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Optimization of Magnetic Neurostimulation Waveforms for Minimum Power Loss

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***Abstract*— Magnetic stimulation is a key tool in experimental brain research and several clinical applications. Whereas coil designs and the spatial field properties have been intensively studied in the literature, the temporal dynamics of the field has received little attention. The available pulse shapes are typically determined by the relatively limited capabilities of commercial stimulation devices instead of efficiency or optimality. Furthermore, magnetic stimulation is relatively inefficient with respect to the required energy compared to other neurostimulation techniques. We therefore analyze and optimize the waveform dynamics with a nonlinear model of a mammalian motor axon for the first time, without any pre-definition of waveform candidates. We implemented an unbiased and stable numerical algorithm using variational calculus in combination with a global optimization method. This approach yields very stable results with comprehensible characteristic properties, such as a first phase which reduces ohmic losses in the subsequent pulse phase. We compare the energy loss of these optimal waveforms with the waveforms generated by existing magnetic stimulation devices.**

I. INTRODUCTION

Neurostimulation techiques have been an essential toolbox in neurophysiology and neuroscience [1]. Electric and magnetic brain stimulation find wide use in clinical applications and basic research studies in both the central and the peripheral nervous system.

Electric stimulation is technically relatively simple, but usually causes high distress if administered nonivasively. In applications where pain is the limiting factor, magnetic stimulation is usually preferred because it causes less activation of (sub)dermal nociceptors. Technically, however, magnetic stimulation requires current pulses in a stimulation coil with amplitudes of several thousand ampères and is very energy inefficient. In early devices, known as monophasic stimulators, the full pulse energy was dissipated as heat. Modern biphasic pulse sources allow a recovery of a substantial fraction of the magnetic energy of the coil after a pulse.

Nevertheless, efficiency is still a limiting aspect in many applications. Furthermore, heating of the coil—which touches the patient—limits the maximum allowed session duration for high stimulation amplitudes and pulse repetition rates [2]–[4]. The pulse source has to provide this lost power compounding the energy inefficiency. Whereas in stationary clinical setups, powerful bulky stimulators are not problematic, home-care or mobile devices for rehabilitation present different requirements. Smaller or even portable units for repetitive stimulation are impossible at the moment due to the immense power demand.

Whereas the influence of coil design on efficiency is extensively studied in the literature [4]–[8], pulse waveforms have been determined only by the available technology.

The role of waveforms and their interaction with neuron dynamics is better established in electric stimulation. Many forward studies have analyzed predefined pulses [9]–[17]. The first systematic optimization of waveforms for electric stimulation dates back to 1946 and promotes the so-called rising exponential pulse [18]. However, from a modern perspective, this approach is based on an overly simple linear leaky integrate-and-fire neuron. The optimality of this result could not be demonstrated either using present nonlinear neuron models [19] or in experiments [12].

In contrast to linear descriptions, nonlinear models of neuron dynamics are generally not invertible. An analytic optimization is therefore usually not possible. The first numerical approach for the optimization of electric stimulation converged on approximately Gaussian waveforms [20].

For magnetic stimulation, there are few experimental studies of different waveforms [21]–[24]. Numerical evaluations cover only a very limited subset of predefined pulse shapes [25]–[28]. Thus, the question of the optimality of magnetic stimulation waveforms remains largely unanswered.

We aim to optimize the waveform of magnetic stimulation for minimum power loss with a nonlinear neuron model and without constraining the pulses shape parameters unnecessarily. In addition, we analyze the unexplored waveform space systematically and compare it to several existing pulse shapes.

II. METHODS

A. Objective

The analysis is based on a nonlinear model of a human motor axon from the literature [29]–[33]. For exploration of the waveform space and for systematic optimization, this model was connected to a global optimization framework.

For the analysis, the induced electric field of several existing stimulation devices (Magstim Rapid, MagVenture MagPro, Neuronetics 2100 CRS, and cTMS [34]) was

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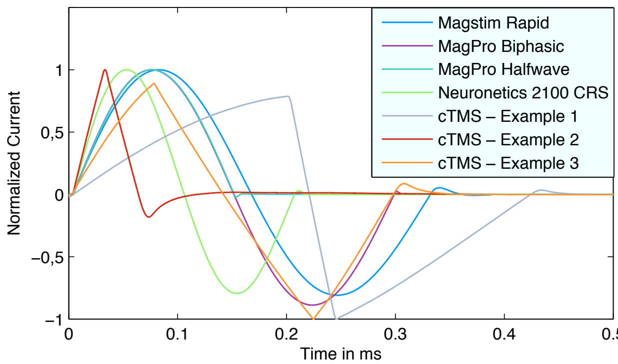

Fig. 1. Representative current waveforms of several magnetic stimulation devices.

recorded with a search coil and integrated, which yields the coil current (see Figure 1).

Magnetic stimulation pulses can be rated and optimized with respect to several performance aspects. The key objective here is the energy loss of a pulse shape at the threshold for eliciting an action potential. Independent from the technological implementation, the unavoidable minimum loss is dominated by ohmic loss due to the high coil current $i(t)$. This leads to the objective $\int_{\mathbb{R}^+} i^2(t)\,\mathrm{d}t$ for the optimization and comparison of pulses at their individual excitation threshold for the nonlinear neuron model. Therefore, we optimize the current waveform.

The energy loss objective, however, is not sufficient for optimization because it does not uniquely define minima. It is known that within the feasible voltage and frequency range the loss can be reduced by using shorter pulses, although the required peak voltage increases. From a technological perspective, the required pulse voltage is an important design constraint. It is limited by the voltage rating of the components and the insulation. Furthermore, high pulse voltages in the coil are also a safety issue. In the optimization algorithm, we therefore constrain the peak voltage for both positive and negative polarity.

Whereas the forward analysis is fully defined with the given elements, for coupling the neuron model to an optimization algorithm, the problem has to be mapped into a limited parameter space that can be handled by a numerical method.

B. *Parametrization of the Waveform Coordinate System*

In the most advanced work on optimization of electric stimulation [20], the waveform is specified by all its sample points, which were passed to an optimization method as parameters. In the context of magnetic stimulation, electromagnetic induction increases the complexity of the problem notably by introducing a differential relationship between the coil current and the induced electrical field. Feeding all sample points of the waveform as degrees of freedom into an optimization algorithm risks high instabilities and, at present, infeasible computing times. Therefore, a problem-specific framework for the magnetic stimulation case was set up.

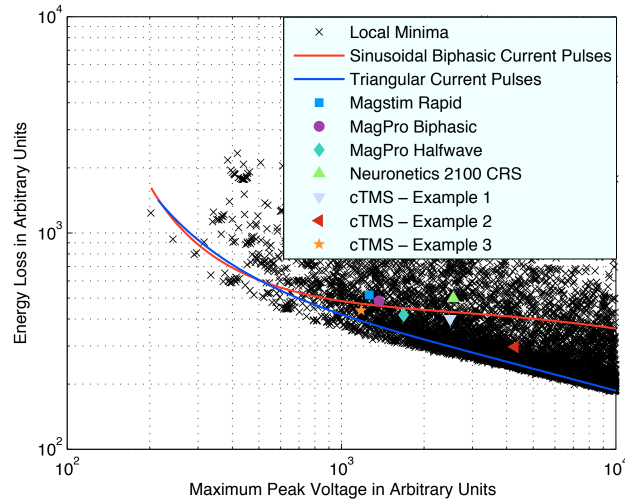

Fig. 2. Energy loss versus peak coil voltage for various current waveforms.

In essence, the question is a variational problem: For a nonlinear differential equation, a continuous solution is to be derived such that an objective, formed by the energy loss, is minimized and all constraints are fulfilled, i.e., an action potential is elicited and the peak coil voltage is within the specified limit. The waveform is defined by an ansatz function in such a way that all representatives are mathematically dense in the full waveform space and accordingly do not bias the solution. The ansatz function here incorporates in parallel spline polynomials and Fourier series. Their parameters, in turn, are the degrees of freedom for the optimization algorithm. The representations and the number of degrees of freedom can be changed dynamically. The underlying mathematical approach is well known in numerics. A certain type of it, for instance, outlines the basis in the finite element method (FEM) [35].

The ansatz function describes the coil current $i(t)$. The temporal shape of the induced electric field is evaluated by differentiating the current. For the optimization, a hybrid global method—a particle swarm method [36] with a local hill-climbing algorithm [37], [38]—was used. Up to 1000 degrees of freedom were used for refining the solutions.

III. Results

Figure 2 shows the energy loss as a function of the maximum device voltage at the action-potential threshold. The graph includes all minima from the exploration of the waveform space. Each point corresponds to a certain waveform. In addition, data for the standard symmetric biphasic sinusoidal pulse (red line) and symmetric monophasic triangular current pulses (blue line, corresponding to symmetric biphasic rectangular voltage pulses) were added. The recorded waveforms of existing devices are represented by single colored symbols. The voltage on the x-axis is in relative units and not calibrated to a specific threshold, but it lies in the range of real devices. Calibration of the x-axis can be easily accomplished based on available threshold data for conventional pulses.

Figure 2 demonstrates that for conventional sinusoidal waveforms the losses can be reduced if shorter pulses are used, which leads to higher coil voltages. This confirms observations from the literature [26], [39]. Triangular current waveforms as well as all pulse shapes that form the lower edge of the scatter of local minima have a steeper slope of loss versus voltage than sinusoidal pulses, indicating that the gain in efficiency at higher voltages is larger for the former than for the latter.

This lower edge of the solution space in Figure 2 is smooth and appears to represent the same class of stimuli. At a relative voltage level of about 1500, which is representative for conventional devices, the loss of the symmetric triangular pulses lies approximately at the midpoint between the solutions space's lower edge and the sinusoidal biphasic pulses. The model predicts that at this voltage level the optimal pulses at the lower edge of the solution space have almost half the loss of the sinusoidal biphasic pulses.

All sinusoidal waveforms of existing devices are within a 15 % band around the symmetric sinusoidal line. The deviation especially of the biphasic pulses can be explained by different damping that reduces the symmetry and decreases the induced peak electric field strength.

The example cTMS waveforms are below the line of the biphasic sinusoidal pulses. The corresponding device can generate nearly triangular current waveforms (with almost rectangular voltage pulses) and gives control over the rising and falling slopes (see Fig. 1). Example cTMS waveforms 2 and 3 are relatively close to the line of symmetric triangular current pulses in Fig. 2. The shorter cTMS pulse 2 is the most efficient representative within this group, but requires a higher voltage of the pulse source than the other two cTMS pulses as shown in Fig. 2. cTMS pulse 1 has notably shorter falling than rising edges which leads to a voltage shape with dominant amplitude in one direction. However, this pulse has relatively long high-amplitude current phases that increase the loss. In addition, due to its voltage unidirectionality, this waveform does not use the full available voltage range given by the constraint. Accordingly, the asymmetry in the rising and falling slopes is responsible for the lower performance in the voltage-loss space.

An optimization at different voltages in the range of conventional devices reveals the exact shapes of waveforms from the lower edge of the solution space; the results are shown in Fig. 3. The optimized current shape seems to be an assembly of three parts or phases: a slow first phase, a triangular main pulse, and a decay to zero. The initial part of the pulse constitutes a slow current slope which is hardly distinguishable from the baseline in the coil-voltage plot. The voltage shape of the second phase is almost rectangular.

The voltage is mainly shaped by the limits given by the constraints. The optimizer uses the maximum allowed voltage level for the total duration of the second phase which leads to the positive rectangular voltage swing. The subsequent negative swing reduces the current as quickly as possible after the peak to prevent high ohmic loss.

The initial slow negative current phase can be explained in a similar way. It biases the onset of the second, depolarizing phase with minimum effect on the neural dynamics. Due to this shift of the starting point, the second phase can sustain a longer rising slope and still avoid very high current as well as the associated high loss levels. The initial phase contributes to the energy loss, but the amplitude is relatively low. Since heating depends on the squared current, the initial phase saves more energy due the reduced current in the second phase than the additional loss it contributes itself.

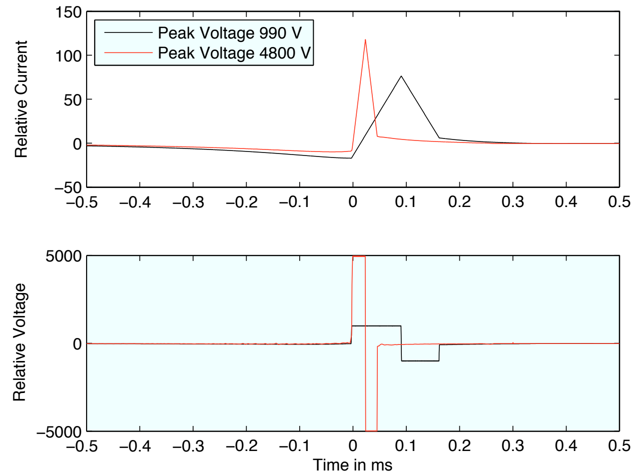

Fig. 3. Examples of magnetic stimulation with minimum power loss

IV. Conclusions

We implemented numerical optimization of the pulse shape for inductive stimulation of a motor axon with the objective of minimizing energy loss. The loss was quantified by the integral of the squared coil current. The optimization constraints were reduced to a minimum for a practically unconstrained search of the pulse shape space.

The results were very consistent across various initial conditions. The structure of these pulses is intuitively reasonable from the perspective of reducing the energy loss. Still, exploration and optimization were performed in a model system. Although the used nonlinear model is very sophisticated and may be a relatively good approximation of neuron dynamics, it cannot claim absolute truth. Therefore, as with any model-based approach, the results should be considered only as a starting point for experimental studies.

Improving the energy efficiency is critical to functional and portable magnetic stimulation systems. In other applications of magnetic stimulation, however, optimizing the pulse shape to effect best selectivity of stimulation of specific neural populations may be of particular importance. Provided appropriately formulated objectives, this optimization framework, which allows nearly unconstrained waveform representation and optimization, is well suited for such alternative optimization goals.

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