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Noise Reduction using Anisotropic Diffusion Filter in Inverse Electrocardiology

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Abstract—**Filtering has been widely used in biomedical signal processing and image processing applications to cancel noise effects in signals recorded from the body. However, it is important to keep the desired characteristics of the physiological signal of interest while suppressing the noise characteristics. In this study, we used anisotropic diffusion filter (ADF) to cancel the noise on the body surface potentials measurements (BSPM) with the goal of improving the corresponding solutions of the inverse problem of electrocardiology (ECG). ADFs have been applied to image processing and they have the advantage of preserving sharp edges while rejecting the noise, thus we have chosen ADFs instead of more conventional filtering techniques. We used unfiltered and filtered BSPMs to estimate the epicardial potential distributions. We compared Tikhonov regularization results when the data included measurement noise and geometric errors. In both cases, filtering of BSPMs using the ADF improved our solutions.**

I. Introduction

Inverse problem of electrocardiology (ECG) is the problem of finding the cardiac electrical sources using the potentials measured on the body surface (BSPMs) [1]. An optimum solution for this problem would provide the clinicians with the ability to detect and diagnose cardiac abnormalities or the opportunity to monitor the effects of drugs on cardiac activity.

However, inverse problems most often are ill-posed, which implies that a small amount of noise would make the problem unstable and the solution would be unreliable. To obtain accurate solutions of the inverse ECG problem, the solutions are regularized by including prior information on the spatial and/or temporal characteristics of the epicardial potential distributions. Some of the regularization methods used in inverse ECG literature include Tikhonov regularization [1], truncated singular value decomposition (TSVD)[2], Bayesian maximum a posteriori (MAP) estimation[3], Kalman filtering [4],[5],[6], *etc*. The success of these regularization methods is directly related to the prior constraints employed, the amount of noise in the data, and the accuracy of the geometric model obtained by segmenting medical images.

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There are several studies that aim to reduce the effects of geometric noise in the inverse problem solutions. To name a few, [7] compared the performances of L-curve, CRESO and zero crossing methods in determining the optimum regularization parameter for the zero'th order Tikhonov regularization in the presence of geometric errors. Greensite provided a theoretical formulation to incorporate the effects of geometric noise in the forward transfer matrix [8]. Shou *et.al.* used truncated total least squares (TTLS) algorithm to reduce the effects of geometric errors [9]. In a study by our group, effects of geometric errors were reduced by modeling geometric error as an additional noise parameter [6]. Filtering of additive measurement noise from the BSPMs to enhance the inverse solutions, however, has not attracted much interest. Filtering of ECG signals, on the other hand, is a well-studied subject in the literature. De-noising by Wavelet transformation [10], finite impulse response (FIR) filtering [11], infinite impulse response filtering [12], adaptive filtering [13] are among the most widely used ECG filtering techniques. In this study we have explored the use of anisotropic diffusion filter (ADF) to reduce the noise in the BSPMs that we use in inverse ECG problem solution.ADF was first introduced by Perona and Malik [14] and it has the advantage of filtering the signals or images while keeping, or enhancing sharp features, such as edges. It has been used in MRI image filtering [15], image processing and computer vision[16].It is well-known that the level of noise in the BSPMs significantly affects the accuracy of the solutions [6], thus we aim to improve our inverse solutions using these filtered BSPMs.

II. Methods

A. Problem Definition

When the cardiac sources are represented in terms of epicardial potential distributions (EPDs), the BSPMs are linearly related to these EPDs:

$$\mathbf{y}(i) = \mathbf{A}\mathbf{x}(i) + \mathbf{n}(i) \qquad i = 1, \dots, T \tag{1}$$

where $i$is the time instant,$\mathbf{y}(i) \in \mathbf{R}^{\mathrm{M}\times 1}$ is the vector of BSPMs,$\mathbf{x}(i) \in \mathbf{R}^{\mathrm{N}\times 1}$ is the vector of EPDs,$\mathbf{A} \in \mathbf{R}^{\mathrm{M}\times \mathrm{N}}$ is the forward transfer matrix, and $\mathbf{n}(i) \in \mathbf{R}^{\mathrm{M}\times 1}$ denotes the measurement noise. The aim is to estimate $\mathbf{x}(i)$.

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B. *Anisotropic Diffusion Filter*

When traditional low-pass filtering techniques such as Gaussian filters or linear diffusion are used, the cost of noise suppression is the blurring of edges in the signal. In the nonlinear anisotropic diffusion filtering, this blurring effect is reduced; the signal quality is enhanced while the details such as the signal edges are retained [14]. In this study, we use the nonlinear diffusion approach to signal filtering proposed by Perona and Malik [14]. In their method, the diffusion equation can be written as:

$$\frac{\partial \mathrm{y}}{\partial v} = div(\mathrm{c}(\mathrm{t}, v)\nabla \mathrm{y}) \qquad (2)$$

where *div* is the divergence operator, ∇ is the gradient operator, v is the variance, and$\mathrm{c}(\mathrm{t}, v)$ is the diffusion coefficient, which should be chosen so that intraregion smoothing is favored over interregion smoothing. This can be achieved by choosing $\mathrm{c}(\mathrm{t}, v)$an appropriate function of the image gradient. Perona and Malik proposed various $\mathrm{c}(\mathrm{t}, v)$ choices; for example:

$$\mathrm{c}(\mathrm{t}, v) = \frac{1}{1+(\frac{\nabla \mathrm{y}(\mathrm{t},v)}{\mathrm{K}})^2} \qquad (3)$$

This diffusion coefficient determines the properties of diffusion. When the data gradient is small, the denominator is close to one; therefore smoothing amount is at its maximum. On the other hand, when the data gradient is large (e.g. at the edges),the denominator is large and the diffusion coefficient is small, thus less smoothing is performed. The constant K is chosen by the user and fixed during the application.

C. *Tikhonov Regularization*

In this approach, the solution at each time instant is found by minimizing the cost function:

$$\|\mathbf{y}(i) - \mathbf{A}\mathbf{x}(i)\|_2^2 + \lambda^2\|\mathbf{R}\mathbf{x}(i)\|_2^2 \qquad (4)$$

where $\mathbf{R}$ is the spatial regularization matrix and λ is the regularization parameter. The corresponding solution is:

$$\hat{\mathbf{x}}_\lambda(i) = (\mathbf{A}^T\mathbf{A} + \lambda^2\mathbf{R}^T\mathbf{R})^{-1}\mathbf{A}^T\mathbf{y}(i) \qquad (5)$$

Filtered-Tikhonov regularization results are obtained by first applying the ADF to the BSPMs, $\mathbf{y}(i)$, then using the filtered BSPMs, $\mathbf{y}_{filt}(i)$, to find the epicardial potentials.

III. Results

Data used in this study were simulated from epicardial potential measurements from a ventricularly paced canine heart. These epicardial potentials were recorded at the University of Utah Nora Eccles Harrison Cardiovascular Research and Training Institute (CVRTI) by R.S. MacLeod and his co-workers[17]. The number of epicardial leads is 490, and the number of simulated BSPM leads is 771.Sampling rate of the signal is 1000 Hz. The forward problem was solved using the boundary element method [18].

The BSPMs were simulated by multiplying the epicardial potentials by the calculated forward transfer matrix, and then adding Gaussian distributed noise. Torso geometry used for simulating the BSPMs included the heart, the lung and the torso surfaces. A homogeneous model was used to obtain the forward matrix used in the inverse calculations. The results were assessed numerically using the correlation coefficient (CC) and the relative difference measurement star (RDMS) measures, and qualitatively by comparing iso-potential maps using the map3d visualization software [19].

First we varied the SNR value of the BSPMs and compared the corresponding Tikhonov regularized solutions, before and after ADF application. The average and standard deviation values of CC and RDMS over time are presented in Tables I and II for varying SNR values. For visual inspection, average CC values are also shown in Fig. 1.

These results show that when the SNR is low, filtering the BSPMs using ADF before solving the inverse problem enhances the reconstructed epicardial maps compared to using the unfiltered data. However, as the SNR of the BSPMs increases, the solutions with the filtered and unfiltered data produce similar results.

TABLE I
THE AVERAGE AND STANDARD DEVIATION VALUES OF CC OVER TIME FOR DIFFERENT SNR VALUES OF THE UNFILTERED DATA.

	Tikhonov, with filter	Tikhonov, no filter
5 dB data	0.62± 0.32	0.52± 0.30
10 dB data	0.66± 0.28	0.58± 0.30
15 dB data	0.72± 0.23	0.61±0.27
20 dB data	0.73± 0.21	0.66± 0.25
25dB data	0.73± 0.21	0.70± 0.24
30 dB data	0.74± 0.20	0.74± 0.22

TABLE II
THE AVERAGE AND STANDARD DEVIATION VALUES OF RDMS OVER TIME FOR DIFFERENT SNR VALUES OF THE UNFILTERED DATA.

	Tikhonov, with filter	Tikhonov no filter
5 dB data	0.77± 0.34	0.88± 0.37
10 dB data	0.73± 0.30	0.82± 0.35
15 dB data	0.67± 0.28	0.78± 0.34
20 dB data	0.66± 0.28	0.72± 0.30
25 dB data	0.65± 0.27	0.68± 0.29
30 dB data	0.65± 0.27	0.65 ± 0.27

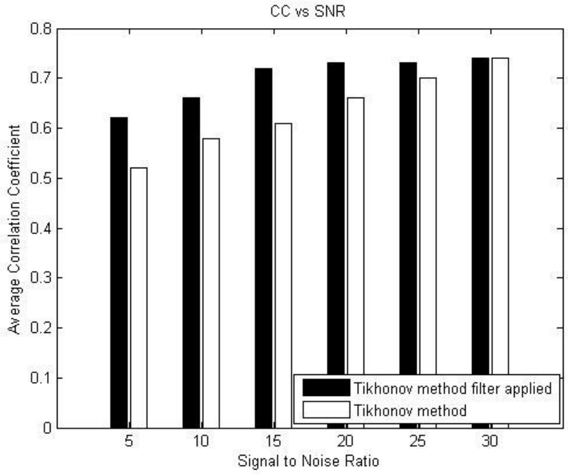

Fig. 1. Comparison of average CC values over time for different SNR values of the unfiltered data. This plot provides a visual representation of the results in Table I.

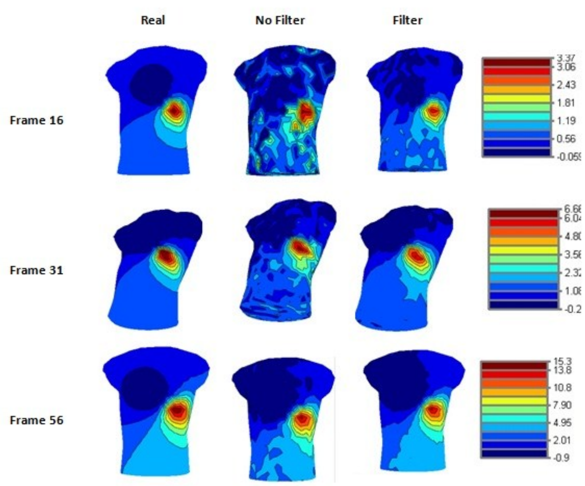

Fig.2. Real (noiseless), noisy-unfiltered, and filtered BSPM maps at different three different time instants. Noise was added to the real BSPMs at 15 dB SNR.

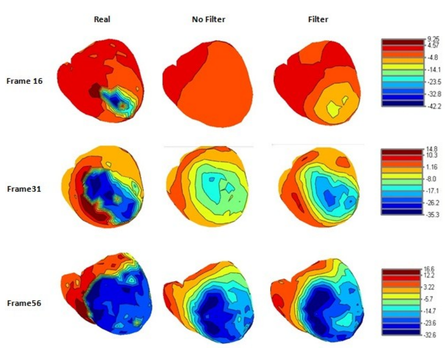

Fig. 3. Epicardial potential distributions corresponding to the BSPMs in Fig. 2. First column of maps correspond to the real epicardial maps, second column maps are the Tikhonov regularized solutions with unfiltered data and the last column maps are the solutions after ADF has been applied.

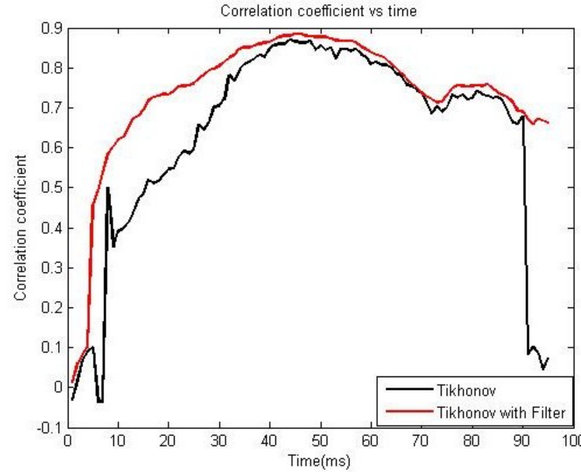

Fig. 4. Correlation coefficient values with respect to time, with and without filtering for the data at 15 dB SNR.

To examine our results more carefully, we examined the epicardial potential distributions over time for the data at 15 dB SNR, which represents a moderate amount of measurement noise. Fig. 2 shows the noiseless, noisy and filtered BSPMs at three different time instants, and Fig. 3 plots the corresponding epicardial potential maps. Note that the 'real' epicardial potential maps are the true value used in the simulations. The CC values between the reconstructed and real epicardial potential distributions at all time instants are plotted with respect to time in Fig. 4. From these results, we observe that during the earlier times of the QRS region where the depolarized parts of the heart are restricted to a smaller region, filtered data produces better reconstructions; the initial stimulation region can be detected earlier with the filtered data than the unfiltered data.

As a final study, we added geometric noise to our forward models by either shifting the position of the heart (in the +y direction) or scaling the size of the heart. In addition, the data include measurement noise at 5 dB and 15 dB. The averages and the standard deviations of the CC values over time for varying amounts of scaling error and shift error are presented in Tables III and IV, respectively. Similar to our results with the additive noise, filtering of the BSPMs using ADF also enhances the reconstructed epicardial potential maps when there are geometric errors in the forward model.

IV. CONCLUSIONS

Here we proposed to use anisotropic diffusion filter to suppress noise in the BSPMs with the goal of enhancing the reconstructed epicardial potential distributions. We applied the ADF to the BSPMs in the temporal dimension. Our efforts continue to implement a spatio-temporal ADF, which we believe would be more effective than the simple one dimensional filter in enhancing the BSPMs.

TABLE III

THE AVERAGE AND STANDARD DEVIATION VALUES OF CC OVER TIME FOR DIFFERENT AMOUNTS OF SCALING ERROR IN THE FORWARD MODEL.

	Scale amount	Tikhonov, with filter	Tikhonov, no filter
Data at 5 dB	%60	0.43±0.32	0.37± 0.30
	%90	0.58±0.28	0.48± 0.30
	%140	0.62 ± 0.23	0.54± 0.27
Data at 15 dB	%60	0.59 ±0.31	0.48± 0.30
	%90	0.68 ±0.24	0.60± 0.27
	%140	0.69±0.18	0.63± 0.22

TABLE IV

THE AVERAGE AND STANDARD DEVIATION VALUES OF CC OVER TIME FOR DIFFERENT AMOUNTS OF SHIFT ERROR IN THE FORWARD MODEL.

	Shift amount	Tikhonov, with filter	Tikhonov, no filter
Data at 5 dB	+6y	0.61±0.20	0.49±0.30
	+10y	0.58±0.28	0.48±0.30
	+15y	0.54±0.23	0.45±0.24
Data at 15 dB	+6y	0.66±0.31	0.61±0.33
	+10y	0.62±0.24	0.57±0.27
	+15y	0.58±0.24	0.54±0.26

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