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Brain Dynamics of Mathematical Problem Solving*

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***Abstract*— The purpose of this study is to examine brain activities of participants solving mental math problems. The research investigated how problem difficulty affected the subjects' responses and electroencephalogram (EEG) in different brain regions. In general, it was found that solution latencies (SL) to the math problems increased with difficulty. The EEG results showed that across subjects, the right-central beta, left-parietal theta, left-occipital theta and alpha, right-parietal alpha and beta, medial-frontal beta and medial central theta power decreased as task difficulty increased. This study further explored the effects of problem-solving performance on the EEG. Slow solvers exhibited greater frontal theta activities in the right hemisphere, whereas an inverse pattern of hemispheric asymmetry was found in fast solvers. Furthermore, analyses of spatio-temporal brain dynamics during problem solving show progressively stronger alpha- and beta-power suppression and theta-power augmentation as subjects were reaching a solution. These findings provide a better understanding of cortical activities mediating math-based problem solving and knowledge acquisition that can ultimately benefit math learning and education.**

I. Introduction

This study explores how individuals solve math problems – both at the behavioral and neurocognitive level. Researchers in the field of problem solving have drawn a distinction between search and insight. In search-based strategies, a solution to a problem is achieved through systematic, analytic evaluation and transformation of problem states. On the other hand, though, a problem may be solved through sudden insight, leading to a phenomenon known as an "Aha!" experience. In such cases, the problem solver often has little awareness of the mental work that led to the solution. The way humans solve problems through insight is an intriguing and important topic. However, there has been relatively little research done in this area, partly due to the difficulty in finding suitable problems. Unlike existing insight research based on anagrams, riddles, and other language puzzles, this study employs math problems that offer the opportunity to manipulate task difficulty in a well-controlled fashion.

This study aims to examine neural dynamics of participants solving intellectually challenging math problems. To do so, this study utilized electroencephalogram (EEG), which reflects brain electrical activity with millisecond temporal resolution. EEG has been widely used to study cognitive processes of the brain. Previously, studies have shown that EEG power in the theta band (4-7 Hz) increases with greater levels of mental effort or cognitive challenge [1][2]. Osaka reported that the peak alpha (around 10 Hz) frequency of the EEG power spectrum increased significantly above resting level while participants performed arithmetic tasks [3]. The frequency shift increased as the difficulty increased. In a related study involving a visual scanning task, Gundel & Wilson found that right parietal alpha and beta activities decreased in inverse relation to task difficulty [4]. At present, it is poorly understood how EEG brain dynamics are modulated with greater demands on insight and search resources engendered by increasing problem difficulty. Further, by studying the relationship between EEG and elements of task performance, such as solution latencies (SLs), it is possible to characterize for the first time variability across individuals in neurocognitive systems recruited to solve math problems.

II. Methods

A. Subjects

Eleven volunteers (9 males, 2 females, age: 15 – 49 years) with normal or corrected-to-normal vision were paid to participate in this math problem-solving experiment, which was approved by the Institutional Review Broad of University of California San Diego. Volunteers did not consume any alcohol, caffeine, or tobacco for 24 hours before their experiments. Volunteers were informed of the experimental procedure and written consent was obtained from each individual prior to the experiment.

B. Experiments

This study adopts an intellectually challenging game that aims to help sharpen students' skills in problem solving, mental math, and patterning. In each trial, four single-digit numbers appeared on a computer screen. Participants were asked to combine the numbers through basic operations (addition, subtraction, division, multiplication) such that the final solution equaled twenty-four. For example, a trial with the numbers 1, 7, 1, 2, could yield the following possible solution: 1+7=8, 2+1=3, 3×8=24. Problems varied in

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difficulty (three levels, level 1 being the easiest and level 3 being the most difficult), but participants were unaware of the difficulty of each problem. Upon reaching a solution, participants pressed a button immediately without taking any time to verify their solutions and then verbally described their solutions to a co-present experimenter. They then pressed another button whenever they were ready to start the next problem. This self-paced experiment lasted one hour.

C. Data collection

1) Behavioral data: Stimulus onsets and participants' button presses were recorded and synchronized with their EEG data.

2) EEG data: EEG measurements provide a non-invasive method for assessing the voltage differences of scalp potentials to examine brain dynamics during math problem solving. In this study, 128-channel EEG data were amplified (BioSemi ActiveTwo EEG system), referenced to the CMS-DRL ground, and sampled at 256 Hz.

D. Data analysis

1) Behavior data: In most of the trials, the subjects arrived at correct solutions. Thus, task performance was mainly characterized by solution latency (SL), which was defined as the time between the stimulus onset and the button press to indicate solution readiness. All incorrectly solved trials were removed from further analysis.

2) EEG data: The acquired EEG signals were first inspected to remove poor-quality EEG channels and trials that were heavily contaminated by movement artifacts. The remaining EEG signals underwent Independent Component Analysis (ICA, [5][6]). ICA is now a widely-used statistical technique to find linear projections of the EEG data that maximize the mutual independences of estimated components, and has been proven as an effective technique to remove EEG artifacts arising from eye blinks, eye movement and muscle activities [7]. EEG signals were analyzed using MATLAB (The Mathworks, Inc.) and the open source toolbox, EEGLAB (Swartz Center for Computational Neuroscience, University of California San Diego, La Jolla, CA; http://www.sccn.ucsd.edu/eeglab).

Fourier transform (STFT) with non-overlapping 1-s Hanning window was then applied to the 2-sec artifact-corrected EEG data immediately before button responses to extract the power spectral density estimates in three frequency bands, including θ (4-7 Hz), α (8-13 Hz) and β (14-30 Hz) over 12 scalp sites (F3, F4, C3, C4, P3, P4, O3, O4, Fz, Cz, Pz and Oz) for the correctly solved trials.

To examine temporal brain dynamics in spectral changes during problem solving, this study employed a visualization tool known as Event-Related Spectral Perturbation (ERSP), proposed by [8]. For each channel, EEG time series during problem solving was transformed into a spectrographic image in a frequency range between 2 and 30 Hz. Spectrographic images were composed into mean ERSP images by converting to log power, averaging across trials, and subtracting the mean log power derived from the 2-s pre-stimulus baseline period of the same trials. Because solution latencies to the mathematical problems varied widely across trials and subjects, a linear time-warping procedure was applied to the power spectral density of each trial to align the duration of each trail to the median latency. For detailed procedure of time-warping, please see the on-line tutorial at http://www.sccn.ucsd.edu/eeglab.

This study also assessed the hemispheric asymmetry of EEG power using the laterality index, which is computed by subtracting the EEG band power in the right hemisphere from the band power in the left hemisphere and dividing by the sum of these two values (LH – RH / LH + RH). The EEG band power was averaged from the data collected during the time between stimulus presentation and solution readiness, indicated by the subject pressing the button. Less EEG power in one hemisphere relative to the other reflects greater relative activation in that hemisphere [9].

4) Statistical analyses: To assess the changes in SL under different performance levels across two groups, Wilcoxon rank sum test was used to assess the statistical significance of the differences of the SL at different performance levels. The level of significance was set at $p < 0.05$.

III. RESULTS

A. Effects of Task Difficulty on Subject Performance

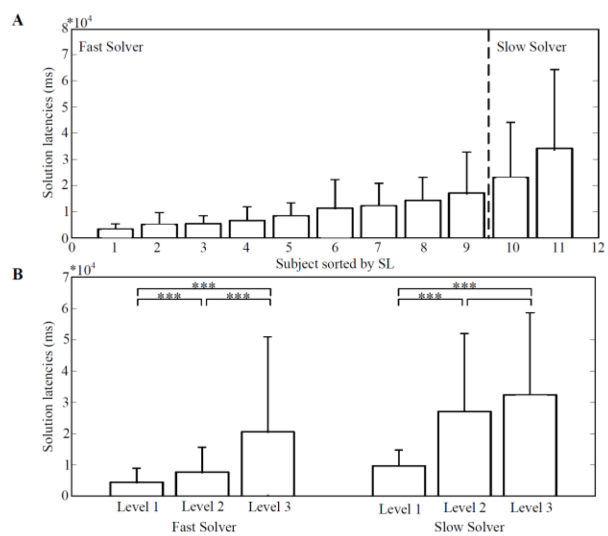

Fig. 1. (A) Distributions of solution latencies to problems at intermediate difficulty. (B) Solution latencies of the problems with increasing difficulty levels. Level 1 is the easiest and level 3 is the most difficult problems. Statistical test results are noted as * $p < 0.05$; ** $p < 0.01$; *** $p < 0.001$.

This study first examined the performance levels of subjects, which was determined by their solution latencies. The quantity of problems solved within an hour varied widely (66-188), but since this was a self-paced experiment, sheer quantity of correct items might not be a good index of task performance. We thus focused on solution latencies to the math problems as a metric for characterizing task performance. Fig. 1 (top panel) shows the distribution of SLs at level 2. Two of the subjects performed considerably more slowly than the remaining subjects (mean SL: 27±24s vs. 7.6±7.9s), especially on more difficult problems (level 3, not shown). In light of the large performance variability, the subjects were divided into two groups, fast and slow problem solvers. We then explored the effects of task difficulty on the SL for each of these groups. Fig. 1 (lower panel) shows the solution latencies of the problems with increasing difficulty levels. As expected, SL in general increased with task difficulty in both subject groups ($p < 0.001$, as determined by Wilcoxon rank sum test, except Level 2 versus Level 3 in slow solvers).

B. EEG Correlates of Task Difficulty

The next focus was the effects of task difficulty on the EEG power in different frequency bands and at different scalp locations. Fig. 2 shows the EEG band power in response to math problems at different difficulty levels. The EEG band power was computed from the data between 2.5 and 0.5 seconds prior to the button press and normalized to the resting spectra by subtracting the logarithmic EEG spectra at resting from the EEG power at performance. Only consistent changes in EEG band power across subjects are plotted in the figure. In most of the subjects, the right-central (C4) beta, left-parietal (P3) theta, right-parietal (P4) alpha and beta, left-occipital (O3) theta and alpha, medial-frontal (Fz) beta and medial central (Cz) theta power decreased as the task difficulty increased.

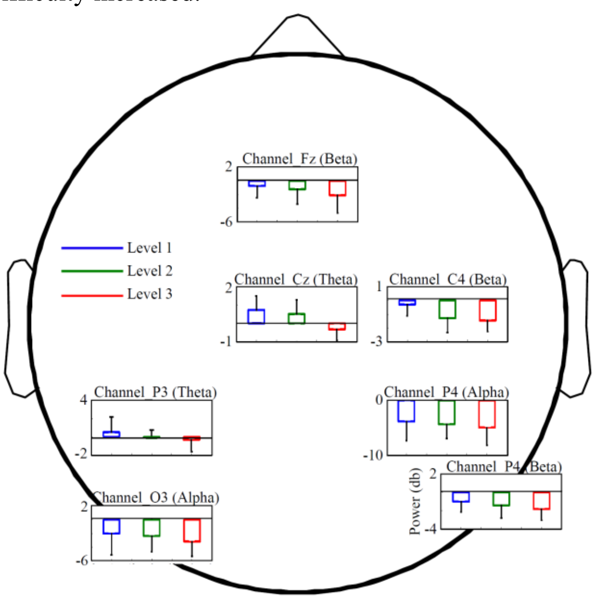

Fig. 2. EEG band power as a function of increasing task difficulty across different scalp locations. Only consistent band power changes across subjects are shown here.

C. EEG Correlates of Task Performance

Next the relationship between task performance and EEG dynamics across different scalp locations was examined. Fig. 3 shows the frontal theta asymmetry of SL-sorted subjects solving math problems with intermediate difficulty. Slow problem solvers exhibited greater frontal theta activities in the right than in the left hemisphere (indicated by the negative laterality index value), while the fast problem solvers exhibited either no lateralization or relatively greater theta power in the left than in the right hemisphere. The two subject groups did not exhibit any appreciable differences in the EEG power in other frequency bands or at scalp locations.

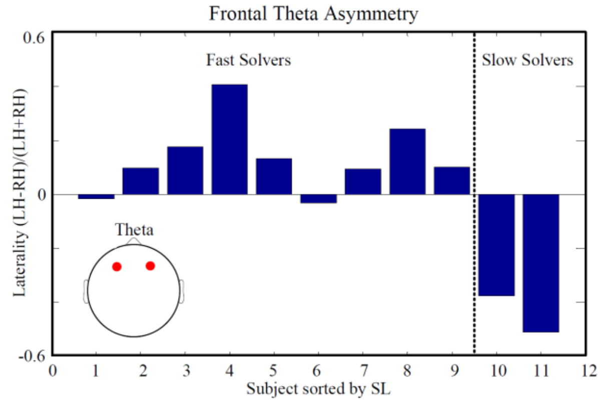

Fig. 3. Ratio of frontal theta power in the left and right hemispheres (LH – RH / LH + RH). Power (in dB) is plotted on the y-axis. Positive values indicate greater power in the LH versus RH; negative values indicate the reverse. Subjects are sorted in ascending solution latency (slow solvers on the right).

D. EEG Dynamics during Mathematical Problem Solving

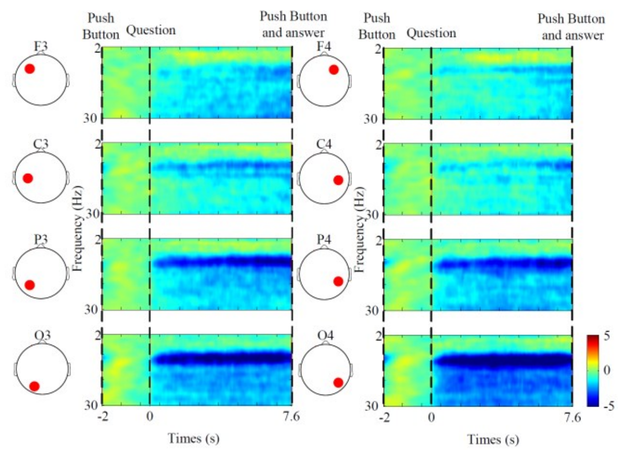

Fig. 4. Event-related spectral activities obtained from eight scalp recording sites between stimulus presentation and solution readiness. Power (in dB) is color coded with increases up to 5 dB in red and decreases down to -5 dB in blue relative to the pre-stimulus baseline. Time is plotted on the x-axis in 1000 ms increments. Frequencies (2 to 30 Hz) are plotted on the y-axis (high to low).

Event-related spectral perturbations (ERSP) were employed to explore the temporal dynamics of the EEG during mathematical problem solving. Fig. 4 shows the averaged ERSPs from the nine fast solvers at eight representative scalp locations for intermediate-level problems (level2). The values used were from the data collected between the times of stimulus presentation and solution readiness. The vertical lines at the right edge of each plot represent the moment of button press, indicating readiness to give a solution. A broadly distributed reduction in alpha and beta activities is visible throughout most of the epoch. Around five seconds before readiness to state a solution, a progressive event-related synchronization (ERS) in the theta frequency range is visible over fronto-central recording sites.

IV. Discussion

In this study, EEG was recorded as healthy adults solved mental math problems. When participants were sorted according to solution latency to problems of intermediate difficulty, the fast problem solvers exhibited greater left-frontal theta activity while the slow problem solvers showed greater right-frontal theta activity. Research on insight problems has implicated the right hemisphere in diffuse attention [10], unconscious processing [11], and broad semantic representation [12][13]. These aspects of cognition are thought to contribute to the restructuring necessary to connect problem elements in such a way that an "Aha!" solution is suddenly achieved. By analogy, the left or right hemispheric dominance of theta ERS across Math24 players may reflect preferences for distinct problem solving styles. Fast problem solvers may have developed efficient search-based strategies, whereas slow solvers may have relied more on insight-based approaches.

In regards to the effects of task difficulty on the topographic EEG power, right parietal alpha and beta activity were found to consistently become attenuated as task difficulty increased -- which may be in part attributed to increased visual scanning of the four numbers to find a solution [4]. Our results were also consistent with Wertheim whose work indicated retinal involvement of oculomotor control as the cause of reduced alpha power [14]. However, it is worth noting that Earle and Pikus have also studied the effects of task difficulty on the EEG alpha activity as individuals performed arithmetic tasks with eyes closed [15]. The study showed that more difficult tasks induced lower alpha power, compared to a simple counting task. Their results suggested that the reduction of alpha power could occur without increasing visual scanning. Our future work will include an additional protocol in which subjects will be instructed to solve mathematical problems with their eyes closed.

Event-related spectral perturbations, a time-frequency analytical method, evaluated averaged dynamic changes in amplitudes of the broad band EEG spectrum as a function of time following the presentations of mathematical problems. ERSP images showed progressively stronger alpha- and beta-power suppression and theta-power augmentation during the course of problem solving, consistent with previous EEG studies in mental arithmetic tasks [16][17]. Studies have also shown that alpha suppression occurred when a task became more demanding and required greater cognitive effort [1][4]. However, ERSPs reported here provide new insights into the detailed spatio-temporal dynamics of spectral changes during problem solving, which was not available in previous studies. For instance, the fact that posterior alpha- and beta-suppressions are detectable from virtually the onset of a problem to the moment of a solution suggests an important role for sustained mental effort in math-based problem solving.

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