Parameter Tuning of Hybrid Nature-Inspired Intelligent Metaheuristics for Solving Financial Portfolio Optimization Problems

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Abstract. In previous studies, nature-inspired algorithms have been implemented in order to tackle hard NP-optimization problems, in the financial domain. Specifically, the task of finding optimal combination of assets with the aim of efficiently allocating your available capital is of major concern. One of the main reasons, which justifies the difficulties entailed in this problem, is the high level of uncertainty in the financial markets and not only. As mentioned above, artificial intelligent algorithms may provide a solution to this task. However, there is one major drawback concerning these techniques: the large number of open parameters. The aim of this study is twofold. Firstly, results from extended simulations are presented regarding the application of a specific hybrid nature-inspired metaheuristic in a particular formulation of the financial portfolio optimization problem. The main focus is on presenting comparative results regarding the performance of the proposed scheme for various configuration settings. Secondly, it is our intend to enhance the hybrid scheme's performance by incorporating intelligent searching components such as other metaheuristics (simulated annealing).

Keywords: parameter tuning, genetic algorithm, portfolio optimization, hybrid Nature-Inspired Intelligent (NII) algorithm.

1 Introduction

Nowadays, a non-trivial task for investment managers, as well as investors in general, is to find efficient ways to allocate capital. By doing so, the level of risk decreases and in the same time the potential investor's goal is achieved. However, the question remains: which is a proper way to select assets for my portfolio? There are several ways of doing so. Some decision makers may invest on a financial index (such as S&P's 500), or in individual stocks. Nevertheless, there exist more intelligent approaches.

Portfolio optimization problems are concerned with finding the optimal combination of assets, as well as their corresponding weights, i.e optimization in two search spaces: one discrete (for assets) and one continuous (for weights). This kind of problems are considered as NP-hard, i.e. there is no deterministic algorithm known that can find an exact solution within polynomial time. Exhaustive search algorithms,

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or other traditional approaches from the field of operational research, are inefficient to find the optimal solution or, in the best case, they get stuck in local optima [1]. A potential solution is the introduction of intelligent metaheuristics.

Nature-inspired algorithms in the field of artificial intelligence correspond to techniques that are based on how biological systems and natural networks deal with real-world situations in nature [2]. The main advantage of nature-inspired intelligent algorithms over traditional methodologies which deal with optimization problems is their searching ability. Finally, hybrid schemes combine unique characteristics of two or more intelligent methods so as to enhance searching of the solution space.

The scope of this paper is to present a statistical analysis regarding various combinations of hybrid algorithm's parameter settings. Also, the overall performance of the algorithm is of great importance, as well. In order of achieve this, we incorporate additional searching components in the main strategy of the genetic algorithm. In this study, we provide results regarding the incorporation of a simulated annealing algorithm. As far as the application domain is concerned, the objective function is to maximize a non-linear financial ratio which takes into account both the risk and the expected return of the portfolio. The main contribution of this work lies in detecting useful trends regarding the hybrid algorithm's parameters. This will provide an assistance tool for further investigation in the portfolio optimization domain.

The structure of this paper is as follows. In section 1, an introduction to some main concepts is given. In section 2, findings from the literature review are presented in brief. In section 3, the basic methodological issues are shown. In section 4, the mathematical formulation of the optimization problem is presented. Computational results and a brief discussion are presented in section 5. Finally, in section 6 some basic conclusions and future research potentials are presented.

2 Literature Review

In this section, evidence from the literature is provided regarding the application of nature-inspired algorithms for the portfolio optimization problem. For convenient reasons, the main findings are presented in brief. Studies in this field are limited, and only a selection of them is presented here.

Table 1. Basic studies from the literature

Reference	Applied Methodology	Portfolio Optimization Problem
[1]	Ant Colony Optimization Algorithm & Firefly algorithm (hybrid)	Maximize Sortino ratio with constraint in tracking error volatility
[3]	Evolutionary Algorithm & Quadratic Programming (hybrid)	Minimize tracking error volatility
[4]	- Genetic Algorithms - Evolutionary Algorithms - Memetic Algorithms	Minimize portfolio's risk
[5]	Genetic Algorithm & Levenberg-Marquardt algorithm	Maximize Sortino ratio with constraint in tracking error volatility
[6]	Particle Swarm Optimization	Minimize portfolio's risk Constraint on portfolio's expected return
[7]	Particle Swarm Optimization	Maximize excess return Constraint on tracking error volatility
[8]	Ant Colony Optimization & non linear programming algorithm (hybrid)	Minimize probability of tracking error falling below a threshold

In what follows, interesting points from the literature survey are presented:

- These studies highlight the significance of nature-inspired metaheuristics.
- Another important aspect is the use of combined methodologies (hybrids) in order to deal with the complexities of the financial portfolio management problem.
- In some of these studies, preliminary results regarding the influence of various configuration settings in the performance of hybrid schemes are included.
- To sum up, findings from the literature review highlight the importance of using hybrid NII techniques in order to solve the portfolio optimization problem under the passive and active management framework. Particularly, new, more complex formulations of the problem, offer new challenges to the academia. The combination of unique characteristics from two or even more NII algorithms is encouraged.

3 Methodological Issues

In this section, the implemented hybrid schemes are briefly presented. In this point, it is important to note that the portfolio optimization problem can be divided into two separate optimization tasks. The first task is to find optimal combination of financial assets (stocks) from a specific market (discrete optimization). The second task is to optimally allocate the available capital into the selected assets (continuous optimization). The common characteristic of both hybrid algorithms is that they deal with the optimization problem separately, as described above. The first hybrid method comprises a genetic-based algorithm [9], which deals with the discrete optimization part, and a mathematical optimization technique, namely the Levenberg – Marquardt method[1] [12], which optimally allocates the available capital. The benchmark hybrid scheme applies the same technique in the discrete optimization task, whereas for the continuous optimization a simulated annealing algorithm is implemented [11].

In what follows, pseudocode of both hybrid algorithms is presented.

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Function Genetic Algorithm – Levenberg_Marquardt
Parameter Initialization
Population Initialization
Calculation of Weights and Fitness Value(Levenberg_Marquardt)
For i=1:generations
  Randomly choose genetic operator
  Apply genetic selection (choose n-best members of population)
  Apply Crossover or Mutation for producing new members
  Calculate weights/evaluate fitness value(Levenberg_Marquardt)
  Adjust population in order to keep best members
End
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Fig. 1. Hybrid Algorithm 1

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[1] This is a local search procedure based on a non-linear programming methodology which combines the Gauss – Newton and the steepest descent method.

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Function Genetic Algorithm - Simulated Annealing
Parameter Initialization
Population Initialization
Calculation of Weights and Fitness Value(Simulated Annealing)
For i=1:generations
  Randomly choose genetic operator
  Apply genetic selection (choose n-best members of population)
  Apply Crossover or Mutation for producing new members
  Calculate weights/evaluate fitness value(Simulated Annealing)
  Adjust population in order to keep best members
End
```

Fig. 2. Hybrid Algorithm 2

4 Application Domain

The portfolio optimization problem deals with finding a combination of assets, as well as the corresponding amount of capital invested in them, with the aim of optimizing a given objective function (investor's goal) under certain constraints. The first person who provided a complete framework for this kind of problem was Harry M. Markowitz, with his seminal paper [10].

In this paper, the objective of the portfolio optimization problem is to maximize a financial ratio, namely the Sortino ratio [5]. Sortino ratio is based on the preliminary work of Sharpe (Sharpe ratio) [5], who developed a reward-to-risk ratio.

The formulation of the financial optimization problem is presented below:

$$Maximize\ Sortino\ Ratio = \frac{E(r_P) - r_f}{\theta_0(r_P)} \tag{1}$$

s.t.

$$\sum_{i=1}^{k} w_i = 1 \tag{2}$$

$$-1 \le w_i \le 1 \tag{3}$$

$$k = 10 \tag{4}$$

where,

$E(r_P)$, is the portfolio's expected return, defined as follows: $E(r_P) = \sum_{i=1}^{k} w_i * E(r_i)$

r_f, is the risk-free return, considered as the market's 'safest' asset

$\theta_0(r_P)$, is the volatility of returns which fall below a certain threshold and equals

$$\theta_0(r_P) = \sqrt{\int_{-\infty}^{0} (0 - r_P)^2 * f(r_P) dr_P} \tag{5}$$

w_i, is the percentage of capital invested in the ith asset
k, is the total number of assets contained in a portfolio[2]
r_P, is the daily return of the portfolio, defined as follows: $r_P = \sum_{i=1}^{k} w_i * r_i$
$f(r_P)$, is the probability density function of the portfolio's returns. Assuming that portfolio's returns follow a normal distribution, the probability density function can be defined as: $f(r_P) = \frac{e^{\frac{-(r_P - E(r_P))^2}{2*\sigma^2}}}{\sigma * \sqrt{2*\pi}}$

5 Computational Study

In this section, results regarding the performance of the hybrid algorithms in various configuration settings are presented. The dataset comprised of 93 daily returns, corresponding to the period 04/01/2010 – 28/05/2010, of 49 stocks of the FTSE/ASE40 Index. In this point, it has to be mentioned that all stocks of the Index have been taken into consideration (even those stocks corresponding to firms which have been excluded the Index). The reason for doing this is to eliminate the effect of survivorship bias[3].

In the next table, values for both algorithms' basic parameters are presented.

Table 2. Parameters for hybrid schemes

Parameters for Genetic Algorithm	
Population	100
Generations	20/30/50
Crossover Probability	0,10/0,90
Mutation Probability	0,10/0,90
Percentage of best members for selection (for *n-best* members selection)	10%
Parameters for Simulated Annealing	
Population	100
Generations	100

[2] In this study, the number of assets included in the portfolio is 10.

[3] Tendency for failed companies to be excluded from performance indices mainly because they no longer exist. This effect often causes the results of the studies to skew higher because only companies which were successful enough to survive until the end of the time period of the study are included.

These configuration settings represent a range of possible values, which are commonly used in the literature [9]. As far as the experimentation set-up is concerned, due to the stochastic behavior of the nature-inspired intelligent metaheuristics a number of independent simulations (100) were executed for each set of configurations. The aim was to produce a range of solutions in order to draw a distribution of the results. Due to space limitations, a specific statistical measure, namely the quantiles of the distribution, was calculated. As far as the distribution of objective function's values, it is desirable to have two basic properties:

– Fat right tails (large number of quantiles in large confidence levels), which indicate high probability of finding portfolios with large Sortino ratios.

Thin left tails (small number of quantiles in small confidence levels), which indicate low probability of finding portfolios with small Sortino ratios.

Table 3. Statistical results for hybrid schemes (numbers in cells represent Sortino ratios)

	pop=100, crossover probability= 0.90, mutation probability=0.10				
gen=20	**Percentiles of distribution**				
	0.025	***0.25***	***0.50***	***0.75***	***0.975***
GA – LMA	1.9246	2.2555	2.4228	2.5859	2.8664
GA - SA	1.9550	2.4550	2.6890	2.8520	2.9950
gen=50					
	0.025	***0.25***	***0.50***	***0.75***	***0.975***
GA – LMA	2.2492	2.5246	2.6533	2.8228	3.1622
GA - SA	2.4890	2.8880	3.0500	3.1573	3.4597
	pop=100, crossover probability= 0.10, mutation probability=0.90				
gen=20	**Percentiles of distribution**				
	0.025	***0.25***	***0.50***	***0.75***	***0.975***
GA – LMA	1.9856	2.3698	2.4558	2.7589	2.9010
GA - SA	2.1580	2.3607	2.5897	2.9897	3.2540
gen=50					
	0.025	***0.25***	***0.50***	***0.75***	***0.975***
GA – LMA	2.2005	2.3969	2.7859	2.9569	3.0056
GA - SA	2.5860	2.7580	2.9950	3.2530	3.5550
	pop=100, crossover probability= 0.10, mutation probability=0.10				
gen=20	**Percentiles of distribution**				
	0.025	***0.25***	***0.50***	***0.75***	***0.975***
GA – LMA	1.7580	1.8560	1.9580	2.0050	2.1250
GA - SA	1.8050	1.9560	1.9990	2.0150	2.1450
gen=50					
	0.025	***0.25***	***0.50***	***0.75***	***0.975***
GA – LMA	1.9057	1.9840	2.0146	2.1980	2.3057
GA - SA	1.9730	1.9960	2.1897	2.3580	2.4760
	pop=100, crossover probability= 0.90, mutation probability=0.90				
gen=20	**Percentiles of distribution**				
	0.025	***0.25***	***0.50***	***0.75***	***0.975***
GA – LMA	2.1050	2.2480	2.3183	2.4097	2.6897
GA - SA	2.2057	2.3840	2.4747	2.8894	3.0013
gen=50					
	0.025	***0.25***	***0.50***	***0.75***	***0.975***
GA – LMA	2.3546	2.5489	2.6563	2.8597	3.0001
GA - SA	2.4982	2.6290	2.8570	3.0052	3.1551

Regarding the results presented in Table 3, above, the following important remarks can be stated. First of all, for each set of configurations, as the number of generations increases, the distribution of results improves. This is quite sensible, due to the fact that for more generations, the algorithm explores the solution space in a great extend. Another, more important finding, is that the hybrid scheme consisting of the genetic algorithm and the simulated annealing process, yields better distributions of results. This may be attributed to the fact that both the GA and SA components have stochastic, and not deterministic, elements which provide them, in a way, better exploration ability. It seems that the incorporation of an intelligent metaheuristic, such as the SA algorithm, provides a better searching strategy of the weight optimization domain (continuous solution space), thus guiding the GA component towards better solutions in the discrete space. Finally, another interesting conclusion concerns the hybrid schemes' behavior for various values of the crossover and mutation probability. These genetic operators play a vital role in the exploration and exploitation of the solution space. Based on the table's results, the best results are obtained in the case where the crossover probability is set to 0.10, whereas the mutation probability is set to 0.90 (this means that the randomness of the GA component rises). This contrasts to many studies, where the mutation probability is set to low values.

6 Conclusion and Future Research

In this study a hybrid NII scheme, which combined a genetic algorithm and the Levenberg-Marquardt algorithm, was proposed for solving a certain formulation of the constrained portfolio optimization problem. More specifically, the objective was to maximize a financial ratio, namely the Sortino ratio. What is more, for benchmarking reasons, a hybrid scheme consisting of a genetic algorithm and a simulated annealing technique, was applied. The main difference between these two schemes is the component that optimizes the amount of capital invested in each asset of the selected portfolio. Mainly, the first technique is based on a deterministic procedure, whereas the second is a stochastic metaheuristic. The focus of this work was twofold. Firstly, our goal was to provide evidence regarding the performance of the hybrid nature-inspired algorithm for various configuration settings. An important task for the decision-maker is to identify 'good' values for the configuration parameters, in a way that high-quality solution spaces are reached. Secondly, our aim was to compare a deterministic with a stochastic component for this kind of problems. It was our firm belief that the intelligent metaheuristic was going to achieve better results.

Results from this study are not directly comparable to other studies from the literature, due to the fact that the formulation of the optimization problem differs. However, based on our findings, it seems that there is a controversial result: in most studies the mutation probability is set to low values, in order to avoid including more randomness in the algorithm. In our case, setting this probability in large values, provides better results. This may indicate that our algorithm approximates a random search procedure. However, in order to draw safer conclusions more sets of simulations have to be executed. Also, the nature of the solution space itself may provide an explanation to this issue. In this point it has to be mentioned that the simulation results are both preliminary and limited. More simulations have been scheduled, as future research.

Finally, some future research directions might be the following: firstly, other, hybrid or not, NII algorithms should be applied. What is more, further simulations are required in order to come up with safer conclusions about the functionality of the proposed alternative mechanisms in this study. As far as the application domain is concerned, other formulations of the portfolio optimization problem should be investigated, specifically these which reflect up-to-date objectives.

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