34th Annual International Conference of the IEEE EMBS
San Diego, California USA, 28 August - 1 September, 2012

A NARMAX Method for the Identification of Time-Varying Joint Stiffness.

Diego L. Guarin[1], *Student Member, IEEE,* and Robert E. Kearney[1], *Fellow, IEEE,*

***Abstract*— Dynamic joint stiffness defines the dynamic relationship between the position of a joint and the torque acting about it and can be separated into intrinsic and reflex components. Under stationary conditions, these can be identified using a nonlinear parallel-cascade algorithm that models intrinsic stiffness and reflex stiffness as parallel pathways. Experimental results demonstrate that both intrinsic and reflex stiffness depend strongly on the operating point defined by mean joint position and the activation level. Consequently, both intrinsic and reflex stiffness will appear to be time-varying (TV) whenever the operating point changes, as during movement. This paper describes and validates a new method for identification of TV ankle stiffness. The method is based on the TV nonlinear autorregresive, moving average exogenous (NARMAX) model class. Simulation results demonstrated that the algorithm can accurately estimate the TV parameters of the ankle stiffness. We conclude that the algorithm is potentially a powerful new tool for the study of joint stiffness during TV conditions.**

I. INTRODUCTION

Joint stiffness can be defined as the dynamic relationship between the angular position of a joint and the torque acting bout it [1]. Therefore, it plays a vital role in the control of posture, where it defines the response of the joint to external perturbations; it is also important in movement control, since it is the torque generated by the muscle that controls the final position of the joint.

At the ankle, dynamic joint stiffness is composed of two components that can be described by the parallel cascade model shown in Fig. 1 [2]. Intrinsic stiffness is generated by the viscoelastic properties of the joint, passive tissue, and active muscle fibres. For small perturbations about a fixed operating point, it can be described as a second order linear system. Reflex stiffness is generated by the active muscle contraction in response to reflex activation from stretch receptors in the muscle. At the ankle, it can be modelled as a linear-nonlinear-linear (NLN) block structured model, comprising a series connection of a delay, a differentiator, a static nonlinearity and a second order low-pass system.

Intrinsic and reflex torques cannot be measured separately experimentally; only their sum can be measured. Consequently, intrinsic and reflex stiffness cannot be estimated separately. To overcome this, our laboratory has developed several system identification methods to separate the intrinsic and reflex torques analytically under stationary conditions [2], [3]. Using these methods we have shown that joint stiffness is highly dependent on joint position and torque

[1]D. L. Guarin and R. E. Kearney are with the Department of Biomedical Engineering, McGill University, Montreal, QC H3A 2T5, Canada. diego.guarinlopez at mail.mcgill.ca, robert.kearney at mcgill.ca

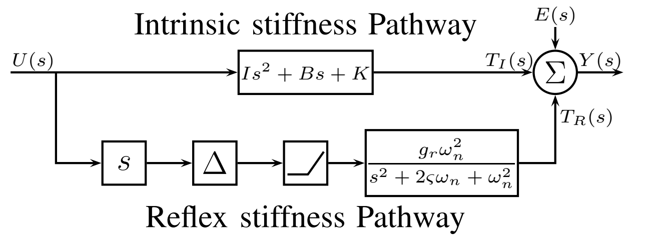

Fig. 1. Ankle stiffness model

[4]; this will result in large changes in intrinsic and reflex stiffness during movement. Thus, a time-variant (TV) identification method is needed to fully understand the time course of changes in stiffness when the angular position of the joint and/or the torque vary with time. Recently, an ensemble-based TV ankle stiffness identification method was developed in our laboratory [5]. One drawback of this method is that it requires a large ensemble of input-output data in which each realization undergoes the same time-varying behaviour. Acquiring such as ensemble of responses is not always feasible with experimental subjects and it can be difficult to ensure that each trial is performed in the same way.

This paper introduces a new TV ankle stiffness identification method, based on a temporal expansion of the unknown TV-coefficients of a TV-NARMAX (nonlinear autoregressive moving average with exogenous inputs) model [6] derived from the parallel cascade structure shown in Fig. 1. This method is different to previously presented algorithms in that requires only one realization of the input-output data.

The organization of this paper is as follows. The NARMAX representation of the ankle stiffness and the TV extension are presented in Section II. Section III introduces a new algorithm for the identification of TV ankle stiffness. In Section IV we present the parameters used in the simulations and describe how we evaluate the accuracy of the identified models. Section V presents the results of identifying TV ankle stiffness and finally, Section VI provides some concluding remarks.

II. NARMAX REPRESENTATION

The input/output relationship of many nonlinear dynamic systems can be modelled as a NARMAX system [7]:

$$\begin{aligned} y(n) =& F^l\left[y(n-1), \ldots, y(n-n_y), u(n), u(n-1), \ldots,\right. \\ & \left. u(n-n_u), e(n-1), \ldots, e(n-n_e)\right] + e(n), \end{aligned} \tag{1}$$

where F^l is a nonlinear mapping, $u(n)$ is the controlled (i.e., exogenous) input, $y(n)$ is the output and $e(n)$ is the uncontrolled input.

978-1-4577-1787-1/12/$26.00 ©2012 IEEE

A. Ankle stiffness NARMAX representation

Kukreja [3] showed that the continuous time parallel cascade ankle stiffness model can be transformed into a NARMAX representation by means of the following approximations:

- For system elements with no poles (i.e., the intrinsic path and the derivative), the Laplace variable s is transformed to the discrete time through Euler's backward formula:
$$s = \frac{1 - z^{-1}}{T}, \tag{2}$$
where T is the sampling time.
- For elements with poles (i.e., the dynamic part of the reflex path), s is transformed to the discrete time through the bilinear transformation:
$$s = \frac{2}{T}\left(\frac{1 - z^{-1}}{1 + z^{-1}}\right). \tag{3}$$
- The continuous time delay (Δ) can be converted to discrete-time as:
$$\tau = \left\lceil \frac{\Delta}{T} \right\rceil. \tag{4}$$
- The half-wave rectifier, in the continuous model, can be approximated by a second order static polynomial
$$c_0 + c_1 x(n) + c_2 x^2(n). \tag{5}$$

Additionally we considered the model output to be corrupted by additive noise

$$y(n) = \omega(n) + e(n), \tag{6}$$

where $\omega(n)$ is the unmeasured noise free output, $e(n)$ is white zero mean noise and $y(n)$ is the measured output.
After applying all the approximations, the nonlinear model can be represented by [3]:

$$\begin{aligned} y(n) =& \phi_1 y(n-1) + \phi_2 y(n-2) + \phi_3 u(n) + \phi_4 u(n-1) \\ &+ \phi_5 u(n-2) + \phi_6 u(n-3) + \phi_7 u(n-4) \\ &+ \phi_8 \gamma(n) - \phi_1 e(n-1) - \phi_2 e(n-2) + e(n), \end{aligned} \tag{7}$$

where

$$\begin{aligned} \gamma(n) = & 4c_0 + \frac{c_1}{T}\left[u(n-\tau) + u(n-1-\tau) - u(n-2-\tau)\right. \\ & \left. -u(n-3-\tau)\right] + \frac{c_2}{T^2}\left[u^2(n-\tau) + 3u^2(n-1-\tau)\right. \\ & +3u^2(n-2-\tau) + u^2(n-3-\tau) \\ & -2u(n-\tau)u(n-1-\tau) \\ & -4u(n-1-\tau)u(n-2-\tau) \\ & \left.-2u(n-2-\tau)u(n-3-\tau)\right]. \end{aligned} \tag{8}$$

The relationship between the coefficients ϕ_i and the parameters of the original model were developed in [3]; the inverse relations are given by:

$$B = T\left(2\frac{\phi_7}{\phi_2} - \phi_4 - \phi_1\phi_3\right), \tag{9}$$

$$I = -T^2\frac{\phi_7}{\phi_2}, \tag{10}$$

$$K = \phi_3(\phi_1 + 1) + \phi_4 - \frac{\phi_7}{\phi_2}, \tag{11}$$

$$\omega_n^2 = \frac{4}{T^2}\left(\frac{-\phi_1 - \phi_2 + 1}{\phi_1 - \phi_2 + 1}\right), \tag{12}$$

$$\varsigma = \frac{\phi_1 + 1}{\sqrt{-\phi_1 + (\phi_2 - 1)^2}}, \tag{13}$$

$$g_r = 4\left(\frac{\phi_8}{-\phi_1 - \phi_2 + 1}\right). \tag{14}$$

Thus, identifying the coefficients ϕ_i, $i = 1, \ldots, 8$ in (7) achieves the identification of the continuous-time model parameters.

B. TV extension

Is possible to create a TV-NARMAX representation of the ankle stiffness model by making the coefficients in (7) functions of the discrete time

$$\begin{aligned} y(n) = & \phi_1(n)y(n-1) + \phi_2(n)y(n-2) + \phi_3(n)u(n) \\ & + \phi_4(n)u(n-1) + \phi_5(n)u(n-2) + \phi_6(n)u(n-3) \\ & + \phi_7(n)u(n-4) + \phi_8(n)\gamma(n) - \phi_1(n)e(n-1) \\ & - \phi_2(n)e(n-2) + e(n). \end{aligned} \tag{15}$$

Basically there are two ways of identifying this kind of system. One approach would be to acquire a large ensemble of input output realizations and then use an ensemble identification method (e.g., [8]) to estimate the parameters as a function of time. Here we consider the other possibility which is to express the time-varying coefficients $\phi_i(n)$ as a function of some (known) basis functions [6]:

$$\phi_i(n) = \sum_{k=0}^{M_i} \alpha_{ik}\pi_k(n), \tag{16}$$

where α_{ik} are some coefficients independent of time. Inserting (16) in (15) gives a time-invariant (TIV) approximation of the TV equations:

$$\begin{aligned} y(n) = & \sum_{i=1}^{2}\sum_{k=0}^{M_i} \alpha_{ik}\pi_k(n)y(n-i) \\ & + \sum_{i=0}^{3}\sum_{k=0}^{M_i} \alpha_{ik}\pi_k(n)(n)u(n-i) \\ & + \sum_{k=0}^{M_8} \alpha_{8k}\pi_k(n)\gamma(n) \\ & + w_1 e(n-1) + w_2 e(n-2) + e(n). \end{aligned} \tag{17}$$

Note that for simplicity we will assume that the parameters of the uncontrolled input are time-invariant.

III. TV SYSTEM IDENTIFICATION

Given the controlled input ($u(n)$), the output ($y(n)$), the basis functions ($\pi_{ik}(n)$) and the uncontrolled input ($e(n)$) it is possible to estimate the coefficients α_{ik} in (17) using a least square approach. Unfortunately the problem in this case is not straightforward because we do not have access to $e(n)$. In addition, the maximum number of basis functions needed to correctly represent the different time-varying coefficients is not known. Given these problems we propose the following algorithm for TV ankle stiffness identification:

1) Assume for the moment that $M_i = M$, $i = 1, \ldots, 8$. Where M is an arbitrarily large number of basis functions (e.g., 15 or 20).
2) Treat the system as if there were no uncontrolled input.
3) Estimate the parameters of the model, $\hat{\Theta}$, by solving the equation

$$\mathbf{y} = \Phi\Theta. \tag{18}$$

To increase the accuracy of the estimation is better to avoid any matrix inversion, therefore a method such as Ortogonal Least Squares (OLS) is recomended [7] (in this case we used an algorithm termed Fast Recursive Algorithm [FRA], which is faster and more stable than OLS [9]).

4) Use the estimated parameters $\hat{\Theta}$ to compute the 1-step prediction as

$$\hat{\mathbf{y}} = \Phi\hat{\Theta}, \tag{19}$$

estimate the residuals

$$\hat{\mathbf{e}} = \mathbf{y} - \hat{\mathbf{y}}. \tag{20}$$

and their variance

$$\sigma^2 = \frac{1}{N}\hat{\mathbf{e}}^T\hat{\mathbf{e}},$$

where $\hat{\mathbf{e}}^T$ is the transpose of the column vector and N is the number of elements.

5) Treat the residuals as the uncontrolled input, and then go to 3. Keep iterating until the variance of the residuals fails to decrease.
6) Calculate the cost function for each parameter ($\delta\alpha_{ik}$), which measures the contribution of that parameter to the variance of the observed signal accounted for the model (see [7] and [9] for information on the cost function and how to compute it). Organize the cost functions according to

$$\begin{bmatrix} \delta\alpha_{11} & \cdots & \delta\alpha_{i1} & \cdots & \delta\alpha_{81} \\ \vdots & & \vdots & & \vdots \\ \delta\alpha_{1M} & \cdots & \delta\alpha_{iM} & \cdots & \delta\alpha_{8M} \end{bmatrix}. \tag{21}$$

And normalize each column by its largest element.

7) Define a threshold to determine which basis functions are most relevant (in all our simulations a threshold equal to 0.001 gave excellent results).
8) Next, for each index i, $i = 1 \ldots, 8$, select only the relevant basis functions π_k (i.e., the basis functions associated with cost functions larger that the selected threshold).
9) Go to 2, but this time use only the selected basis functions. The process ends when the variance of the residuals fails to decrease.

IV. SIMULATIONS

A TV model of ankle stiffness (Fig. 1) was simulated using Simulink (The Mathworks, Inc.) for 60 s. Theoretically all the parameters could be TV but in this study only K (elasticity) and g_r (reflex gain) were varied with time, because previous studies demonstrated that these parameters varied most with changes in position and/or activation level [4]. In this experiment the initial values of K and g_r were chosen from a uniform random distribution, between 50 and 130 Nm/rad for K and 0.5 and 20 Nm/rad/s for g_r. These values were held constant for 7.5 s. New values were selected every 7.5 s so K and g_r had 8 different values.

The input signal was a white Gaussian noise with zero mean, maximum amplitude of ± 0.04 rad and filtered by a second order low pass filter with cut off frequency of 30 Hz to represent the low-pass properties of the ankle actuator used in our laboratory.

Additionally, white Gaussian noise was added to the noise free output to simulate measurement noise; in this experiment the standard deviation of the noise was selected such that the Signal to Noise Ratio (SNR) was equal to 20 dB, a SNR lower than the expected in real experiments [5].

As the input had power up to 30 Hz and the nonlinearity was of second-order, we expect frequencies up to 60 Hz in the output. Therefore, the sampling frequency was selected as 200 Hz (actually the input had power at frequencies larger than 30 Hz, that is why we choose a sampling frequency 3.3 times larger than the highest expected frequency). Fig. 2. shows input-output signals for a typical simulation run (all the parameters, except K, gr and the second-order polynomial, were the same as in [3]). Note that the mean value of the position (given by the thick red line) is constant and centred at zero, while the mean value of the observed torque varies with time as a result of the changes in reflex gain and the intrinsic elasticity.

In all simulations the position input was between ± 0.04 rad and the operating range of velocity was between ± 10 rad/s. The coefficients of the second-order polynomial were selected as the best fit (in the least square sense) to a half-wave rectifier whose domain is between -10 and $+10$ and co-domain between 0 and $+10$. If the range of the input (and of velocity) were to change then the coefficients of the second-order polynomial would have to be modified as well.

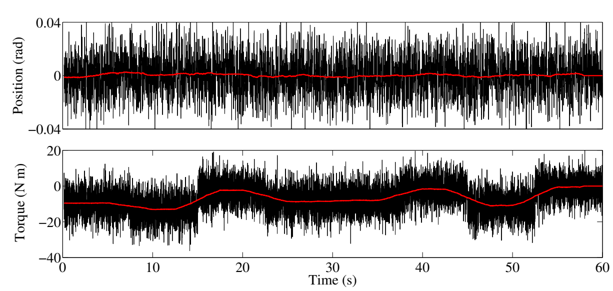

Fig. 2. Top - Input position. Bottom - Simulated torque. Red line is the mean value calculated over 100 data points.

A. Model validation

To determine the accuracy of the model, the identified NARMAX model output ($\hat{y}(n)$) was compared with the noise-free output of the continuous-time simulation ($y(n)$) by computing the percent of variance accounted for ($\%VAF$) as

$$\%VAF = \left(1 - \frac{\sum_{n=1}^{N}(y(n) - \hat{y}(n))^2}{\sum_{n=1}^{N}(y(n))^2}\right) \times 100. \quad (22)$$

V. RESULTS

A. TIV NARMAX model

When modelling TV systems it is important to determine whether a time-invariant model is needed. To do so a TIV NARMAX model was estimated from the input-output data of Fig. 2 by using only one basis function that was time-invariant (i.e., a constant term). After estimating the model parameters we performed a free-run of the model without noise (contrary to the 1-step prediction used in the identification step) with the same input used for identification. The predicted TIV torque did not have the time-dependent properties of the original (observed) torque. Moreover, the $\%VAF$ was less than 60 $\%$ confirming that a TV model was needed.

B. TV NARMAX model

We then performed a TV NARMAX identification using a basis set consisting of Walsh functions from order 0 to 15, which are well suited to represent piecewise constant functions [10]. After performing the best basis selection, we ended up using from 1 to 13 basis functions to represent each coefficient. Fig. 3. shows how the parameters K (elasticity) and g_r (reflex gain) used in the simulation (black line) changed with time. The parameters estimated by the new identification procedure are superimposed on the original ones (red dotted line).The estimated parameters followed the original parameters very closely, showing only small discrepancies at the sharp transitions. Note that the other identified parameters (i.e., $\hat{I}$, $\hat{B}$, $\hat{\varsigma}$ and $\hat{\omega_n}$) were also found to vary with time, but they showed only small variations around a fixed point that was near the true value.

After identifying the model we conducted a free-run of the TV NARMAX model using the same input as the used for identification, in this opportunity we obtained $98.91\%VAF$. Next, we performed cross-validation, using a new input (with the same characteristics), but driving the TV NARMAX model with the parameters identified before. Fig. 4. shows the input (upper panel) and output (lower panel, black line) of the continuous-time system along with the output of the TV NARMAX model (lower panel, red line), as observed, there is almost no difference between the output of the continuous and discrete-time models ($98.69\%VAF$)

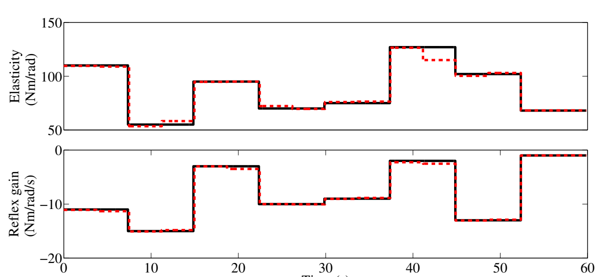

Fig. 3. Simulated (black line) and identified (dotted red line) parameters

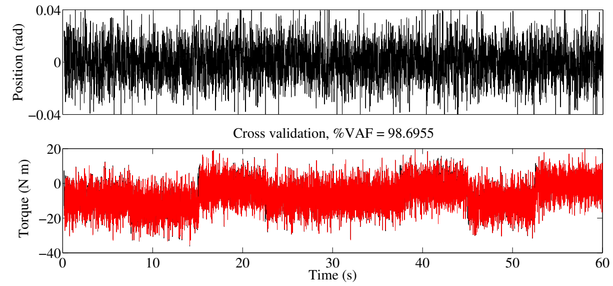

Fig. 4. Position and torque. Simulated (black line) and identified (red line)

VI. CONCLUSIONS

We have presented a new TV ankle stiffness identification method which takes advantage of the fact that a TV-NARMAX model can be approximated by a TIV-NARMAX model by using a set of basis functions to represent the TV parameters. We have also developed a method that uses OLS (or in this case FRA, a modified version of OLS) to select the most relevant basis functions. This prevents over-parametrizing the model and should make the identification more robust. Simulations showed that the proposed method is able to correctly identify the time course of the parameters even when the output is contaminated by noise. The new method shows great promise for use as a tool to examine the time-varying properties of joint stiffness. Another important aspect of the method is that as the uncontrolled input is modelled during the identification procedure, is not necessary to make any assumptions about the nature of noise.

REFERENCES

[1] R. E. Kearney and I. W. Hunter, *Crit. Rev. Biomed. Eng*, vol. 18, no. 1, pp. 55 – 87, 1990.
[2] R. E. Kearney, R. B. Stein, and L. Parameswaran, *IEEE Trans. on Biomedical Eng*, vol. 44, no. 6, pp. 493 – 504, 1997.
[3] S. L. Kukreja, H. L. Galiana, and R. E. Kearney, *IEEE Trans. on Biomedical Eng*, vol. 50, no. 1, pp. 70 – 81, 2003.
[4] M. M. Mirbagheri, H. Barbeau, and R. E. Kearney, *Exp Brain Res*, vol. 135, no. 4, pp. 423 – 436, 2000.
[5] D. Ludvig, T. S. Visser, H. Giesbrecht, and R. E. Kearney, *IEEE Trans. on Biomedical Eng*, vol. 58, no. 6, pp. 1715 – 1723, 2011.
[6] V. Z. Marmarelis, *Advances methods of physiological system modelling*. Biomedical Simulations Resource, 1987, vol. I, ch. Recent Advances in nonlinear and nonstationary analysis, pp. 323 – 336.
[7] M. Korenberg, S. A. Billings, Y. P. Liu, and P. J. McIlroy, *Int. J. of Control*, vol. 48, no. 1, pp. 193 – 210, 1988.
[8] M. Lortie and R. E. Kearney, *Ann. Biomed. Eng*, vol. 29, pp. 619 – 635, 2001.
[9] K. Li, J.-X. Peng, and G. W. Irwin, *IEEE Trans. on Automatic Control*, vol. 50, no. 8, pp. 1211 – 1216, 2005.
[10] J. L. Walsh, *American J. of Mathematics*, vol. 45, no. 1, pp. 5– 24, 1923.