An Indirect System Identification Technique for Stable Estimation of Continuous-Time Parameters of the Vestibulo-Ocular Reflex (VOR)

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Abstract— The vestibulo-ocular reflex (VOR) is a well-known dual mode bifurcating system that consists of slow and fast modes associated with nystagmus and saccade, respectively. Estimation of continuous-time parameters of nystagmus and saccade models are known to be sensitive to estimation methodology, noise and sampling rate. The stable and accurate estimation of these parameters are critical for accurate disease modelling, clinical diagnosis, robotic control strategies, mission planning for space exploration and pilot safety, etc.

This paper presents a novel indirect system identification method for the estimation of continuous-time parameters of VOR employing standardised least-squares with dual sampling rates in a sparse structure. This approach permits the stable and simultaneous estimation of both nystagmus and saccade data. The efficacy of this approach is demonstrated via simulation of a continuous-time model of VOR with typical parameters found in clinical studies and in the presence of output additive noise.

I. INTRODUCTION

The eyes play a critical role in maintaining balance. They are directly connected to organs of equilibrium, most importantly the inner ear. Paired structures called semicircular canals behind the ears sense motion and relay information to balance control centres in the brain. Health of the semicircular canals can be monitored by studying the vestibulo-ocular reflex (VOR). The VOR is a reflexive eye movement that stabilises images on the retina during head movement. Ocular responses to head perturbations consist of intermingled segments classified as "slow" (nystagmus) or "fast" (saccade), according to their average speed characteristics. Changes in VOR activity can be an indication of serious brain or head trauma, which can negatively impact an astronaut or pilot's ability to successfully achieve critical mission objectives.

NASA's Advanced Exploration Systems and Human Research Programs are critically interested in pioneering new capabilities allowing future human missions beyond Earth orbit [1], [2]. The objective quantification of mechanisms that control VOR will enhance the capabilities of NASA's human exploration missions by developing techniques that permit prediction of spatial disorientation and development of individualised countermeasures for astronaut crews that must perform critical space operations under varied gravito-inertial conditions such as launch, landing and orbital maneuvering [3].

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It has been shown that the VOR can be accurately modelled by the NARMAX (Non-linear AutoRegressive, Moving Average eXogenous) structure [4]. Moreover, a modified extended least-squares (MELS) algorithm was developed to estimate unbiased parameter values of non-linear Hammerstein structure multimode systems. This two-step process adds overhead to data analysis, which can be significant for sufficiently long data records and batch processing. However, it can be reduced to one step by exploiting the nature and efficiency of sparse matrices.

Recently, it was demonstrated that the utilisation of a sparse matrix approach is ideally suited for system identification of switched systems with application to the VOR [5]. This study demonstrated that a sparse matrix approach is an efficient technique for the simultaneous analysis of nystagmus and saccade dynamics. However, several questions remain unsolved in this modelling and identification approach.

The problem of continuous-time system identification from sampled input-output data can be divided into two broad approaches: (i) indirect methods, where a discrete-time model is estimated from sampled data; then an equivalent continuous-time model is calculated and (ii) direct methods, where a continuous-time model is obtained directly without going through the intermediate step of first determining a discrete-time model; based on concepts of approximate numerical integration to recreate time-derivatives needed in continuous-time formulations [6].

We propose to investigate the problem of continuous-time system identification by developing methodology using an indirect approach. One such approach relies on matrix preconditioning techniques, such as standardised least-squares, to improve the spectral properties of the regressor matrix [7], [8]. Often when the clustered spectrum is away from zero it results in rapid and robust convergence, especially when the preconditioned matrix is close to normal. We deem this will provide more robust solutions and greater consistency for non-linear biological systems by circumventing issues of implementing numerical derivatives and filter selection required by direct techniques, which are more challenging in a non-linear framework. Here, we focus on one critical issue, namely, that of mapping the underlying continuous-time system to discrete-time for estimation, then inverse mapping back to continuous-time to provide physiological relevance and insight.

The identification of continuous-time VOR parameters is further challenged due to the slow and fast speed characteristics of the system. This corresponds to one pole close to

the $j\omega$-axis in continuous-time or the unit circle in discrete-time whilst the other pole is located significantly inside the left hand plane/unit circle. To avoid aliasing of the fast mode, the system is typically oversampled 5 to 10 times the highest known dynamics. This results in the slow mode moving close to the unit circle as the sampling interval approaches zero, leading to numerical instability for parameter estimation. To avoid the numerical instability problem we propose using a dual sampling rate technique and, thus, permitting stable and robust continuous-time parameter estimation.

The organisation of this paper is as follows. In §II we formulate the identification problem addressed here. Section III introduces a dual sampling rate, standardised least-squares method in a sparse structure, which permits stable indirect estimation of continuous-time VOR parameters. This sparse matrix formulation is capable of simultaneously quantifying both nystagmus and saccade data. Section IV provides results of the proposed algorithm on a simulated VOR model whilst §V provides a discussion of our findings. Section VI summarises the conclusions of our study.

II. PROBLEM STATEMENT

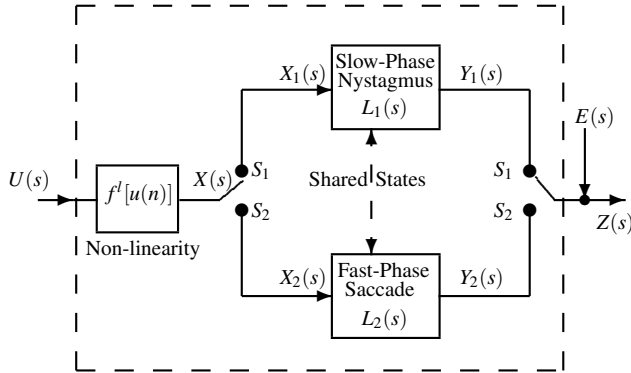Consider the continuous-time VOR system illustrated in Fig. 1 and characterised by Eqn. 1 as

Fig. 1. System describing vestibular dynamics. Switch position S_1: nystagmus. Switch position S_2: saccade

$$
\begin{aligned}
f^l(\cdot) &= a + bu(n) + cu^2(n) + du^3(n), &(1)\\
Y_1(s) &= \frac{K_1}{\tau_1 s + 1}X_1(s) + \frac{\tau_1 y_1(t_i)}{\tau_1 s + 1}\\
&= \frac{G_1}{s + p_1}X_1(s) + \frac{y_1(t_i)}{s + p_1}; \ i = 1, \cdots, q,\\
Y_2(s) &= \frac{K_2}{\tau_2 s + 1}X_2(s) + \frac{\tau_2 y_2(t_\ell)}{\tau_2 s + 1}\\
&= \frac{G_2}{s + p_2}X_2(s) + \frac{y_2(t_\ell)}{s + p_2}; \ \ell = 1, \cdots, r
\end{aligned}
$$

where $Y_1(s)$ and $Y_2(s)$ are first-order dynamics for slow and fast-phase modes, $y_1(t_i)$ and $y_2(t_\ell)$ represent the initial values at each switch time and q, r are the number of switches [4], [5]. It has been demonstrated that a NARMAX description

of VOR slow and fast-phases is [4]

$$
\begin{aligned}
y(n) &= \begin{cases} y_1(n) & \text{Dynamic Mode } S_1 \\ y_2(n) & \text{Dynamic Mode } S_2 \end{cases} &(2)\\
y_1(n) &= \beta_1 + \beta_2 y_1(n-1) + \beta_3[u(n) + u(n-1)]\\
&+ \beta_4[u^2(n) + u^2(n-1)] + \beta_5[u^3(n) + u^3(n-1)]\\
&+ \kappa_1 \delta_{11}(n-j) + \cdots + \kappa_i \delta_{1i}(n - j_i); \ i = 1, \cdots, q\\
y_2(n) &= \vartheta_1 + \vartheta_2 y_2(n-1) + \vartheta_3[u(n) + u(n-1)]\\
&+ \vartheta_4[u^2(n) + u^2(n-1)] + \vartheta_5[u^3(n) + u^3(n-1)]\\
&+ \lambda_1 \delta_{11}(n-k) + \cdots + \lambda_\ell \delta_{1\ell}(n - k_\ell); \ \ell = 1, \cdots, r
\end{aligned}
$$

where δ is the Kronecker impulse function, j, k are the lags on the δ_{1i}th and $\delta_{2\ell}$th impulse where the lag times are assumed known and q, r are the number of data segments of sub-system one and two, respectively. The unknown model parameters are compactly represented as $\theta_{1,2} = [\beta_1, \vartheta_1 \ \beta_2, \vartheta_2 \ \beta_3, \vartheta_3 \ \beta_4, \vartheta_4 \ \beta_5, \vartheta_5 \ \kappa_1, \lambda_1 \cdots \kappa_i, \lambda_\ell]^T$.

Table I illustrates the relationship of the discrete-time (DT) parameters in Eqn. 2 to the underlying continuous-time (CT) parameters in Eqn. 1 and their inverse. Notice

TABLE I

(LEFT) FORWARD AND (RIGHT) INVERSE RELATIONSHIP OF NARMAX MODEL PARAMETERS TO UNDERLYING CONTINUOUS-TIME PARAMETERS.

DT Coefficient	CT Relationship	CT Coefficient	DT Relationship
β_1, ϑ_1	$\frac{(2G_{1,2}aT)}{2 + p_{1,2}T}$	$\tau_{1,2}$	$\frac{-(\beta_2, \vartheta_2)T - T}{-2 + 2(\beta_2, \vartheta_2)}$
β_2, ϑ_2	$\frac{-(-2 + p_{1,2}T)}{2 + p_{1,2}T}$	$G_{1,2}a$	$\frac{2(\beta_1, \vartheta_1)}{(\beta_2, \vartheta_2)T + T}$
β_3, ϑ_3	$\frac{(G_{1,2}bT)}{2 + p_{1,2}T}$	$G_{1,2u}b$	$\frac{4(\beta_3, \vartheta_3)}{(\beta_2, \vartheta_2)T + T}$
β_4, ϑ_4	$\frac{(G_{1,2}cT)}{2 + p_{1,2}T}$	$G_{1,2}c$	$\frac{4(\beta_4, \vartheta_4)}{(\beta_2, \vartheta_2)T + T}$
β_5, ϑ_5	$\frac{(G_{1,2}dT)}{2 + p_{1,2}T}$	$G_{1,2}d$	$\frac{4(\beta_5, \vartheta_5)}{(\beta_2, \vartheta_2)T + T}$
κ_i, λ_ℓ	$\frac{(Ty_{1,2}(t_{i,\ell}))}{2 + p_{1,2}T}$		

that the estimated overall gain is a product of the linear system and static non-linearity gain. Clearly, the system and its parameter mapping in the forward and inverse direction are non-linear and governed by $z = \frac{(2 + Ts)}{(2 - Ts)}$ and $s = \frac{2}{T}\frac{(z-1)}{(z+1)}$ where T is the sampling interval.

Given this framework, the goal is to estimate the NARMAX model parameters as

$$
\min_{\theta_{1,2}} \frac{1}{2} \left\| (\mathbf{Z_{1,2}} - \phi_{1,2}\theta_{1,2}) \right\|_2^2 \tag{3}
$$

where $\mathbf{Z}_{1,2} \in \mathbb{R}^{N \times 1}$ is a vector of outputs, $\phi_{1,2} \in \mathbb{R}^{N \times p}$ is a matrix of regressors and $\theta_{1,2,} \in \mathbb{R}^{p \times 1}$ is a vector of unknown coefficients. Using the estimate $\theta_{1,2}$ and the relationship on the RHS of Table I we calculate the continuous-time VOR parameters. This approach is known as indirect system identification of continuous-time parameters.

In addition, notice the expression in Eqn. 3 is a non-sparse matrix approach, requiring both fast and slow-phase to be analysed separately. However, using a sparse matrix approach by constructing a diagonal block-oriented data matrix and exploiting its natural sparseness as

$$
\begin{bmatrix} \mathbf{Z}_1 \\ \mathbf{Z}_2 \end{bmatrix} = \begin{bmatrix} \phi_1 & \mathbf{0} \\ \mathbf{0} & \phi_2 \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} \tag{4}
$$

it is possible to simultaneously estimate both nystagmus and saccade parameters.

III. STANDARDISED LEAST-SQUARES WITH DUAL SAMPLING

In a clinical setting the VOR is appropriately sampled taking into consideration fast-phase dynamics. However, a suitable sampling rate for saccade results in slow-phase dynamics being highly oversampled. Although it is desirable to oversample system dynamics to avoid aliasing, too high a sampling rate can lead to numerical instability in system identification resulting in biased estimates [9], [10]. To avoid numerical instability we propose using a dual sampling rate approach to analyse both modes of VOR. This dual sampling rate method is easily achieved by modifying the sparse structure in Eqn. 4 as

$$\begin{bmatrix} \mathbf{Z}_1^* \\ \mathbf{Z}_2 \end{bmatrix} = \begin{bmatrix} \phi_1^* & \mathbf{0} \\ \mathbf{0} & \phi_2 \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} \tag{5}$$

where the superscript "*" denotes that the nystagmus signal has been down sampled to achieve better numerical results. Here, we assume that the fast mode has been appropriately sampled and does not need to be down sampled since the user has control over the sampling rate. However, this result is easily generalisable allowing both signals to be analysed at appropriately down sampled rates.

In many non-linear systems there are large numerical differences of the regressors due to the basis function(s) used to estimate the static map. The large differences lead to the regressor matrix being ill-conditioned and results in unstable matrix inversion and poor parameter estimates [11]. To alleviate ill-conditioning we propose using standardised least-squares (SLS) in combination with the dual sampling approach proposed in Eqn. 5.

Given a matrix of independent variables ϕ and of dependent variables $\mathbf{Z}$ compute the mean and standard deviation of each variable, and replace ϕ and $\mathbf{Z}$ with the centred and standardised variate as

$$\widetilde{\phi} = \left(\phi - \mu_\phi \right) \Sigma_\phi^{-1} \quad \text{and} \quad \widetilde{\mathbf{Z}} = (\mathbf{Z} - \mu_Z) \Sigma_Z^{-1} \tag{6}$$

where Σ_ϕ is a diagonal matrix of standard deviations with $\Sigma_{\phi k}$ denoting the standard deviation of the kth column of ϕ and μ_ϕ is a matrix who's kth column has all entries equal to the mean of column k of ϕ. Substituting Eqn. 6 into 5 yields a dual sampling rate SLS formulation in a sparse structure.

$$\begin{bmatrix} \widetilde{\mathbf{Z}}_1^* \\ \widetilde{\mathbf{Z}}_2 \end{bmatrix} = \begin{bmatrix} \widetilde{\phi}_1^* & \mathbf{0} \\ \mathbf{0} & \widetilde{\phi}_2 \end{bmatrix} \begin{bmatrix} \widetilde{\theta}_1 \\ \widetilde{\theta}_2 \end{bmatrix} \tag{7}$$

In the sequel we use the formulation in Eqn. 7 to estimate continuous-time parameters of nystagmus and saccade dynamics.

IV. SIMULATIONS & RESULTS

The accuracy of the dual sampling rate SLS technique with sparsity was validated by simulating a continuous-time VOR model (e.g. Fig. 1) using Simulink. The parameters used in

TABLE II

LEFT: CONTINUOUS-TIME COEFFICIENT VALUES. τ_1: SLOW-PHASE TIME-CONSTANT, τ_2: FAST-PHASE TIME-CONSTANT, K_1: SLOW-PHASE GAIN, K_2: FAST-PHASE GAIN AND T: SAMPLING INTERVAL. RIGHT: COEFFICIENT VALUES OF STATIC NON-LINEARITY. b: LINEAR TERM, c: SQUARED TERM AND d CUBIC TERM.

CT System Coeff.	Value	CT NL Coeff.	Value
τ_1	15.0 s		
τ_2	50.0 ms	b	1.20
K_1	-9.43	c	-3.00×10^{-3}
K_2	0.222	d	-1.50×10^{-5}
T	1.00×10^{-2} s		

the simulation are shown in Table II and representative of typical values found in experiments [4].

One hundred Monte-Carlo simulations were generated in which the input-output realisation was the same but had a unique Gaussian white, zero-mean, noise sequence added to the output. Excluding the noise free (NF) case the signal-to-noise ratio (SNR) of the noise sequence was decreased from $20 - 0$ dB in increments of 5 dB. The system was perturbed using a sinusoid input (1/6 Hz frequency and 188 deg/s amplitude). A sinusoid input was used because it is the type of perturbation used in clinical settings. The system input-output was sampled at 100 Hz.

The system parameters were estimated as outlined in Eqns. 4-7. Specifically, for each input-output realisation, we analysed the VOR model as follows:

1) *Single Sampling Rate, Non-Standardised Least-Squares*: Both VOR modes were analysed using a single sampling rate and identified simultaneously using a sparse structure using traditional least-squares (Eqn. 4),

2) *Dual Sampling Rate, Non-Standardised Least-Squares*: fast-phase was sampled at the original rate (100 Hz) whilst slow-phase was down sampled by 10 (10 Hz) and identified simultaneously using a sparse structure using traditional least-squares (Eqn. 5),

3) *Dual Sampling Rate, Standardised Least-Squares*: fast-phase was sampled at the original rate (100 Hz) whilst slow-phase was down sampled by 10 (10 Hz) and identified simultaneously using a sparse structure using SLS (Eqn. 7).

The continuous-time parameters of slow and fast-phase dynamics were estimated using the theoretical relationships in Table I. Notice that we consider the linear system to have unity gain and translate the overall gain onto the polynomial basis function giving an estimate as a product of the linear system and static non-linearity gain. This is necessary because it is impossible to measure the signal at the output of the static non-linearity. Therefore, we deem that the best estimate of the linear system gain is a product of the linear system gain and linear coefficient of the static non-linearity, i.e. $G_{1,2}b$.

The results of this study are shown in Fig. 2. The panels of the left column illustrate our findings for a non-standardised (traditional) least-squares approach using a single sampling rate to analyse both modes. The results show the nystagmus
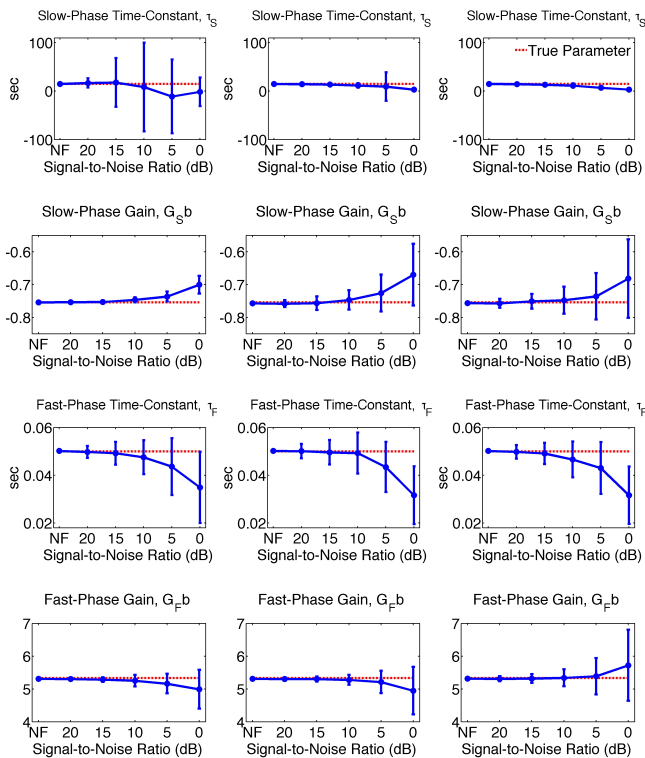
Fig. 2. Left column: Single Sampling Rate, Non-Standardised Least-Squares. Centre column: Dual Sampling Rate, Non-Standardised Least-Squares (100 & 10 Hz). Right Column: Dual Sampling Rate, Standardised Least-Squares (100 & 10 Hz). Ordinate: STD about mean. Abscissa: Output SNR = NF, 20, 15, 10, 5, 0 dB, where NF denotes noise free case. (Note that the abscissa is shown in decreasing SNR which corresponds to increasing noise amplitude.)

time constant is significantly biased with large variance whilst the nystagmus gain and saccade parameters are as theoretically expected with increasing bias and variance for decreasing SNR. The centre column displays the results of using a dual sampling rate with a non-standardised least-squares technique. The panels show this method improves bias and variance estimates for decreasing SNR since it yields theoretically expected trends. However, the non-monotonic increase in variance for the slow-phase time constant (e.g. 10-0 dB) suggests that the regressor matrix tends to becomes ill-conditioned for decreasing SNR. The right column displays the results of using a dual sampling rate with a standardised least-squares approach. These results demonstrate that a dual sampling rate approach in combination with SLS significantly improves bias and variance estimates to expected trends. Nevertheless, the bias and variance of the slow phase gain increase with dual sampling rate and standardisation. This result is expected since in these two cases the gain parameter is estimated with 10 times less data. Hence, using SLS the slow-phase variance monotonically increases for decreasing SNR and agrees with theoretically expected results.

V. DISCUSSION

Simulation results presented in §IV demonstrate that a dual sampling rate approach provides improved results to single sampling rate techniques. In addition, when combined with preconditioning offered by SLS, estimates are further improved. Hence, our dual sampling rate SLS technique is a robust and easily applicable methodology for the analysis of VOR dynamics.

In many continuous and discrete-time parameter estimation problems the estimates may have incorrect sign due to high variance, possibly due to numerical ill-conditioning. Our proposed technique alleviates this problem by using matrix preconditioning (compare Fig. 2 (a)-(b) with (c)). This result may have general applicability for many estimation problems in biology such as for the analysis of ankle dynamics [12].

The central problem with VOR analysis is due to its time-constants (or poles) being separated by many orders of magnitude. In our example with a clinically relevant parameter set, the system poles are separated by 30 orders of magnitude, which yields in slow-phase being highly oversampled and leading to numerical instability.

The approach presented here is generalisable to most bifurcating systems since they have poles that are typically several orders of magnitude apart, resulting in highly oversampled dynamics and unstable parameter estimates.

VI. CONCLUSIONS

The results demonstrate that our dual sampling rate SLS approach with sparsity is a robust and easily implementable technique for the analysis or VOR data. These results suggest that clinical analysis of nystagmus (and saccade) may be improved, providing clinicians more accurate information for diagnosis, disease modelling, etc. In addition, the robust estimation of VOR may lead to improved mission planning for astronauts and test-pilots of advanced aircraft.

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