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Assessment of conceptual inconsistencies in the hybrid reservoir-wave model

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***Abstract*— The reservoir-wave paradigm separates pressure into windkessel-related 'reservoir' and wave-related 'excess' components, however the conceptual validity of this approach has not been sufficiently scrutinized. This paper assesses two logical implications of the reservoir-wave concept. First, parameters defining the reservoir (resistance and compliance) should be independent of wave effects. Second, wave analysis performed using excess pressure should provide a more accurate and physically intuitive representation of wave propagation and reflection in a vascular system, compared with the traditional wave analysis based on unseparated pressure. These issues were investigated with one-dimensional numerical models. Using a single vessel model, reservoir parameters were shown to be highly influenced by wave propagation effects. In a single bifurcation model, wave analysis based on excess pressure underestimated the reflection coefficient of the known impedance mismatch at the junction, overestimated the distance to this reflection site, and exhibited backward expansion waves suggestive of multiple negative impedance mismatches that did not exist in the system. Traditional wave analysis accurately and intuitively described waves. The identified conceptual inconsistencies in the reservoir-wave paradigm may arise from the use of hybrid (0D and 1D) dimensionality, rather than a hierarchical approach to model dimensionality.**

I. INTRODUCTION

Reduced order models form the foundation of our understanding and language of haemodynamics. In the clinic, terms such as 'pressure' and 'flow' usually refer to cross-sectionally averaged or integrated quantities (a 1D concept), while 'resistance' and 'compliance' refer to parameters that have been lumped over some vascular territory (a 0D concept). Historically, all reduced order descriptions of haemodynamics in a given vascular territory have been based on either wave (1D) or windkessel (0D) models. Recently, however, the reservoir-wave paradigm was proposed as a new reduced order framework for analysing haemodynamics [1], [2], adopting the unique approach of representing a single vascular territory (e.g. the systemic arteries) with hybrid (0D *and* 1D) dimensionality[1]. This is achieved by separating pressure (P) into two components, a 0D 'reservoir pressure' (P_{res}) and a 1D 'excess pressure' (P_{ex}) as follows:

$$P(x,t) = P_{\mathrm{res}}(t) + P_{\mathrm{ex}}(x,t) \tag{1}$$

Based on the classical two-element windkessel (2Wk, Fig. 1A), P_{res} represents the nominally space-independent pressure arising from changes in arterial reservoir volume. Excess pressure is essentially the component of pressure that is unexplained by the 2Wk. Since 2Wk neglects wave effects, P_{ex} has been interpreted to be a component of pressure caused exclusively by waves, with P_{res} therefore being a 'waveless' pressure [1], [2]. Fig. 1B shows an example of the reservoir-wave pressure decomposition.

This paper revisits and extends my recent work [3] by investigating the conceptual robustness of the reservoir-wave paradigm in two respects. First, the estimation of arterial compliance has historically been confounded by the presence of wave propagation effects [4]; a logical implication of the reservoir-wave paradigm is that reservoir compliance (C_{res}) calculated via P_{res} should not suffer from this issue. Similarly, reservoir resistance (R_{res}) should be insensitive to wave effects. Second, according to the reservoir-wave paradigm, performing wave analysis with the wave-related P_{ex} should more accurately represent the wave propagation and reflection properties of a vascular system compared with the traditional analysis based on P [1], [2].

II. METHODS

A. *Estimation of reservoir parameters*

In the reservoir-wave approach, waves are assumed to be negligible during diastole when pressure is decaying exponentially (Fig. 1B). As in [1], [3], P_{res} was therefore calculated via the following 2Wk governing equation by iteratively adjusting the reservoir parameters (R_{res}, C_{res} and P_{∞}) to minimise the difference between P_{res} and P during this nominal 'wave-free' diastolic period (i.e. after the vertical arrow in Fig. 1B).

$$P_{\mathrm{res}} = P_{\infty} + e^{-t/\tau}\left[(P_0 - P_{\infty}) + \frac{1}{C_{\mathrm{res}}}\int_0^t Q_{\mathrm{in}} e^{t'/\tau} dt'\right] \tag{2}$$

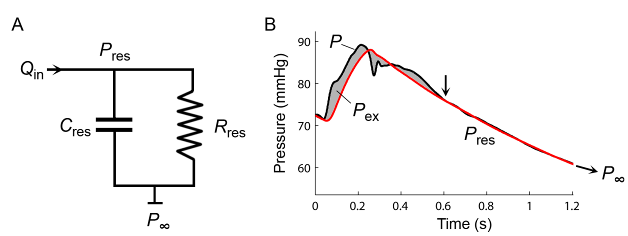

Fig. 1. (A) Two-element windkessel describing the arterial reservoir, composed of resistance (R_{res}), compliance (C_{res}) and asymptotic pressure (P_{∞}); (B) measured pressure (P) and reservoir pressure (P_{res}) from an adult sheep, the shaded area (i.e. $P - P_{\mathrm{res}}$) is excess pressure (P_{ex}). After the time indicated by the vertical arrow, pressure decreases in an approximately exponential fashion and $P_{\mathrm{res}} \approx P$.

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[1]This is not to be confused with multi-scale models that employ different dimensionality for distinct vascular territories, e.g. 1D large arteries and 0D microvasculature.

where $Q_{\rm in}$ is inflow, $\tau = R_{\rm res}C_{\rm res}$ and a zero subscript here and below refers to an end-diastolic value.

To assess the accuracy of reservoir parameters estimated with this technique, two numerical models were employed, a single 1D vessel (wave speed 6.3 m/s, cross-sectional area 2 cm^2) and an anatomically-based 1D model of the systemic arteries [3], [5]. The three-element windkessel (3Wk) was used for all outlet boundary conditions (with $P_\infty = 35$ mmHg [1]) and a forward component of left ventricular pressure was imposed via a numerical valve at the inlet [3], [5], [6]. Total (i.e. actual) compliance and resistance ($C_{\rm tot}$ and $R_{\rm tot}$) were calculated as the sum of all contributions from outlet 3Wk compartments and 1D segments [7].

The sensitivity of calculated reservoir parameters to wave propagation effects was assessed in the single segment model by varying its length between 0.1 and 50 cm while concurrently adjusting 3Wk compliance to maintain a constant value of $C_{\rm tot}$. This procedure was performed for physiologically relevant [8] values of $C_{\rm tot}$ (0.5, 1.0 and 1.5 mL/mmHg) and $R_{\rm tot}$ (0.5, 1.0 and 1.5 mmHg.s/mL) and all nine combinations thereof. For the systemic arterial model, $C_{\rm tot}$ and $R_{\rm tot}$ were manipulated by uniformly scaling 1D segment wave speeds and 3Wk resistances respectively.

B. Wave analysis

To investigate whether the modified ($P_{\rm ex}$-based) wave analysis more accurately identifies reflected waves compared with the traditional (P-based) analysis, a simple one-bifurcation model was adopted, with the same inlet/outlet boundary conditions as described in the previous section. The distance to the bifurcation junction ($L_{\rm bif}$) was varied, keeping total model length constant. Three well-defined reflection sites exist in this model. The first is the bifurcation junction which, based on the characteristic impedances of the 1D segments [3], produces a pressure reflection coefficient of 0.27 (see legend of Fig. 3 for segment parameters). In addition, the impedance mismatches at the distal 1D-3Wk interfaces constitute frequency-dependent reflection sites. Standard wave separation was used to separate P and $P_{\rm ex}$ into forward and backward components ($P_\pm$ and $P_{\rm ex\pm}$).

III. RESULTS

A. Reservoir parameters

With the single segment model, $R_{\rm res}$ and $C_{\rm res}$ matched $R_{\rm tot}$ and $C_{\rm tot}$ when segment length was negligible, confirming the validity of the equation and iterative algorithm in the absence of wave propagation effects. However, as length was increased, $R_{\rm res}$ became progressively greater than $R_{\rm tot}$ and $C_{\rm res}$ progressively less than $C_{\rm tot}$, with percentage errors of up to 45% and -21% respectively depending on $R_{\rm tot}$ and $C_{\rm tot}$ values (Fig. 2, left panels). In all cases, errors in estimated P_∞ were less than 1%, likely due to the use of long diastoles in the analysis. Similar results were observed with the systemic arterial model (Fig. 2, right panels), although unlike for the single segment model, both reservoir parameters overestimated actual system parameters in some cases and underestimated them in other cases.

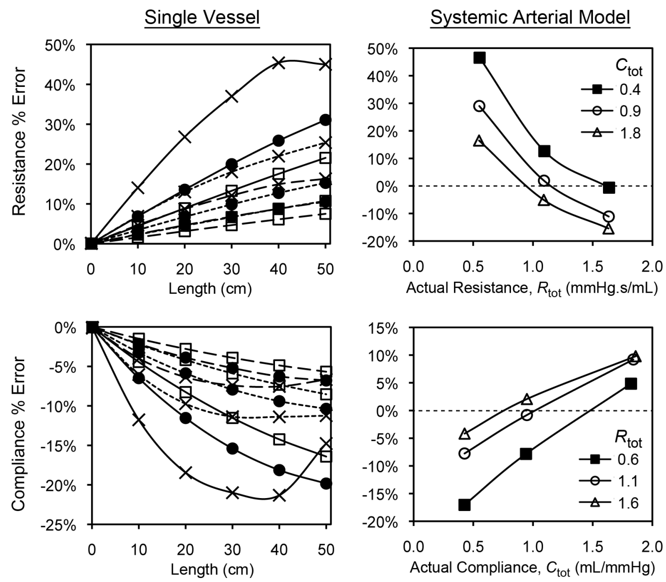

Fig. 2. Errors in reservoir resistance ($R_{\rm res}$) and compliance ($C_{\rm res}$) (compared with actual values, $R_{\rm tot}$ and $C_{\rm tot}$) calculated from a 2Wk model for a single vessel and a full systemic arterial model. In the left panels, actual compliance values tested were 0.5 (crosses), 1.0 (filled circles) and 1.5 (open squares) mL/mmHg, while actual resistance values were 0.5 (solid lines), 1.0 (short dashed lines) and 1.5 (long dashed lines) mmHg.s/mL.

B. Traditional vs modified wave analysis

Fig. 3A shows pressure and flow waveforms (which were similar to in-vivo aortic waveforms) and separated components of P and $P_{\rm ex}$ at the inlet of the single bifurcation model. Fig. 3B-D shows the effect of decreasing $L_{\rm bif}$ from 30 cm to 15 cm and 1 cm on P_- and $P_{\rm ex-}$. During systole, when reflected waves are easy to interpret since only one forward (incident) wave exists, shades of red and green in Fig. 3B-D denote periods when P_- or $P_{\rm ex-}$ are increasing or decreasing, signifying backward-running compression waves or expansion waves (BCW or BEW) respectively.

With traditional wave separation, two BCWs were apparent. As $L_{\rm bif}$ was decreased (Fig. 3B-D), the first BCW arrived earlier at the inlet, with the calculated distances to the reflection site (based on wave speed and the time delay between the feet of P_+ and P_-) no different to $L_{\rm bif}$. The later BCW arrived slightly earlier as $L_{\rm bif}$ was decreased, as was expected because wave speed in the distal segments was higher than in the proximal segment. Reflection coefficient for the first BCW, calculated via pressure changes of incident and reflected waves ($\Delta P_-/\Delta P_+$), was 0.27 regardless of $L_{\rm bif}$ and agreed precisely with the theoretically-predicted reflection coefficient. Global reflection coefficient, calculated as overall P_- amplitude divided by P_+ amplitude, decreased slightly from 0.55 to 0.47 as $L_{\rm bif}$ was decreased.

Using the reservoir-wave approach, the two BCWs were still present but their pressure effect was reduced, with calculated reflection coefficients for the first BCW of 0.16, 0.14 and 0.22 as $L_{\rm bif}$ was decreased, or 19–48% underestimation of the theoretical reflection coefficient. Calculated $L_{\rm bif}$ was overestimated by 3–4 cm when actual $L_{\rm bif}$ was 15 or 30 cm

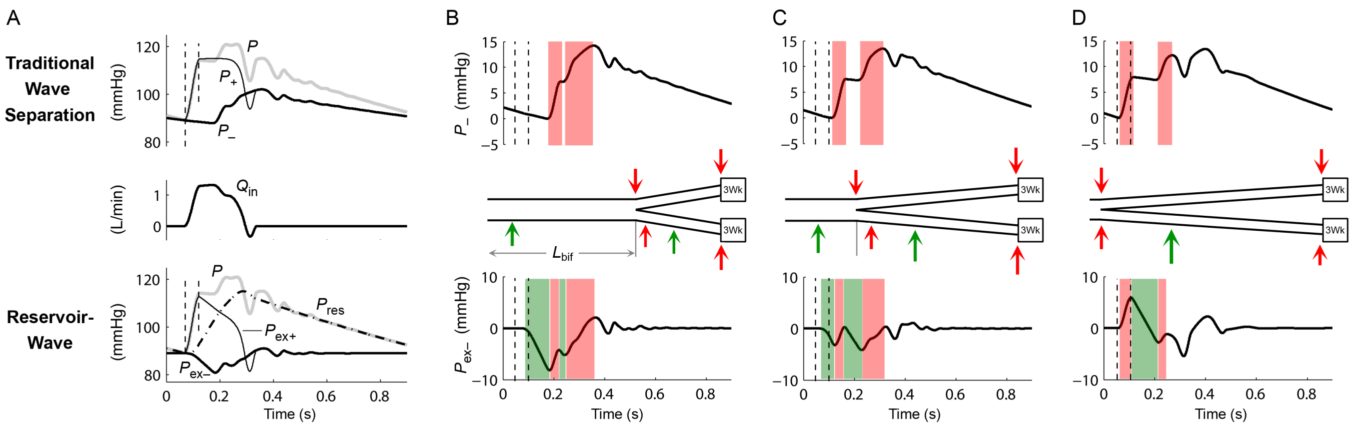

Fig. 3. (A) Wave separation using P (the traditional approach, top panel) and P_{ex} (the reservoir-wave approach, bottom panel) at the inlet of the single bifurcation model (L_{bif} = 30 cm), producing pressure components related to forward-running (P_+ or $P_{\mathrm{ex}+}$) and backward-running (P_- or $P_{\mathrm{ex}-}$) waves. Inflow (Q_{in}) is shown for reference. (B-D) P_- and $P_{\mathrm{ex}-}$ for L_{bif} = 30, 15 and 1 cm. Total length (50 cm) was constant. Arrows on the top side of the model schematics indicate the location of reflection sites predicted by the traditional approach, arrows on the bottoms side indicate locations predicted by the reservoir wave approach. Green and red arrows indicate negative and positive reflections respectively. Dashed vertical lines indicate the time span of the initial forward compression wave. Proximal/distal segment values of c and A were 5.90/7.16 m/s and 6.06/2.10 cm^2.

(Fig. 3B,C). In addition, two BEWs were also present (one when $L_{\mathrm{bif}} = 1$ cm), suggesting that negative reflection sites existed proximal and distal to the junction (green shading and arrows in Fig. 3B-D). The fall in $P_{\mathrm{ex}-}$ associated with these BEWs was highly dependent on L_{bif}. Using the maximum excursion of $P_{\mathrm{ex}-}$ and $P_{\mathrm{ex}+}$, global reflection coefficient was -0.34, -0.17 and 0.22 with decreasing L_{bif}.

IV. DISCUSSION

This study has demonstrated two significant conceptual inconsistencies in the reservoir-wave paradigm related to the reservoir parameters and the modified wave analysis.

A. Reservoir parameters

Both single segment and full systemic arterial models suggested that reservoir parameters (C_{res} and R_{res}), even when estimated using noise-free data with extended diastoles, may not correspond with the physiological quantities of interest (C_{tot} and R_{tot}). Calculated C_{res} and R_{res} were dependent on wave propagation effects, even though a logical implication of the reservoir-wave concept is that these parameters should be insensitive to wave effects. The disagreement between physiological and estimated values of compliance was affected by R_{tot}, and similarly the disagreement between resistance values was affected by C_{tot}, an interdependence that suggests flaws in the estimation technique (in the presence of wave propagation effects).

It has been shown previously that estimating arterial compliance on the basis of windkessel models is confounded by wave effects. For example, in [9], when arterial compliance was estimated from a range of methods based on 2Wk or 3Wk, errors were greater at higher heart rates when the 'windkesselness' of the system was lower [10]. The estimation method proposed by Wang et al [1] differs from those employed in [9] in that R_{res} is a free parameter in the iterative fitting procedure and the 2Wk incorporated a non-zero P_∞; however, these differences do not appear to ameliorate the confounding effect of wave propagation on compliance estimation that have been addressed in detail in [4], [11]. Note also that, when applied to physiological data, this conceptual problem is likely to be compounded by challenges in reliably fitting an exponential curve to diastolic pressure (especially in young individuals) and/or maintaining P_∞ as a free parameter in the absence of long diastoles [12].

B. Modified wave analysis

A key feature of the reservoir-wave paradigm is that wave phenomena (i.e. transients that propagate in space) are solely attributed to P_{ex}, as indicated by the space-independence of P_{res} in Eq. (1). Logically, therefore, wave analyses such as wave separation and wave intensity analysis should be performed using P_{ex}. However the claim that this approach is more correct than the traditional approach employing P [2], [13] has not been validated. In this study, a simple test of this issue was performed using a single bifurcation model. Results were consistent with and extended findings of [3]. The main findings related to the modified (P_{ex}-based) wave analysis were as follows (focusing on the bifurcation junction as the reflection site of interest).

1) The calculated local reflection coefficient was less than the value calculated from the known impedance mismatch.
2) Changing L_{bif} altered the calculated reflection coefficient, despite the impedance mismatch being unchanged.
3) Distance to the reflection site was overestimated.
4) Two BEWs suggested the existence of negative reflection sites, but none existed in the system.
5) Although changing L_{bif} should have had a minor influence on global reflection coefficient (a measure of overall wave reflection caused by junction and terminal 3Wk impedance mismatches), a substantial (-0.34 to 0.22) and qualitative (negative to positive) effect on global reflection coefficient was observed.

Note that in [3], several other problems were demonstrated using slightly more complex models:

6) Despite a prescribed absence of forward waves during mid-systole, modified wave analysis exhibited a forward expansion wave during mid-systole (this also occurred with the single bifurcation model, see Fig. 3A).
7) Changing the reflection coefficient of a distal impedance mismatch caused changes to proximal wave profiles at earlier times than would be physically possible given the finite wave speed of the vessels.

Importantly, none of these anomalies occurred with traditional (P-based) wave analysis.

C. Hybrid versus hierarchical paradigms

Windkessel and wave models are often presented in the literature as having complementing strengths and weaknesses [10], [12]. Windkessel models provide an intuitive explanation for the diastolic pressure decay when inflow is zero, but are unsatisfactory during systole when wave models appear more appropriate. Conversely, doubt has been expressed about the ability of wave theory to explain the diastolic pressure decay [1], [2]. Although the intention of the reservoir-wave paradigm was to combine the strengths of these two models into one unified model, the current study and [3] cast doubt on the validity of this approach.

The most novel but also most concerning feature of the reservoir-wave paradigm is that it treats a single vascular domain with hybrid dimensionality, with nominal 1D waves considered independent of nominal 0D reservoir function (Fig. 4, left panel). However, mathematically and conceptually, it would appear more valid to consider the 0D windkessel model as a *reduction* or *simplification* of the 1D wave model. Indeed, the windkessel equations may be easily derived by spatially integrating the linearised 1D governing equations [14], which in turn may be derived via the non-linear 1D equations from the 3D Navier-Stokes equations [14], [15]. In this hierarchical paradigm (Fig. 4, right panel), all phenomena described by a lower dimensional model (e.g. 0D windkessel) must also lie within the explanatory power of a higher dimensional model (e.g. 1D wave model) [3]. Thus one of the main 'problems' that motivated the reservoir-wave paradigm, that the diastolic pressure decay cannot be explained by a wave model [1], [2], would seem unfounded.

Given the ambiguous physical meaning of P_{res} and P_{ex} as conceived in the reservoir-wave paradigm, in that reservoir parameters are not wave-independent, P_{res} exhibits wave propagation behaviour [3] and P_{ex} is defective in describing waves, a return to Lighthill's original concept of P_{res} and P_{ex} may be warranted, namely that of a constant reservoir pressure (e.g. $P_{\mathrm{res}} \equiv P_0$) upon which any perturbations (P_{ex}) are considered due to the passage of waves [16]. In this paradigm, windkessel phenomena (such as the diastolic pressure decay) may be understood to arise as a result of 'long waves' in which spatial pressure differences and the frequency of pressure fluctuations are small.

V. CONCLUSIONS

Close scrutiny of the reservoir-wave paradigm has revealed a number of conceptual inconsistencies that may lead to inaccurate estimates of global vascular properties (resistance and compliance) and incorrect conclusions about wave propagation and reflection. Measured pressure (not excess pressure) should be used when performing wave analysis.

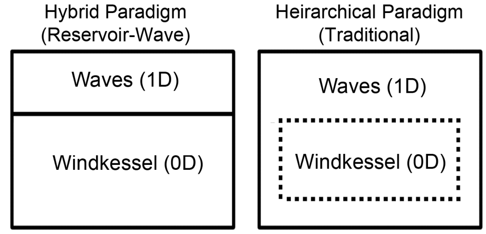

Fig. 4. Dimensionally hybrid vs hierarchical paradigms.

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