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Estimation of neural activity including informative priors in Kalman filter based approach: A Simulation Study

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***Abstract*— Present study proposes a methodology for including sets of informative priors, comprising several dynamics present in the data, as prior information to solve the Electroencephalogram (EEG) inverse problem into a Kalman filter based solution. To this aim, spatial, temporal, and frequential signatures of EEG recordings are used to infer multiple priors for the EEG source reconstruction of neural activity, when there are active sources generating complex dynamics at the sensors. Attained results using physiological-based simulations show that including more informative *s-f-t* priors along with a temporal-based solution, the reconstruction of neural activity can be improved achieving an average localization error of $4\,mm$, compared to $47\,mm$ that can be achieved using the baseline approach.**

I. INTRODUCTION

Electroencephalogram (EEG) is a long-standing neuroimaging technique for the analysis of neural activity consisting in measuring the electric potential on the surface of the head with an array of sensors. The EEG is known for having a higher temporal resolution compared to other methods as the functional magnetic resonance imaging (fMRI), being the former the neuroimaging approach most commonly used. EEG neural source reconstruction methods estimate the activity inside the brain that better fits the potentials measured over the scalp (inverse problem solution). However, EEG-based reconstruction methods device an ill-posed problem due to the infinite number of possible source activity that may generate the same potentials.

Usually, solution of the inverse problem using methods, like Low resolution tomography (LORETA), is calculated using only the measurement at one single time instant. However, neural activity has strong spatial and temporal dynamics inherent to its nature, so in solution of the inverse problem it is necessary to consider the dynamic variability of the neural activity, therefore, the accurate estimation of neural activity is highly dependent on the inclusion of such information in the inverse problem solution. In this regard, the Kalman filter is a useful tool for including such temporal information [1], [2]. Moreover, under this framework, prior information has to be included to obtain an optimal unique solution. Typically, the prior information is included in the form of pre-fixed covariance matrices, e.g., an identity matrix, in the case of minimum norm estimates (MNE), or a matrix based on a discretized Laplacian operator [1]. Nevertheless, these approaches do not include adequate priors since such covariance matrices are not based on the information available on data. Thus, several ways of obtaining informative constraints have been proposed. For example, the multiple sparse priors method, where multiple covariance matrices are computed from potentially activated areas of the brain [3]. Those covariance matrices are linearly combined to obtain a final, potentially, more informative prior. Another approach to obtain the prior covariance matrix is to use a Linear Constrained Minimum Variance (LCMV) spatial filter, termed beamformer, as discussed in [4]. Nevertheless, the EEG data has a dynamic information inherent in its nature that cannot be fully exploited if the raw data is used to extract the covariance matrix. In this regard, *t-f* representations can be considered as an alternative technique that allows to extract additional information about the signal dynamics [5]. The precision of the neural activity reconstruction can be improved by adequate decomposing the recordings into well-defined *s-f-t* components, without making very restrictive assumptions about the data, i.e., linearity or statistical independence of the sources [5], [6].

Aimed to obtain a set of multiple priors comprising the underlying dynamic behaviors hidden in the raw EEG recordings, the present study proposes the usage of the enhanced signal representation, introducing *s-t-f* decomposition of the data. Afterwards, obtained spatial, temporal and frequential signatures of data are mapped into the sources space, as a result providing a set of priors directly related with the different dynamics present in the data. Accomplished *s-f-t* based priors are to be further included within the Kalman filter solution framework.

II. BACKGROUND

A. Formulation of the EEG inverse problem

The magnitude of electromagnetic fields measured at the scalp with EEG can be obtained from the quasi-static approximation of Maxwell equations and Poisson equation. This allows writing the following general linear model:

$$\boldsymbol{Y} = \boldsymbol{L}\boldsymbol{J} + \boldsymbol{\epsilon}, \qquad (1)$$

where $\boldsymbol{Y} \in \mathbb{R}^{N_c \times N_t}$ is the EEG dataset of N_c sensors and N_t time samples, $\boldsymbol{J} \in \mathbb{R}^{N_d \times N_t}$ is the amplitude of N_d current dipoles distributed through the cortical surface with fixed orientation perpendicular to it. Both data and sources are related by the gain matrix $\boldsymbol{L} \in \mathbb{R}^{N_c \times N_d}$ (also known as the lead field matrix), and the obtained measurements

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are affected by zero mean Gaussian noise $\boldsymbol{\epsilon} \in \mathbb{R}^{N_c \times N_t}$ with covariance $\operatorname{cov}(\boldsymbol{\epsilon}) = \tau^2 \boldsymbol{Q}_\epsilon \in \mathbb{R}^{N_c \times N_c}$, $\tau \in \mathbb{R}$. The selection of a distributed approach ($N_d \gg N_c$) means that the lead field matrix $\boldsymbol{L}$ is non-invertible, and that the source estimation $\widehat{\boldsymbol{J}}$ can not be directly recovered.

Moreover, the problem can be rewritten in a state space model representation, as follows [1] :

$$\boldsymbol{j}_{k+1} = \boldsymbol{G}\boldsymbol{j}_k + \boldsymbol{\eta}_k, \tag{2a}$$

$$\boldsymbol{y}_k = \boldsymbol{L}\boldsymbol{j}_k + \boldsymbol{\epsilon}_k, \tag{2b}$$

where $\boldsymbol{\eta}_k \in \mathbb{R}^{N_d \times 1}$ is the gaussian process noise with covariance matrix $\operatorname{cov}(\boldsymbol{\eta}_k) = \alpha^2 \boldsymbol{Q} \in \mathbb{R}^{N_d \times N_d}$, being $\alpha \in \mathbb{R}$, and $\boldsymbol{G} \in \mathbb{R}^{N_d \times N_d}$ the state transition matrix. Finally, $\boldsymbol{j}_k \in \mathbb{R}^{N_d \times 1}$ and $\boldsymbol{y}_k \in \mathbb{R}^{N_c \times 1}$ are vectors containing the activity of the dipoles and channels, respectively, at time instant k.

Under the state space framework, Kalman filter becomes a suitable method to estimate the brain activity $\boldsymbol{J}$ [1]. However, matrices $\boldsymbol{G}$, $\boldsymbol{Q}$, and $\boldsymbol{Q}_\epsilon$ should be properly selected in advance. In this work, computation of the matrix $\boldsymbol{Q}$ is studied. In general, the dynamic structure of an EEG signal is very complex and cannot be fully described by the deterministic model $\boldsymbol{G}$. Thus, according to the state space model, a lot of information is contained in $\boldsymbol{\eta}_k$, and so in $\boldsymbol{Q}$. However, $\boldsymbol{Q}$ typically does not contain relevant information since normally it is computed without taking into account the information available in $\boldsymbol{Y}$, as discussed in [7].

B. Informative s-f-t priors

A suitable way to obtain an informative prior $\boldsymbol{Q}$ is by introducing constraints derived from different *s-f-t* signatures. This work proposes a multivariate decomposition applied over a three-way time-varying EEG spectrum $\mathbb{S}(s; f, t) \in \mathbb{R}^{N_c \times N_f \times N_t}$, which shows the energy of the s channel at frequency f and time instant t, being N_f the number of frequency-bins, defined as:

$$\mathbb{S}(s; f, t) = |\boldsymbol{v}(t, f) * \boldsymbol{y}(s, t)|^2, \tag{3}$$

where notation $*$ stands for convolution.

Thus, proposed approach tends to obtain the covariance matrix as a weighted sum of N_k priors, each one related to an specific behavior on the *s-f-t* domain, by using the well known beamformer-based approach [4]:

$$d_{ii}^k = \frac{1}{\delta_i} (\tilde{\boldsymbol{l}}_{(\cdot,i)}^\top (\boldsymbol{\Lambda}^k)^{-1} \tilde{\boldsymbol{l}}_{(\cdot,i)})^{-1}, \quad \forall k = 1, \ldots, N_k, \tag{4}$$

where d_{ii}^k stands for the ith diagonal element of the prior matrix $\boldsymbol{D}^k \in \mathbb{R}^{N_d \times N_d}$, $\delta_i = 1/(\tilde{\boldsymbol{l}}_{(\cdot,i)}^\top \tilde{\boldsymbol{l}}_{(\cdot,i)})$ represents a normalization term, and $\tilde{\boldsymbol{l}}_{(\cdot,i)}$ is the ith column of $\tilde{\boldsymbol{L}}$, which is the result of squaring each element of the matrix $\boldsymbol{L}$, representing a linear relation between the spatial signatures of each *s-f-t* decomposition and the spectrum of current sources generating scalp voltages, as recommended in [5] for source spectral imaging approaches. Additionally, matrix $\boldsymbol{\Lambda}^k \in \mathbb{R}^{N_c \times N_c}$ stands for the spatial-covariance matrix (relating to the sensors) on the new space of representation. Thus, to obtain the set of N_k multiple priors, the below way of computing corresponding covariance matrices, is proposed, based on Parallel Factor Analysis, termed Parafac, whose basic structural model for unfolding the data matrix $\mathbb{S}(s; f, t)$ is defined as [5], [8]:

$$\widehat{s}_{sft} = \sum_{k=1}^{N_k} a_{sk} b_{fk} c_{tk} + \varepsilon; \tag{5}$$

being N_k the number of decomposition factors and ε is the reconstruction residual.

The challenge is to find the loading matrices $\boldsymbol{A} \in \mathbb{R}^{N_c \times N_k}$, $\boldsymbol{B} \in \mathbb{R}^{N_f \times N_k}$, and $\boldsymbol{C} \in \mathbb{R}^{N_t \times N_k}$, whose corresponding row vectors $\boldsymbol{a}_k = \{a_{sk}\}$, $\boldsymbol{b}_k = \{b_{fk}\}$ and $\boldsymbol{c}_k = \{c_{tk}\}$, are related to the spatial, spectral, and temporal signatures of each decomposition factor, respectively. The k-th covariance matrix, $\boldsymbol{\Lambda}^k$, stand for the similitude matrix of each spatial signature $\boldsymbol{a}_k$ of the decomposition, as follows:

$$\boldsymbol{\Lambda}^k = \frac{\boldsymbol{a}_k \boldsymbol{a}_k^\top}{||\boldsymbol{a}_k||^2}, \ \boldsymbol{\Lambda}^k \in \mathbb{R}^{N_c \times N_c}, \tag{6}$$

Therefore, because the direct relation among spatial, frequential and temporal factors, each covariance matrix contributes with the information in a specific frequency band, given by the spectral signature $\boldsymbol{b}_k$ and with a particular temporal behavior, represented by $\boldsymbol{c}_k$. So, after obtaining the N_k spatial covariance matrices, the same number of beamformers (*s-f-t* based priors) can be computed by Eq. (4) and used to create the prior covariance matrix $\boldsymbol{Q}$, as follows:

$$\boldsymbol{Q} = \sum_{k=1}^{N_k} h_k \boldsymbol{D}^k, \tag{7}$$

where h_k is the $k-$th weighting hyperparameter. Since the norm of each spatial signature $\boldsymbol{a}_k$ is directly related to the contribution of the k-th decomposition component, the hyperparameter h_k can be defined as $h_k = ||\boldsymbol{a}_k||$.

Lastly, the covariance parameters α and τ are computed using a likelihood maximization algorithm, more specifically, the Akaike Information Criterion, as explained in [1].

III. Experimental Set-Up

A. Database Description

The most common approach to assess the inverse solution performance is using simulated data, since the activity of the underlying sources is known and the methods can be objectively validated. Particularly, one active source randomly located in the cortex, simulated by a realistic model is considered, provided that all the parameters are tuned to simulate normal activity [9]:

$$\begin{aligned} \frac{1}{\varsigma^2} \frac{\partial^2 \varphi(t)}{\partial t^2} + \frac{2}{\varsigma} \frac{\partial \varphi(t)}{\partial t} &= c_1 \varphi(t) + c_2 \varphi(t - t_0) \\ &+ n_2 \varphi(t)^2 + n_3 \varphi(t)^3 + \zeta, \end{aligned} \tag{8}$$

Moreover, to obtain a set of measurements $\boldsymbol{Y}$, Eq. (1) is solved using a lead field matrix calculated by the Boundary Elements Method and discretized using 8194 vertices. The source activity is measured by means of 34 sensors on the scalp. In addition, to evaluate performance of the proposed method, the following two error measurements are carried out, as suggested by [10]:

$$e_1 = ||\boldsymbol{r}(\max_{\forall i}\{\widetilde{m}_i\}) - \boldsymbol{r}_{sim}||^2 \quad (9a)$$

$$e_2 = \sum_{i=1}^{N_d} \widetilde{m}_i ||\boldsymbol{r}_i - r_{sim}||^2 \quad (9b)$$

where $\boldsymbol{r}_{sim}$ are the true spatial coordinates of the active dipole, $\boldsymbol{r}_i$ are the coordinates of the ith dipole, $\widetilde{m}_i$ is the estimated energy of ith dipole, and $\boldsymbol{r}(\max_{\forall i}\{\widetilde{m}_i\})$ are the coordinates of the dipole having the highest estimated energy. The former measurement computes the raw localization error, while the latter one adds a spatial dispersion penalty.

B. Preprocessing and s-f-t representation enhancement

For the sake of simplicity, the *s-f-t* analysis is performed within a range from 1 to 50 *Hz*, with a resolution of 50 bins. Additionally, to perform the *s-t-f* represention, this work considers the complex Morlet Wavelet, $\boldsymbol{\upsilon} \in \mathbb{C}$, that has been used for the analysis of multichannel EEG spectrum:

$$\boldsymbol{\upsilon}(t, f) = \sigma_t^{-1/2}\pi^{-1/4}\exp(2\pi f_0 t)\exp\left(\frac{-t^2}{2\sigma_t^2}\right), \quad (10)$$

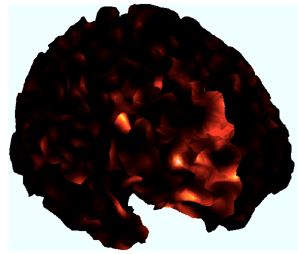
where f_0 is the central frequency and σ_t is the bandwidth.

In computing the Parafac decomposition, the number of factors (N_k) selection is based on the Concordia index [8]. Besides, the factors are computed under non-negativity constrains [5]. For instance, Figure 1 shows the spatial (top), frequential (left-bottom) and temporal (right-bottom) signatures of the Parafac decomposition. It can be clearly seen that most quantity of information is located between 10 and 30 *Hz*, as expected for normal neural activity.

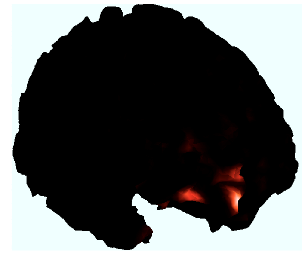
C. Results

The scenario designed to assess performance of considered approach, consists in computing 30 inversions for different randomly selected source locations, generating activity with the dynamical model described in Eq. (8). Furthermore, results of proposed approach are compared against the Kalman filter with process noise covariance matrix $\boldsymbol{Q} = (\Delta^\intercal \Delta)^{-1}$, where $\boldsymbol{\Delta}$ is a Laplacian operator, as suggested in [1]. Regarding to the state transition matrix used, in both filters a random walk model is considered for the sake of simplicity i.e., $\boldsymbol{G} = \boldsymbol{I}_{N_d}$. An example of obtained inversions is shown in Figure 2. Bottom panel shows the simulated activity (Figure 2(c)), while top-left (Figure 2(a)) and top-right (Figure 2(b)) panels show reconstructed activity for LORETA-based priors and *s-f-t* based priors, respectively. It is worth noting that inclusion of more informative priors allows achieving a more accurate reconstruction.

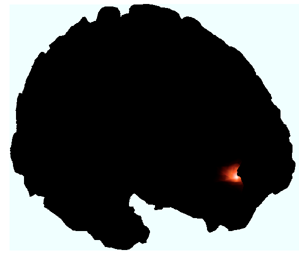
Moreover, the approach is evaluated under different values of *SNR*= $\{10, 20, 30\}$ dB. Figure 3 shows the results for both error measures against the *SNR* level, as the mean and standard deviation computed over 30 realizations of the experiment. As expected, the lower the *SNR* value, the higher the measured error, nevertheless, it can be seen an improvement in the reconstruction, when *s-f-t* priors are used.



(a) Reconstruction using Kalman with LORETA-based priors.

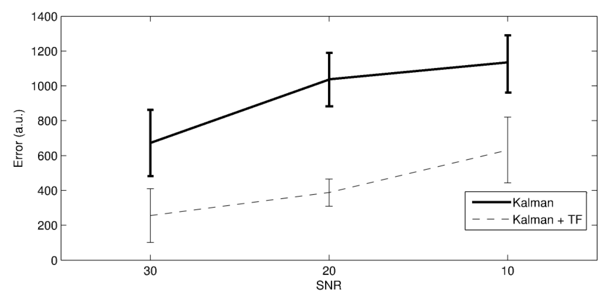


(b) Reconstruction using Kalman with *s-f-t* based priors.

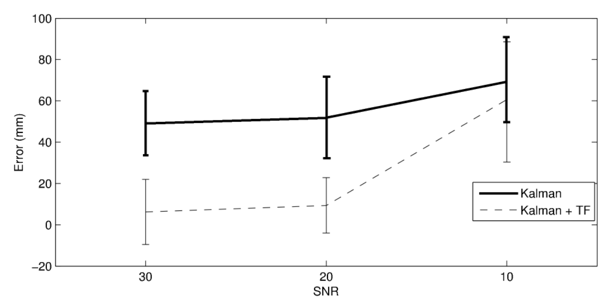


(c) Simulated activity.

Fig. 2. Simulated activity and its corresponding reconstruction using the considered methods.



(a) Weighted distance.



(b) Euclidean distance.

Fig. 3. Error measurements under several Signal-to-Noise Ratio values.

IV. Discussion

The main goal of present study is to develop a methodology for EEG source reconstruction of neural activity, that allows to include informative *s-f-t* priors into an inverse problem solution based on a Kalman filter framework. Several tests are carried out to assess the behavior of proposed approach, which evidence the following aspects to consider:

Present study discusses the introduction of *s-f-t* information in terms of the process noise covariance matrix to improve the EEG source reconstruction. For this aim, Parafac

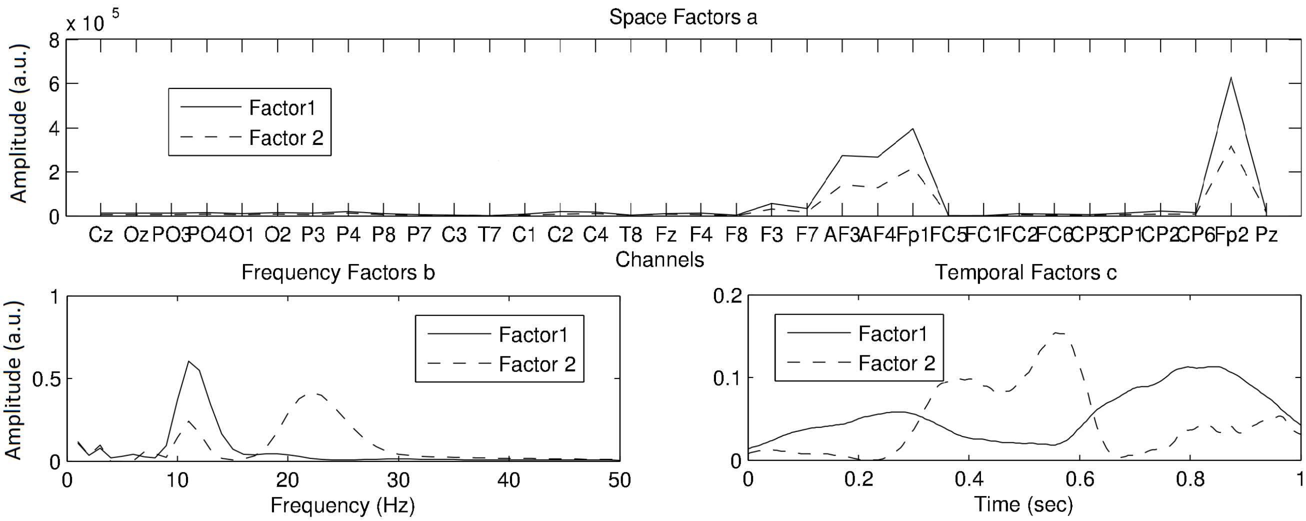


Fig. 1. Factors obtained from the Parafac decomposition over the dataset corresponding to the case presented in Figure 2

multivariate decomposition is considered to extract the *s-f-t* information from the EEG data, which has proved to be appropriate for EEG representation.

Several studies have considered the Kalman filter framework as a suitable tool for solving the EEG inverse problem, due to its ability of including the different dynamics present in the data as part of the solution. Nevertheless, as shown in Figures 2 and 3, inclusion of informative *s-f-t* priors improves the neural activity reconstruction, in comparison with conventional approaches.

In average, achieved results show lower error in the neural activity reconstruction when informative priors are considered, for both error measurements. Nevertheless, the lower the *SNR* value, the higher the error. This fact can be explained due to the lack of a denoising stage. Moreover, as the *SNR* value decreases, the solution obtained using the proposed approach tends to converge to the same solution obtained by Kalman filter using the LORETA-based prior.

In the proposed approach, there is a limitation that should be pointed out: the proposed solution based on *s-f-t* priors should not work properly when several foci of activity with the same frequency are considered. This drawback is conditioned by the fact that the time frequency representation takes into account only the magnitude of the spectra of the channels. Therefore, any information about the phase between the active sources will be lost.

V. Conclusions

A new methodology to generate data-based priors is explored, which is based on the assumption that by including *s-f-t* representation, the obtained priors may contain more suitable information about the neural activity. Additionally, the obtained priors are included into a Kalman filter framework for solving the EEG inverse problem, which allows to include explicitly different dynamics present on the data into the solution. Obtained results show that the *s-f-t* based estimated priors along with a dynamical solution, are able to detect more accurately the source localization. Additionally, discussed estimation approaches supply a better interpretability about the obtained priors, according to the dynamics present on the data.

As a future work, a model that encodes more complex dynamics should be taken into account instead of using the random walks approach. Also, a time-frequency representation that provides information about the phase of the signal should be studied.

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