sparsity in the excitation pattern, the design equation can be transformed into the sparse domain and truncated to reduce the computation load [9, 10]. However, the method with sparse transform can only speedup the design for up to 10 times depending on the sparsity of the target pattern. Another method [11] is reported in that the design of parallel excitation pulses can be significantly accelerated by using the CUDA enabled GPU. However, the design using GPU is limited in size due to the limit available memory on GPU (no more than 2 GB for a single GPU).

In this paper, we propose a very fast pulse design method that addresses both the memory and design speed problem. The two computational expensive matrix-vector multiplications are substituted by two operators, which carry out the same physical functions as the multiplications. However, the computation cost and memory cost are significantly reduced. Simulation results of the proposed method shows that the design speed is improved for 3 times and the memory cost is reduced by 10^3 times with the same excitation error and convergence rate.

II. METHODS

We will first briefly review the conventional spatial domain method [3] for parallel excitation pulse design. Then, a new method for pulse design with gridding CG is proposed in which the computation costly matrix-vector multiplications are substituted by the gridding operators. Finally, a piecewise linear model is provided to incorporate off-resonance in the pulse design with gridding CG.

A. RF pulse design using the spatial domain method

Under small tip angle assumption (STA) [7], the excitation pattern of transverse magnetization and the complex RF pulse are Fourier pairs defined on the chosen k-space trajectory. Parallel excitation pattern of a multi-channel transmit system is the linear sum of the excitation patterns from all the channels weighted by the transmit sensitivity of each individual coil,

$$M(\vec{x}) = i\gamma M_0 \sum_l S_l(\vec{x}) \int_0^T b_l(t) e^{i\vec{x}\vec{k}(t,T)} dt \qquad (1)$$

where $M(\vec{x})$ is the specified spatial target patter, S_l is the B_1^+ map of the $l-$th channel and the excitation trajectory is defined as integral of gradient $\vec{k}(t,T) = -\gamma \int_t^T G(s)ds$. To solve the RF pulse $b_l(t)$, Eq. (2) is discretized in time and in space as,

$$M[\vec{x}_i] = i\gamma M_0 \sum_l S_l[\vec{x}_i] \left(\sum_j b_l[t_j] e^{i\vec{x}_i \vec{k}[t_j]} \right) \qquad (2)$$

In matrix form, it becomes

$$\mathbf{m} = \sum_l \mathbf{S}_l \mathbf{A} \mathbf{b}_l \qquad (3)$$

where $\mathbf{m}$ is the vector form target pattern, $\mathbf{S}_l$ is the sensitivity matrix of the l-th channel with the constant $i\gamma M_0$, $\mathbf{A}$ is

[1]S. Feng and [2]J. Ji are with the Department of Electrical & Computer Engineering, Texas A & M University, College Station, TX 77843-3128 USA (corresponding author: Jim Ji, phone: 979-458-1468; e-mail: jimji@tamu.edu)

the inverse Fourier encoding matrix defined on the k-space trajectory $\vec{k}$, and $\mathbf{b}_l$ is the sampled driving RF waveform vector of the l-th channel to be solved. Bold variables denote the matrices and vectors.

Then the pulse design problem can be formulated as a minimization problem,

$$\arg\min \|\mathbf{m} - \mathbf{S}\mathbf{A}_{full}\mathbf{b}\|_2 \tag{4}$$

where the system matrix is defined as $\mathbf{A}_{full} = \sum_l \mathbf{S}_l\mathbf{A}$ and the $\mathbf{b}$ vector is a stack of $\mathbf{b}_l$ from all the channels. Numerical methods such as CG method can be used to solve the problem. In a typical 3D design with 8-ch transmit array with a target pattern of resolusion $32 \times 32 \times 32$, the system matrix $\mathbf{A}_{full}$ would approximately have $30k \times 10k$ elements for a 8 msec pulse. And it takes about 10 min to solve Eq. (4) on an i7-core computer.

In each iteration of CG, the major computations (more than 90%) are consumed by two matrix-vector multiplications: $\mathbf{A}_{full}\times$ and $\mathbf{A}_{full}^H\times$. Each of these two requires $n_m n_{rf} n_c$ complex scalar multiplications, where n_m, n_{rf} and n_c are the number of pixels in the target pattern, the number of sampled points of the RF pulse for a single channel and the number of channels respectively.

B. RF pulse design with gridding CG

In this section, two operators G_1 and G_2 are introduced to substitute the matrix-vector multiplications without specifying the large system matrix. The operators combine the gridding of k-space data, FFT and the sensitivity modulation. So they are physically equivalent to the matrix-vector multiplications in the process of pulse design. The flow charts of these two matrix-vector multiplications with gridding operators are given in Fig. 1.

The forward operator G_1 on $b_l[t]$ will carry out the same function as the matrix-vector multiplication of the pulse of the l-th channel,

$$\mathbf{S}_l\mathbf{A}\mathbf{b}_l = G_1\{b_l[t], S_l[x], \vec{k}[t]\} \tag{5}$$

The $\mathbf{A}$ matrix is an inverse Fourier encoding matrix that maps $b_l[t]$ from on the non-Cartesian excitation trajectory $\vec{k}$ (e.g. spiral trajectory) to a spatial domain pattern on the Cartesian grid. Thus, it can be replaced by gridding, as in [12], followed by an inverse FFT. In the process of the gridding, the pulse (k-space data) $b_l[t]$ is first convolved with the Kaiser-Bessel kernel and then sampled on the Cartesian grid with doubled resolution corresponding to 2×FOX. The reason of sampling on a grid with finer resolution is to reduce the aliasing artifact caused by the convolution kernel in spatial domain. Then, an inverse FFT of the Cartesian data generates a spatial pattern of size 2×FOX. Then, the pattern is trimmed from the center to size of FOX and divided pixel-by-pixel by the inverse Fourier transform of the convolution kernel to compensate the convolution. After gridding, the pattern is modulated by the transmit sensitivity $S_l(x)$ and reshaped into vector form.

From Eq. (3) and Eq. (5) , the final pattern vector is the linear sum of the pattern vectors from all channels,

$$\mathbf{m} = \sum_l \mathbf{S}_l\mathbf{A}\mathbf{b}_l = \sum_l G_1\{b_l[t], S_l[x], \vec{k}[t]\} \tag{6}$$

Similarly, the backward operator G_2 performed on the spatial pattern M[x] will play the same role as the Hermitian transposed matrix-vector multiplication for the l-th channel,

$$(\mathbf{S}_l\mathbf{A})^H\mathbf{m} = \mathbf{A}^H\mathbf{S}_l^H = G_2\{M[x], S_l[x], \vec{k}[t]\} \tag{7}$$

In this backward operator, the spatial pattern is first modulated by the Hermitian transposed transmit sensitivity of the l-th channel as $\mathbf{S}_l^H\mathbf{m}$. Then, the Fourier encoding matrix $\mathbf{A}^H$, which maps the spatial domain Cartesian pattern to data on the non-Cartesian k-space trajectory $\vec{k}$, is substituted by gridding. In this gridding process, the sensitivity modulated pattern $\mathbf{S}_l^H\mathbf{m}$ is first divided pixel-by-pixel by the inverse Fourier transform of the convolution kernel and zero-padded to the size of 2×FOX. The k-space data on the Cartesian grid is then obtained by the FFT of the spatial pattern. Finally, the k-space data is convolved with the convolution kernel and sampled along the desired trajectory $\vec{k}$.

The result of the Hermitian transpose multiplication of the system matrix is a stack of vectors from individual channel results obtained from Eq. (7) as,

$$\begin{aligned}\mathbf{A}_{full}^H\mathbf{m} &= \left[(\mathbf{S}_1\mathbf{A})^H \cdots (\mathbf{S}_L\mathbf{A})^H\right]\mathbf{m} \\ &= \left[G_2\{M[x], S_1[x], \vec{k}[t]\} \ldots G_2\{M[x], S_L[x], \vec{k}[t]\}\right]\end{aligned} \tag{8}$$

Finally, the same CG method as in the conventional method will be used to solve the pulse design problem. In the steps of CG, the two multiplications are substituted by Eq. (6) and Eq. (8).

The approximate computation cost (number of complex scalar multiplications) of the operator G_1 and the direct matrix multiplication are compared in Table I. Parameter $\epsilon = 2$ denotes the factor of oversampling/zero-padding and $w = 6$ is the size of convolution kernel. In the general design setup, the magnitude of n_s and n_m are on the similar level in order to satisfy the Nyquist rate without pTx acceleration. So the computation cost is approximately reduced by the factor of $\frac{n_s}{w^2 + \epsilon^2 \log_2(\epsilon^2 n_m) + 2}$. Note that only the amount of multiplications is counted here.

For a pulse with $n_s = 1024$ to excite a target pattern defined on a grid with $n_m = 1024$ points, the computation cost of $\mathbf{A}_{full}\mathbf{b}$ with operator G_1 for $n_c = 8$ channels is about 12 times less than the direct multiplication. As a dual pair, the computation of $\mathbf{A}_{full}^H\mathbf{m}$ with operator G_2 shares the same computation gain versus the direction corresponding matrix multiplication.

The savings in memory cost is much more significant. In the pulse design gridding CG, only several matrices of size $n_m n_c$ need to be saved. In the direction matrix multiplication, the system matrix $\mathbf{A}_{full}$ of size $n_s n_m n_c$ need to be stored. In the above example, the memory cost is reduced by about 3 magnitudes using the gridding CG method.

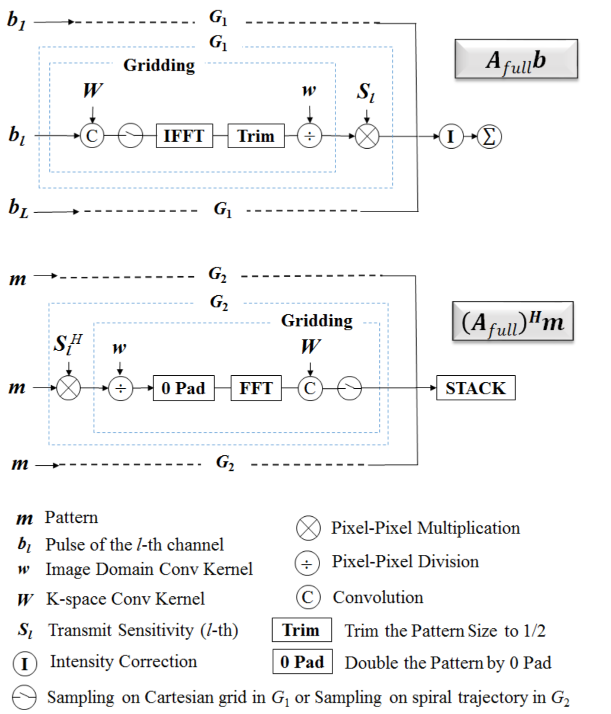

Fig. 1. Flow chart of the two matrix-vector multiplications substituted by the two operators G_1 and G_2

TABLE I

COMPUTATION COSTS OF $\mathbf{A}_{full}\mathbf{b}$ WITH OPERATOR G_1 AND THE DIRECT MATRIX MULTIPLICATION (NUMBER OF COMPLEX MULTIPLICATIONS)

$\mathbf{A}_{full}\mathbf{b}$ with operator G_1			$\mathbf{A}_{full}\mathbf{b}$
$\otimes$	FFT	$\times$, $\div$	
$w^2 n_s n_c$	$\epsilon^2 \log_2(\epsilon^2 n_m) n_m n_c$	$2 n_m n_c$	$n_s n_m n_c$

III. EXPERIMENTS

To evaluate the performance of the proposed design method, a 2-D tailored pulse will be designed to excite a 2-D pattern as in Fig. 2(a) over a $20 \times 20\text{cm}^2$ FOX. An 8-ch linear transmit array and a spiral trajectory as in Fig. 2(b) with $2\times$ pTx acceleration are used for the design using the proposed method and the conventional spatial domain method. The total pulse length is $5.3m$sec with a dwell time of $0.0026m$sec. It is assumed that there is no off-resonance effect in this experiment. And both methods are performed with exactly the same setup, including parameters, transmit sensitivities and the target pattern. The residual of each CG iteration is measured by the l_2 norm of the current residual vector which can show the convergence. And the quantitative difference between the two methods in term of designed pulse, excitation error, design time and memory cost are compared. The excitation patterns for excitation error measurement are obtained from the Bloch simulator.

The proposed pulse design method with gridding CG is used to design pTx pulses with the off-resonance information incorporated. The conventional spatial domain method is also used to design pulses for comparison. The time cost of the pulse designs are recorded and the final excitation patterns of the designed pulses are evaluated using the Bloch simulator.

All simulations are performed in Matlab 2011b (Math Works, Natick, MA) on a desktop with 2.67GHz i-7 CPU and 9 GB memory.

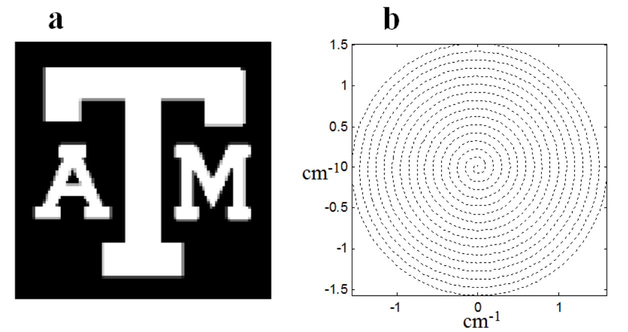

Fig. 2. (a) The target pattern for the pulse design and (b) the excitation k-space trajectory with acceration of $R = 2$

IV. RESULTS

The pulse design result of the experiment on designing 2D ptx in the absence of off-resonance is given in Fig. 3. The residual curve of the proposed method is shown in Fig.3(a). And its relative difference in residuals comparing to the conventional design is shown in Fig.3(b). As can be seen, the CG in the proposed method converges towards zero at the same rate as the conventional method and the relative difference is within 0.5%. The excitation patterns from the Bloch simulator are shown in Fig.3(c). Both the methods lead to an normalized root mean square error of 5.65% as expected, because the maximum error of a single gridding step is controlled below 0.1%. Thus, the proposed method can achieve the same accuracy as the conventional method.

The time consumed by the matrix-vector multiplication with operator G_1 and the direct multiplication in the conventional design are 2.3 sec and 18 sec respectively in 100 times of iteration. Similar gain is observed for the Hermitian transposed matrix-vector multiplication with G_2. And the total design time is reduced by about 10 fold using the proposed method.

The system matrix $\mathbf{A}_{full}$ alone requires 1012 GB memory in the conventional method. In the pulse design with gridding CG, it requires no more than 5 MB memory in total. Thus, the memory cost of the proposed method is improved by about 3 magnitudes.

V. CONCLUSION AND DISCUSSION

In this work, we proposed a very fast pulse design method based on the spatial domain method with gridding CG. The matrix-vector multiplications, which are computational expensive in the conventional method, are substituted by two operators which involves with FFT and gridding only. The design speed can be improved by about 10 times theoretically and validated by 2 times in the experiment.

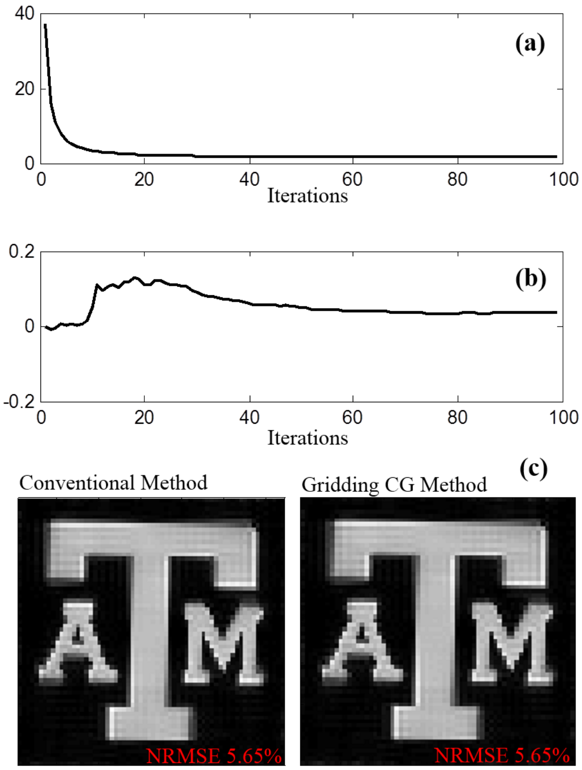

Fig. 3. Results of pulse design using gridding CG: (a) The residuals of each step in the gridding CG and (b) its relative difference from CG with the matrix-vector multiplications. (c) The excitation patterns

The memory cost can be reduced by 10^3 times by using the proposed method. This eases the memory burden of designing longer pTx pulses with more transmit channels or exciting a pattern defined on a grid with finer resolution. Meanwhile, the memory bottleneck of implementing the pulse designs on GPU is completely broken. The proposed operators are implemented on a channel by channel base and can be easily paralleled. All these promise a further speedup of 20 times of the proposed method. Future work will include develop the technique to include off-resonance term in the proposed pulse design method and implement this method on GPU to achieve highest design speed.

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