34th Annual International Conference of the IEEE EMBS
San Diego, California USA, 28 August - 1 September, 2012

Measurement of the Thermal Relaxation Time in Agar-gelled Water

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Abstract— **In this study, we presented an experiment to obtain the thermal relaxation time which is necessary to model heat conduction by the hyperbolic heat equation. This experiment was evaluated by finite element simulation to acquire reliably this parameter for biological tissue. Besides that, we measured the thermal relaxation time of agar-gelled water with 2% of concentration at 25^oC. The average value of thermal relaxation time for the gel was 7.9630s with standard deviation of 1.4562.**

I. INTRODUCTION

Thermal ablation is widely used in medical procedures and, among the ablation techniques, radiofrequency ablation is the most commonly used. Such procedures are guided by imaging equipment and supported by impedance or temperature measurements. However, none of those technologies provides accurate data to physicians perform safely the procedure. Therefore, the physicians do not know exactly how much tissue is being heated. In this way the health problem can still remain if it heats less than necessary, or affecting other tissues with a higher than desired heating.

Nowadays, the heat conduction is modeled by the Fourier's heat conduction law. Combining Fourier's law equation with energy balance equation, we will have the classical parabolic heat conduction equation:

$$\frac{dT}{dt} = \alpha \bigtriangledown^2 T \tag{1}$$

where α is the thermal diffusivity coefficient of a material.

This classical theory is based on un-physical property that heat propagates at infinite speed. Therefore, a thermal disturbance at a point in a medium is measured at all points in that medium instantaneously. However, heat conduction is due to microscopic motion and collisions of electrons and phonons and then Fourier condition of propagation speed cannot be sustained.

For high thermal conduction material, where the thermal relaxation time ranges from 10^{-8}s to 10^{-12}s, the heat wave has high propagation speed and the thermal propagation in these materials can be well represented by Fourier model. In contrast, for low thermal conduction material, like human body, the propagation velocity is lower and this un-physical property becomes visible.

Several studies tried to model the heat conduction more precisely [6], [11], [7], [8], [9], [10], [14]. The most acceptable heat conduction model was proposed by Vernotte [15] and Cattaneo [4]. Their heat equation is given by:

$$\tau \frac{d^2T}{d^2t} + \frac{dT}{dt} = \alpha \bigtriangledown^2 T \tag{2}$$

This equation is known as the hyperbolic heat equation, where τ is a coefficient called thermal relaxation time. The term $\frac{d^2T}{dt^2}$ included represents the wave damping and the term $\frac{dT}{dt}$ accounts for wave propagation. The hyperbolic equation reduces to parabolic equation when the thermal relaxation time goes to zero, i.e. in steady-state conditions.

Some studies analyzed the influence of thermal relaxation time in one specific area [7], [8], [9], [14], [1], [13], [12], [2], but, in most of them, there is not a single thermal relaxation time value been used. This occurs because there are not reliable values of thermal relaxation time in literature [11].

This study aimed to propose and to evaluate a method in order to achieve reliable thermal relaxation time measurements. We studied and analyzed the method using finite element simulations. Once the experiment was adjusted, we measured the thermal relaxation time of the agar-gelled water which is a commonly used material to simulate biological tissues.

II. THEORETICAL SOLUTION

After evaluation of the methods currently been used to acquire thermal relaxation time of materials, we decided to adapt the methodology proposed by Roetzel [11]. This methodology explores the main difference between the equations of Fourier and non-Fourier (hyperbolic), which is the wave speed of propagation.

In this methodology, a periodic signal is used as heat input of the system to exploit the speed of propagation through a material of semi-infinite geometry. The choice of this type of geometry is a way to ensure that the periodic signal will be attenuated in several wavelengths. In addition, the material was considered linear and homogeneous, in order to facilitate the analysis of these parameters.

As the input signal, a periodic signal was chosen because only this type of signal maintains the effects of hyperbolic heat conduction for an extended period [11]. Therefore, we used a cosine as the input signal, propagating in the semi-infinite medium.

Hence, to resolve equation 2, we input a exponential complex. Therefore, in rectangular coordinate system, the temperature signal can be expressed by:

$$T_{(x,y,z,t)} = T_{a(x,y,z)} e^{j(\omega t - \beta_{(x,y,z)})} \tag{3}$$

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where T_a is the amplitude temperature (^{o}C), i.e. the measured temperature (T) minus initial temperature of the material (T_m), ω is the angular frequency (rad/s), t is the time (s) and β is the phase (rad).

Using equation 3 in the hyperbolic equation 2, it becomes:

$$\bigtriangledown^2 T_{(x,y,z)} + \left(\sqrt{\frac{\tau\omega^2 - j\omega}{\alpha}}\right)^2 T_{(x,y,z)} = 0 \quad (4)$$

Equation 4 is the wave equation (Helmholtz equation) [3], where the propagation constant is the term $\sqrt{\frac{\tau\omega^2 - j\omega}{\alpha}}$.

Considering only one dimension, the equation becomes:

$$\frac{d^2 T_x}{dx^2} + \left(\sqrt{\frac{\tau\omega^2 - j\omega}{\alpha}}\right)^2 T_x = 0 \quad (5)$$

where T_x depends only on dimension x.

The general solution of equation 5 is:

$$T_x = \{C_1 e^{-x\sqrt{\frac{\tau\omega^2 - j\omega}{\alpha}}} + C_2 e^{x\sqrt{\frac{\tau\omega^2 - j\omega}{\alpha}}}\} e^{j\omega t} \quad (6)$$

To solve equation 6, we considered the boundary conditions, discussed before. We choose the cosine as input periodic signal propagating in a semi-infinite medium. Then, the boundary conditions were:

$$T_{x(x \to +\infty, t)} = 0 \quad (7)$$

$$T_{x(x=0,t)} = T_{ax} \cos(\omega t) \quad (8)$$

where T_{ax} is the amplitude temperature (^oC) in only x dimension.

With conditions 7 and 8, the solution of equation 6 is:

$$T_x = T_{ax} e^{-x\sqrt{\frac{\tau\omega^2 - j\omega}{\alpha}}} \cos(\omega t) \quad (9)$$

Taking only the real part of equation 9:

$$Re\{T_x\} = T_{ax} e^{-x\sqrt{\frac{\omega}{2\alpha}\mu}} \cos\left(\omega t + x\sqrt{\frac{\omega}{2\alpha}\frac{1}{\mu}}\right) \quad (10)$$

where $\mu = \sqrt{(\tau\omega)^2 + 1} - \tau\omega$.

The method used to obtain the thermal relaxation time is based on the temperature signal of two fixed point in the same material. Calculating the amplitude ratio and the phase difference of the temperature wave measured in these two points, the following equations are obtained:

$$A = \frac{T_{ax} e^{-x_1\sqrt{\frac{\omega}{2\alpha}\mu}}}{T_{ax} e^{-x_2\sqrt{\frac{\omega}{2\alpha}\mu}}} = e^{(x_2 - x_1)\sqrt{\frac{\omega}{2\alpha}\mu}} \quad (11)$$

and

$$P = x_2\sqrt{\frac{\omega}{2\alpha}\frac{1}{\mu}} - x_1\sqrt{\frac{\omega}{2\alpha}\frac{1}{\mu}} = (x_2 - x_1)\sqrt{\frac{\omega}{2\alpha}\frac{1}{\mu}} \quad (12)$$

where x_1 and x_2 are the positions of the two points (m). Isolating μ of equations 11 and 12, we will have:

$$\mu = \frac{\ln A}{P} \quad (13)$$

$$\mu = \left(\frac{\ln A}{x_2 - x_1}\right)^2 \left(\frac{2\alpha}{\omega}\right) \quad (14)$$

$$\mu = \left(\frac{x_2 - x_1}{P}\right)^2 \left(\frac{\omega}{2\alpha}\right) \quad (15)$$

Since $\mu = \sqrt{(\tau\omega)^2 + 1} - \tau\omega$, we have three different forms to obtain the thermal relaxation time. Note that the equations 14 and 15 only can be used if the thermal diffusivity of the material and the distance between the two points are known. Therefore, we used the equation 13 and we obtained:

$$\tau = \frac{1 - \mu^2}{2\omega\mu} = \frac{P^2 - (\ln A)^2}{2\omega P \ln A} \quad (16)$$

If the thermal diffusivity is not known, it can be obtained, after using the equation 13, by the following equations:

$$\alpha = \left(\frac{x_2 - x_1}{\ln A}\right)^2 \frac{\omega\mu}{2} = \left(\frac{x_2 - x_1}{P}\right)^2 \frac{\omega}{2\mu} = \frac{\omega\,(x_2 - x_1)^2}{2P\ln A} \quad (17)$$

III. EXPERIMENTAL APPARATUS

With the mathematical method demonstrated, it was possible to build a thermal system that uses the proposed method to obtain a reliable value of the thermal relaxation time of the sample. Therefore, this system needed to reproduce the conditions established to solve hyperbolic equation, i.e. cosine as input signal in a semi-infinite, homogeneous and linear material.

We constructed an isolated thermal box, inspired on the experiment proposed by Roezel in [11], as shown in two dimensions on figure 1. It is made of stainless steel (1, 2, 4) to be resistant and, internally, it has an acrylic parallelepiped (3), separated from the steel plates by polyurethane (2). Inside this acrylic parallelepiped, it is placed a sample of material being tested (7). Thus, we wanted to mechanically and thermally insulate the material to be tested in the external environment.

At the bottom of acrylic parallelepiped, there is a copper plate to improve heat transfer between the material sample and the peltier device (5), which is the component that generates heat signal. To function properly, the peltier needs a stable temperature in one side. Thus, at the bottom of peltier, it was placed a hollow steel parallelepiped (4) which serves as a container for fluid to establish the temperature.

Through the isolated thermal box, there are 7 holes (6), where thermocouples are placed for measurement of thermal waves. The signals from the thermocouples are the signals to be worked out by the method proposed.

To maintain the correct operation of the isolated thermal box, an auxiliary system was organized. A simple scheme of the auxiliary system proposed is shown in figure 2.

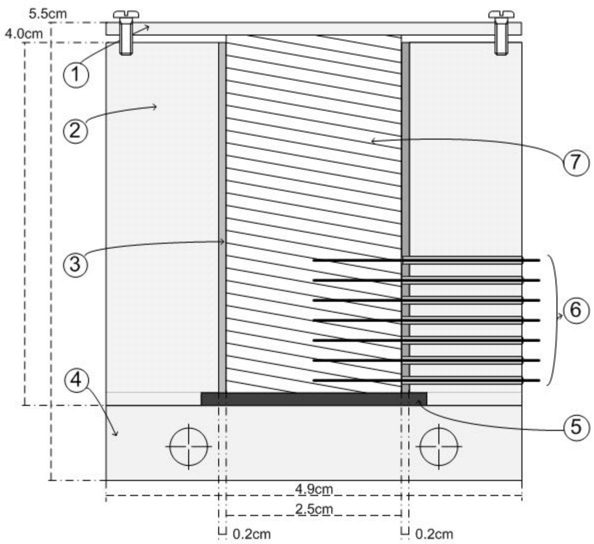

Fig. 1. Isolated thermal box design.

The two thermal baths (a), in figure 2, had the function to establish through water the temperature of two parts of the insulated box. The first thermal bath kept the temperature inside the hollow steel parallelepiped to keep the peltier working properly and the second thermal bath kept the environmental temperature of the system. Thus, the isolated thermal box was covered with a plastic bag and inserted into the water of the second thermal bath. The first thermal bath maintained its temperature at 18^oC and the second was kept at 25^oC.

The collection of the temperature signal was performed by the thermocouples, positioned in the holes of the insulated box. These thermocouples are type K and have 0.1^0C accuracy. The thermocouples were connected on the acquisition module (b), which is composed by NI 9213 module and CompactDAQ module, both from National Instruments. The temperatures collected by the thermocouples were sent to the NI 9213 module. The NI 9213 module was programmed to scan a sample per second ($1Hz$) with a precision of 16 bits/sample. This module was attached in a CompactDAQ module, which was responsible to protocol the digital data and to send them via USB to the computer (c).

In the computer, data acquisition software was developed to extract the digital data from NI CompactDAQ protocol and to store them. The recorded data were then passed to a Matlab program (The MathWorks, Inc), an also computer software, for the method calculation. The Matlab program performed the data regression and the amplitude and phase of the signal extraction. With the amplitude and phase of the signal, it calculated the thermal relaxation time and thermal diffusivity through the equations 16 and 17 respectively.

The data acquisition software was also responsible for generating the heat wave at the terminals of the peltier device. For this purpose, it has implemented a digital PID controller (Proportional Integral Derivative controller). Therefore, it was possible to minimize the occurrence of distortions and errors in signal generation, improving the quality of acquired data.

The output data of the PID controller were sent to the peltier device by the power amplifier module (d). The power amplifier module had a AT90USB128 microcontroller (Atmel Corporate) that received the digital signal through USB and converted it to analog. With operational amplifiers, power amplifier module conditioned the signal with high electrical current (approximately 2A) to be sent to the peltier device.

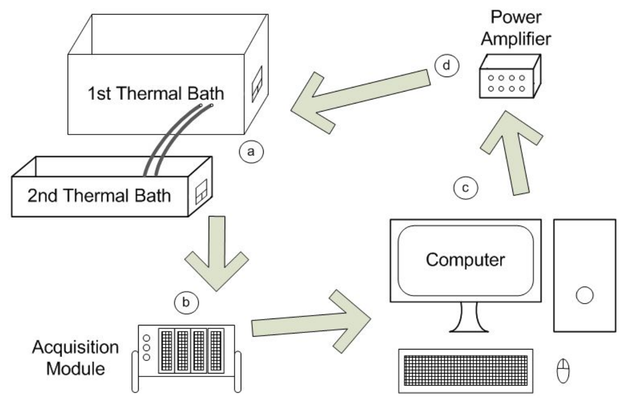

Fig. 2. System overview.

In order to test the apparatus, we used a sample of agar-gelled water with 2% of concentration, which has a known thermal diffusivity, as the material to be searched. Therefore, it was possible to verify if the thermal relaxation time measured presented reliable results, after comparing the thermal diffusivity known and the thermal diffusivity calculated by the method.

IV. METHOD ANALYSIS

To analyze the size of the insulated box and, consequently, the boundary conditions, we simulated the heat system in finite elements. Because the insulated box is symmetric, its structure was designed in 2D on Comsol Multiphysics (Comsol). It was designed with one central material, consisted of agar-gelled water, two materials around the core material, consisted of polyurethane, and one thin material on the top, consisted of stainless steel. The size of each material was taken from the original insulated box.

The heat system was simulated with the heat PDE (partial differential equations). The thermal diffusivity values used for polyurethane plates and stainless steel were $6.574 \cdot 10^{-7} m^2/s$ [5] and $4.2 \cdot 10^{-6} m^2/s$ [5] respectively. On the tested material place, we used $1.4 \cdot 10^{-7} m^2/s$ [5] for the thermal diffusivity and $0s$ for the thermal relaxation time. On the simulation, we used a cosine thermal wave of 5^oC amplitude and 10 minutes period as the peltier signal.

The data collected on the simulation were processed by the Matlab software. As result, the average of the thermal relaxation time was $0.12s$ and its standard deviation was 0.30. In another simulation, we exchanged the thermal relaxation time coefficient in the simulation as $40s$ to see if a thermal relaxation time of that order could affect the boundary conditions. The result obtained was $40.8s$ with standard deviation of 0.7. The thermal diffusivity results presented almost the same aspects of the thermal relaxation time results on the simulation.

Thus, we guarantee that the boundary conditions of experimental heat system were the same of that proposed by the theoretical solution if we use a cosine of 5^oC amplitude and 10 minutes period as the input thermal signal. This guarantee is valid even so the thermal relaxation time of the material varies from $0s$ to $40s$.

V. RESULTS

After examining the proposed method and the experiment, we established a protocol to conduct the experiment sessions. In this protocol, which was tested and modified until its adequacy, a session consisted of two hours of continuous operation of the experiment. Therefore, the sample received a total of 12 periods of the cosine heat wave and rested for 30 minutes.

In this protocol, the first 30 minutes of experiment were retained from the analysis to stabilize the cosine signal in the sample. Thus, for each session of the experiment, we got 9 periods of the cosine signal in each of the thermocouples. In order to lessen the influence of noise in signals, regression was performed on the data and it was excluded from the analysis those signs that the regression errors were above 3%. Thus, it was possible to ensure the quality of thermal waves for the extraction of reliable thermal coefficients.

The samples of water-agar, were performed 13 sessions of the experiment, using two different samples. The objectives of these samples were to evaluate the experiment, because its thermal diffusivity is known, and to get the thermal relaxation time of this material widely used to simulate biological tissues.

After performing the experiment on that protocol, the results of thermal relaxation time, which had up to 3% of error on the regression of the data collected, were removed and 105 thermal relaxation times were obtained. The average of the thermal relaxation time obtained was $7.9630s$ and its standard deviation was 1.4562.

To verify if these results were consistent, the same procedure was done to obtain the thermal diffusivity value of the agar-gelled water. The average of the thermal diffusivity obtained was $1.3951 \cdot 10^{-7} m^2/s$ and its standard deviation was $1.9313 \cdot 10^{-9}$.

The most acceptable value for the water thermal diffusivity at 25^oC is $1.4 \cdot 10^{-7} m^2/s$. When we compare this value to the average obtained on this research, we can see that they are very close. Furthermore, this analysis indicates that the results of thermal relaxation time obtained with this methodology are quite reliable. Therefore, the average value of thermal relaxation time for agar-gelled water with 2% of concentration at 25^oC should be $7.9630s$.

VI. CONCLUSIONS

With this research, we expect to give to the scientific society a reliable methodology and experiment to obtain results of thermal relaxation time. It is expected that the results presented strengthen the need of further studies about the usability of the hyperbolic heat equation in modeling the heat propagation. Furthermore, a reliable thermal relaxation time for agar-gelled water at 25^oC is shown.

The experiment, presented in this study, has obtained $7.9630s$ for the thermal relaxation time of the agar-gelled water with 2% of concentration at 25^oC. Besides that, the thermal diffusivity obtained was $1.3951 \cdot 10^{-7} m^2/s$.

In relation to the ablation or laser procedures, because the agar-gelled water is a commonly used material to simulate biological tissues, it can be investigated if modeling those procedures with hyperbolic equation makes significant difference on results. Even the influence of the thermal relaxation time is already known, nobody had an exact value of this coefficient. In this study, we contribute to evaluate more exactly the influence of the thermal relaxation time on heat conduction.

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