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Mixture Noise Removal In Ultrasound Images Using SCoBeP and Low-rank Matrix Completion

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***Abstract*— Denoising as one of the most significant tools in ultrasound imaging was studied widely in the literature. However, most existing ultrasound image denoising algorithms have assumed the additive white Gaussian noise. In this work, we propose two efficient ultrasound image denoising methods that can handle a noise mixture of various types. Our methods are based on SCoBeP [1] and low-rank matrix completion as follows. In our first method, a noisy image is processed in blockwise manner and for *each* processed block we find candidate match pixels on other images using sparse coding and belief propagation, where in our second algorithm, we use *overlapped* patches to further lower the computation complexity. The blocks centered around these candidate pixels then will stack together and unreliable pixels will be removed using fast matrix completion method [2]. We demonstrate the effectiveness of our algorithm in removing the mixed noise through the results. We also compare with other denoising technique using matrix completion. Our methods results in comparable performance with significantly lower computation complexity.**

I. INTRODUCTION

Medical Imaging has been a useful tool for possible non-invasive examination. However, images from various modalities need to be denoised as a pre-processing step for many planning, navigation, detection, data-fusion and visualization tasks in medical applications [3]–[6].

Among all medical image modalities, ultrasound imaging technique uses sound waves reflected from different organs of the body to give local details and important diagnostic information on the human body. This relatively safe technique is inexpensive for many applications. However, ultrasound images usually suffers from various noise types such as impulsive, Poisson, and Gaussian noises [7].

Many image denoising methods have been proposed in the last few decades, e.g. [7]–[10]. One of the first methods to address the denoising problem was the bilateral filter proposed by Tomasi and Manduchi [9]. However, this method does not perform well under strong noise intensity. Pizurica, *et al.* [7] proposed a wavelet domain method for noise filtering in ultrasound images. They exploit the general knowledge regarding the correlation of significant image features across different resolution scales to perform a preliminary coefficient classification. In [11], an approach was proposed to denoise the Doppler ultrasound signal. Using this method, wavelet coefficients of the Doppler signal at multiple scales were first obtained using the discrete wavelet frame analysis. Then, a soft thresholding-based denoising algorithm was employed to deal with these coefficients to get the denoised signal.

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Recently, sparse representation has been applied on ultrasound image denoising as a powerful tool [12], [13]. For example in [12], a multiplicative speckle noise is converted to an additive one by the logarithm operation and an existing procedure is applied to convert the log-transformed into a WGN process. The K-SVD algorithm is applied with the dictionary derived directly from the noisy image.

However, prior works have been limited to the one specific type of noise, where existence of other types of noise will degrade the performance of the denoising methods. In contrast, our methods do not suffer from this limitation and can even remove a mixture of strong noises from ultrasound images.

In this work, we show that the proposed methods can effectively handle noisy images that suffer from noise mixtures. In our proposed methods, we apply a suboptimal block matching algorithm described in [1], we then incorporate a decomposition approach for matrix completion [2] into the denoising algorithm.

The key intuition of the approach is to keep only the reliable pixels and get rid of all unreliable pixels that are likely to be overwhelmed by strong noise. In our first method, a noisy image is processed in blockwise manner and for each processed block we find candidate match pixels on other images using sparse coding and belief propagation, where in our second algorithm, we use overlapped patches to further lower the computational. The blocks centered around these candidate pixels will be vectorized and then stacked into a matrix. The reliable pixels in the matrix are identified based on their deviation from the mean of all elements in the same row. We will then applying matrix completion on the incomplete matrix and a nearly noise free block will be output. Then, a denoised patch will be constructed as the average value of each row in the completed matrix. Repeating the same procedure for all blocks of reference image will build a denoised image.

The rest of this paper is structured as follows. In Section II, we include the implementation detail of our proposed methods. We show and discuss our simulation results in Section III, followed by a brief conclusion in Section IV.

II. PROPOSED METHOD

Consider an observed noisy ultrasound images $y(x) = z(x) + n(x)$, where $z(x)$ is the original ultrasound images and $n(x)$ is Gaussian/Poisson/Impulsive noise sample. $x = (i, j, k) \in X$ are coordinates in the spatio-temporal $3D$

domain $X \subset \mathbb{Z}^3$, where the first two components (i,j) are the spatial coordinates and the third one k is the time (ultrasound image) index.

In our first algorithm, we register the patches from the ultrasound images to the each extracted patch of the reference image, where in our second algorithm, we only extract overlapped patches in the reference image. The patches have the small overlap with each other, hence, the second algorithm has less number of extracted patches in compare to the first algorithm to further lower the computational. Moreover, in the first algorithm, we only utilize the center of denoised patch, where the whole denoised patch from the second algorithm will be used. These significantly improve the block matching performance that directly links to the overall denoising performance.

To match the extracted features of the reference ultrasound image to the corresponding extracted features of the other ultrasound image, we create a dictionary which contains all feature vectors of the reference ultrasound image and apply sparse coding to the extracted feature of the other ultrasound image. Sparse coding will reconstruct a patch at pixel x as a linear combination of the reference patches. Note that the obtained sparse coefficient vector should be sparse, i.e., it should be 0 for most coefficients. To select the n candidate blocks, we simply pick those corresponding to n largest coefficients in the sparse coefficient vector. We denote a set as an $n \times 2$ matrix storing the locations of these candidate pixels and a probability vector as the length-n vector storing the corresponding values of the sparse coefficient vector. Each coefficient in the probability vector serves as a prior probability of matching the patch at x to a reference patch of the reference ultrasound image taking only local characteristics into accounts but ignoring geometric characteristics of the matches. Finally, to incorporate geometric characteristics, we model the problem by a factor graph and apply belief propagation to update probabilities (for more details, see [1]).

The key steps of the algorithms are summarized in Algorithm 1 and Algorithm 2, and the detail implementation of each step is presented in the following.

Implementation Details:

- $\mathbf{Y} = MakeOverlap(y)$ forms a 3D matrix by stacking the vectorized blocks with v pixel overlap in each direction from each image.
- $\mathbf{Y} = ExtractDenseFeature(y)$ extracts all possible patches of y, where the result is a 3D matrix containing the vectorized 2D blocks. To achieve this purpose, we consider a 2D block of size $S = (2a+1) \times (2b+1)$ containing neighboring pixels around each pixel on a data, where a and b are positive integers. For each pixel p_x in the ultrasound image y, we vectorized a patch centered around the pixel p_x to a feature vector $Y_x \in \mathbb{R}^{S\times 1}$. A 3D ultrasound feature data $\mathbf{Y} \in \mathbb{R}^{M\times N\times S}$ is then constructed from Y_x as follows:

Algorithm 1 Medical Image Denoising using Dense SCoBeP and matrix completion

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Inputs : noisy ultrasound images y
Initialize :

- Set the first image of y as a reference image X

Use SCoBeP to find candidate pixels :

- $\mathbf{Y} = ExtractDenseFeature(y)$
- $\mathbf{X} = ExtractDenseFeature(X)$
- $\mathcal{D} = MakeDic\big(\mathbf{Y}\big)$
- $[S,\rho] = FindTopSCV\big(\mathcal{D}, \mathbf{X}\big)$
- $\hat{\rho} = BP\big(S,\rho\big)$

Find the denoised patches: For each coordinate $x \in X$ do:

(a) $\mathbf{M} = ReassembleSCoBeP\Big(\mathbf{Y}, \hat{\rho}_x, S_x\Big)$
(b) $\hat{\mathcal{Z}_{S_x}} = ReliableElements\Big(\mathbf{M}\Big)$
(c) $\check{\mathcal{Z}}_x = DMC\Big(\hat{\mathcal{Z}}_{S_x}\Big)$
(d) $\hat{z}_x = AVG_{row}(\check{\mathcal{Z}}_x)$
(e) $\mathcal{Z} = \mathcal{Z} + \hat{z}_x$

Output : a denoised ultrasound image $\mathcal{Z}$

$$\mathbf{Y} = \begin{bmatrix} Y_{1,1} & Y_{1,2} & \cdots & Y_{1,N} \\ Y_{2,1} & Y_{2,2} & \cdots & Y_{2,N} \\ \vdots & \vdots & \ddots & \vdots \\ Y_{M,1} & Y_{M,2} & \cdots & Y_{M,N} \end{bmatrix}. \tag{II.1}$$

Note that $\mathbf{X}$ is created in the same manner as $\mathbf{Y}$ but from the reference image X instead.

- $\mathcal{D} = MakeDic\big(\mathbf{Y}\big)$ creates a dictionary $\mathcal{D}$ using the vectors of $\mathbf{Y}$. Later, the dictionary $\mathcal{D}$ is used to match the extracted features of the ultrasound images to the corresponding extracted features of the reference image. We can write $\mathcal{D}$ as

$$\mathcal{D} = [Y_{1,1} \ ... \ Y_{1,2} ... \ Y_{1,N} ... \ Y_{M,N}]\,. \tag{II.2}$$

Note that we normalize dictionary $\mathcal{D}$ to guarantee the norm of each feature vector to be 1.

- $[S,\rho] = FindTopSCV\big(\mathcal{D}, \mathbf{X}\big)$ finds the top candidate match pixels using the sparse coding algorithm, where S_x is an $n \times 2$ matrix stores the locations of these candidate pixels and ρ_x as the length-n vector stores the corresponding values of S_x. Each coefficient in ρ_x serves as a prior probability of matching the reference patch at x to a patch centered around the pixel y_x. Mathematically, we try to solve the following sparse coding problem to find the most sparse coefficient vector $\hat{\alpha}_x$ such that

$$\mathbf{X} = \mathcal{D}\hat{\alpha}_x. \tag{II.3}$$

Although there are several methods to solve (II.3) [14]–[16], in our work, we employ Subspace Pursuit (SP) [15] because of its computational efficiency.

- $\hat{\rho} = BP\big(S,\rho\big)$ models the problem by a factor graph and applies belief propagation [17] to update probability

Algorithm 2 Medical Image Denoising using Overlapped SCoBeP matrix completion

Inputs : noisy ultrasound images y, pixel overlap v
Initialize :

- Set the first image of y as a reference image X
- Set $\mathcal{V}$ and $\mathcal{W}$ to be zero images with the same size as that of an input ultrasound image

Use SCoBeP to find candidate pixels :

- $\mathbf{Y} = MakeOverlap(y)$
- $\mathbf{X} = ExtractDenseFeature(X)$
- $\mathcal{D} = MakeDic\Big(\mathbf{Y}\Big)$
- $[S, \rho] = FindTopSCV\Big(\mathcal{D}, \mathbf{X}\Big)$
- $\hat{\rho} = BP\Big(S, \rho\Big)$

Find the denoised patches: For each coordinate $x \in \Omega$ with v pixel overlap in each direction do:

(a) $\mathbf{M} = ReassembleSCoBeP\Big(\mathbf{Y}, \hat{\rho}_x, S_x\Big)$
(b) $\hat{\mathcal{Z}_{S_x}} = ReliableElements\Big(\mathbf{M}\Big)$
(c) $\tilde{\mathcal{Z}}_x = DMC\Big(\hat{\mathcal{Z}}_{S_x}\Big)$
(d) $\hat{z}_x = AVG_{row}(\check{\mathcal{Z}}_x)$
(e) $\mathcal{V} = \mathcal{V} + \hat{z}_x$
(f) $\mathcal{W} = \mathcal{W} + \hat{w}_x$

Normalize : $\hat{\mathcal{Z}} = \mathcal{V}/\mathcal{W}$
Output : a denoised ultrasound image $\hat{\mathcal{Z}}$

ρ, where $\hat{\rho}$ is the updated probability. In our case, we assign a variable node for each pixel on the reference image and connect each pair of neighboring pixels with a factor node. Also, we introduce one extra factor node to take care of the prior knowledge obtained in the sparse coding step for each pixel of the reference image (for more details, see [1]).

- $\Omega \subset X$ includes the coordinates of the reference blocks. In general, each pixel in the reference image can be covered by several patches, we aggregate overlapped patches by a weighted average at each pixel.
- y_x denotes a block of size $q \times q$ in y, where its center is at x.
- $\mathcal{Z}_{S_x}$ denotes a matrix formed by stacking the vectorized blocks $Z_{x \in S_x}$ together, where Z_x is a block of size $q \times q$ centered at x in y.
- $\mathbf{M} = ReassembleSCoBeP\Big(\mathbf{Y}, \hat{\rho}_x, S_x\Big)$ returns the matrix $\mathbf{M}$ using the probability $\hat{\rho}_x$ and the candidate locations S_x, where the result containing the linear combination of the matched blocks in $\mathbf{Y}$, so $\mathbf{M} = \sum \mathbf{Y}_{\hat{\rho}_x S_x}$
- $ReliableElements\Big(\mathbf{M}\Big)$ discards those matrix elements of $\mathbf{M}$, which are far away from the mean of its corresponding row and indicates them as unreliable elements, and then replaces them by zero. Note that those unreliable elements could be the pixels corrupted by Gaussian/Poisson/Impulsive noise or from mismatched patches obtained from previous step (i.e., block matching). Keeping the reliable elements lets us to recover the full matrix needed for next step.
- $DMC\Big(\hat{\mathcal{Z}}_{S_x}\Big)$ denotes the low-rank matrix completion step aking $\hat{\mathcal{Z}}_{S_x}$ as input and output $\check{\mathcal{Z}}_x$ as a completed matrix with removed noise elements. Recently, many matrix completion methods have been studied [2], [18]–[20], in our work, we use decomposing approach for low-rank matrix completion algorithm [2], because of its computational efficiency.
- $AVG_{row}(\check{Z}_x)$ finds the average value of each row in matrix $\check{\mathcal{Z}}_x$ and convert the obtained vector to a block. Also, $\hat{z}_x$ will be an estimated block of size $q \times q$ centered at x in $\hat{\mathcal{Z}}$.

III. EXPERIMENTAL RESULTS

To evaluate the performance of the proposed methods, we now present some experimental results obtained by applying the proposed methods on some ultrasound images. In this section, we will illustrate some examples that will assess our denoising methods for real ultrasound images. Comparison will be made against the state-of-the-art technique: the wavelet domain image denoising algorithm [7].

We also replaced our proposed decomposition matrix completion with OptSpace [20] to compare the result and time consumption (Table I). It can be seen that our methods perform notably faster than OptSpace [20].

All tests in this section were processed in the following manner: All 30 images were involved in the denoised image. The similar block size used for block matching was 63×63 and was not changed for various tests. We obtained a locally consistent solution by allowing patches to overlap in Algorithm 2, where the overlapped regions (v) were 5 pixels in each direction. Further, for each reference patch, we extract 3 most similar patches used in each image using SCoBeP.

In this work, we apply our proposed methods without any changes or generating noisy images. Note that since there are no published methods that perform denoising on such general images, we choose the wavelet domain image denoising algorithm [7] for comparison, because their source code is available. As for the non-synthetic case, while we do not have the ground truth and thus cannot evaluate the the methods quantitatively using PSNR, the visual comparison illustrates the robustness of our proposed methods when it is applied directly to real images.

As shown in the Figures III.1 and III.2, wavelet domain image denoising algorithm [7] generates severe artifacts at edge areas, while our proposed denoising methods perform remarkably well for the detail structures and are free of these artifacts.

TABLE I: Time comparison for using various matrix completion

	Algorithm 1	**Algorithm 2**	**Denoising method using OptSpace [20]**
Time $(seconds)$	410	120	1398

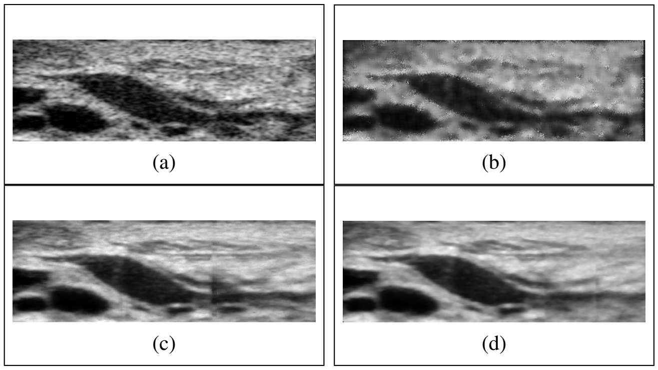

Fig. III.1: Non-synthetic (real) experiment. (a) real ultrasound image; (b) wavelet domain image denoising algorithm [7]; (c) the proposed denoising Algorithm 1; (d) the proposed denoising Algorithm 2.

IV. Conclusion

In conclusion, we have proposed in this paper two efficient patch based ultrasound image denoising methods using SCoBeP [1] and decomposition approach for matrix completion [2], where we keep only reliable pixels and get rid of all unreliable pixels. Our methods can handle a mixture of noises while most of the existing methods have been limited to the one specific type of noise. Quantitative and qualitative experiments have shown that the proposed algorithm outperforms the state-of-the-art methods in handling ultrasound image denoising.

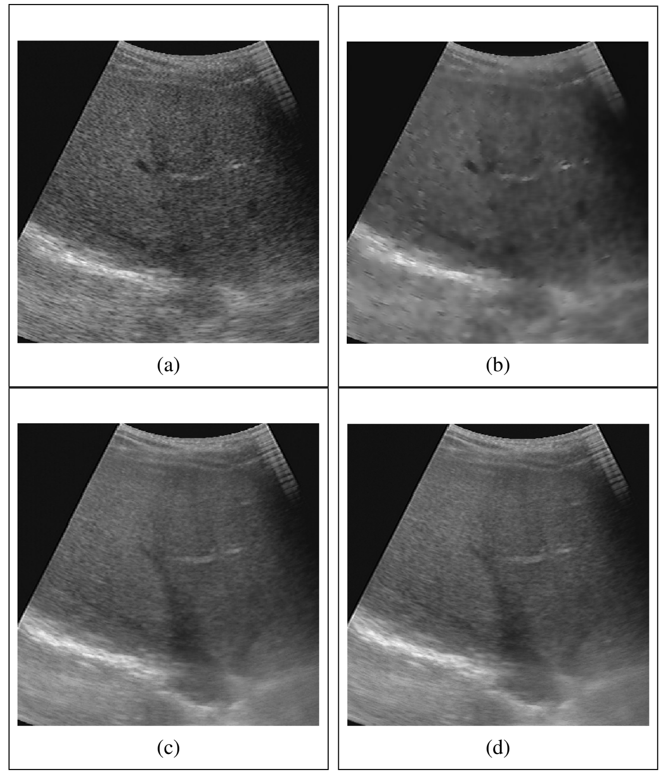

Fig. III.2: Non-synthetic (real) experiment. (a) real ultrasound image; (b) wavelet domain image denoising algorithm [7]; (c) the proposed denoising Algorithm 1; (d) the proposed denoising Algorithm 2.

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